GATEFLIX

CONTROL SYSTEMS

For

ELECTRICAL ENGINEERING

INSTRUMENTATION ENGINEERING

ELECTRONICS & COMMUNICATION ENGINEERING

CONTROL SYSTEMS

SYLLABUS

ELECTRONICS & COMMUNICATION ENGINEERING

Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

ELECTRICAL ENGINEERING

Principles of feedback; transfer function; block diagrams; steady-state errors; Routh and Nyquist techniques; Bode plots; root loci; lag, lead and lead-lag compensation; state space model; state transition matrix, controllability and observability.

INSTRUMENTATION ENGINEERING

Feedback principles. Signal flow graphs. Transient Response, steady-state-errors. Routh and Nyquist criteria. Bode plot, root loci. Time delay systems. Phase and gain margin. State space representation of systems. Mechanical, hydraulic and pneumatic system components. Synchro pair, servo and step motors. On-off, cascade, P, P-I, P-I-D, feed forward and derivative controller, Fuzzy controllers.

ANALYSIS OF GATE PAPERS

	ELECTRONICS			ELECTRICAL			INSTRUMENTATION		
Exam Year	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total
2003	5	8	21	3	7	17	4	10	24
2004	4	7	18	2	9	20	4	8	20
2005	3	10	23	3	6	15	-	10	20
2006	2	7	16	1	3	7	1	8	17
2007	2	8	18	1	7	15	2	6	14
2008	1	8	17	1	8	17	1	9	19
2009	-	6	12	4	5	14	1	6	13
2010	3	3	9	2	3	8	-	4	8
2011	2	4	10	3	3	9	1	6	13
2012	2	3	8	1	4	9	2	6	14
2013	2	6	14	2	4	10	1	4	9
2014 Set-1	5	1	7	2	2	6	2	4	10
2014 Set-2	3	3	9	2	3	8	-	-	-
2014 Set-3	3	2	7	2	3	8	-	-	-
2014 Set-4	3	2	7	-	-	-	-	-	-
2015 Set-1	2	5	12	2	4	10	1	3	7
2015 Set-2	3	4	11	2	3	8	-	-	-
2015 Set-3	3	4	11	-	-	-	-	-	-
2016 Set-1	3	2	7	1	4	9	3	3	9
2016 Set-2	2	4	10	1	4	9	-	-	-
2016 Set-3	3	3	9	-	-	-	-	-	-
2017 Set-1	5	3	11	3	4	11	2	4	10
2017 Set-2	3	4	9	4	3	10	-	-	-
2018	2	3	8	4	2	8	2	3	8

CONTENTS

1 BLOCK DIAGRAMS

1.1 OPEN LOOP SYSTEM

The open loop control system is a non-feedback system in which the control input to the system is determined using only the current state of the system and a model of the system. Control characteristic of such systems are independent of o/p of the system.

$$\frac{C}{R} = G$$

Examples:
1) Automatic coffee server
2) Bottling m/c of cold drink
3) Traffic Signal
4) Electric lift,
5) Automatic washing m/c

1.1.1 ADVANTAGE

1) No stability problem.
2) The open-loop system is simple to construct and is cheap.
3) Open loop systems are generally stable

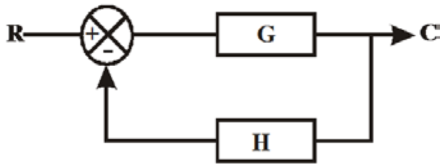
1.1.2 DISADVANTAGE

1) The open loop system is inaccurate & unreliable
2) The effect of parameter variation and external noise is more.
3) The operation of open-loop system is affected due to the presence of non-linearity in its elements.

Note:
- An open loop stable system may become unstable when negative feedback is applied.
- Except oscillators, in positive feedback, we have always unstable systems.

1.2 CLOSED LOOP SYSTEM

The closed loop control system is a system where the actual behavior of the system is sensed and then fed back to the controller and mixed with the reference or desired state of the system to adjust the system to its desired state. Control characteristic of the system depends upon the o/p of the system.



$$\frac{C}{R} = \frac{G}{1+GH}$$ (Negative feedback)

Examples:
1) Electric iron
2) DC motor speed control
3) A missile launching system (direction of missile
4) Changes with the location of target)
5) Radar Tracking system
6) Human Respiratory system
7) A man driving a vehicle (eye-sensor, Brain-controller)
8) Auto pilot system
9) Economic inflation

1.2.1 ADVANTAGES

1) As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.
2) Reduced effect of parameter variation
3) BW of system can be increased
4) Reduced effect of non-linearity

1.2.2 DISADVANTAGES

1) The system is complex and costly
2) Gain of system reduces with negative feedback
3) The closed loop systems can become unstable under certain conditions.

1.3 LAPLACE TRANSFORM

The ability to obtain linear approximations of physical systems allows the analyst to consider the use of the Laplace transformation. The Laplace transform methods substitutes relatively easily solved algebraic equations for the more difficult differential equations.
The Laplace transform exists for linear differential equations for which the transformation integral converges. Therefore, for f (t) to be transformable, it is sufficient that

$$\int_0^{\infty} |f(t)| e^{-st} dt < \infty$$

Signals that are physically realizable always have a Laplace transform. The Laplace transformation for a function of time, f (t), is

$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

1.3.1 IMPORTANT RESULTS

F(t) = F(s)

1) $\delta(t) = 1$

2) $u(t) = \frac{1}{s}$

3) $u(t-T) = \frac{1}{s} e^{-sT}$

4) $t\, u(t) = \frac{1}{s^2}$

5) $\frac{t^2}{2} u(t) = \frac{1}{s^3}$

6) $t^n u(t) = \frac{n!}{s^{n+1}}$

7) $e^{at} u(t) = \frac{1}{s-a}$

8) $e^{-at} u(t) = \frac{1}{s+a}$

9) $te^{at} u(t) = \frac{1}{(s-a)^2}$

10) $te^{-at} u(t) = \frac{1}{(s+a)^2}$

11) $t^n e^{-at} u(t) = \frac{n!}{(s+a)^{n+1}}$

12) $\sin\omega t\, u(t) = \frac{\omega}{s^2+\omega^2}$

13) $\cos\omega t u(t) = \frac{s}{s^2+\omega^2}$

14) $e^{-at}\sin\omega t\, u(t) = \frac{\omega}{(s+a)^2+\omega^2}$

15) $e^{-at}\cos\omega t\, u(t) = \frac{s+a}{(s+a)^2+\omega^2}$

16) $\sinh \omega t\, u(t) = \frac{\omega}{s^2-\omega^2}$

17) $\cosh \omega t\, u(t) = \frac{s}{s^2-\omega^2}$

1.3.2 PROPERTIES OF LAPLACE TRANSFORM

1) If the Laplace transform of f (t) is F(s), then
   - $L\left(\frac{df(t)}{dt}\right) = sF(s) - f(0^+)$
   - $L\left(\frac{d^2f(t)}{dt^2}\right) = s^2F(s) - sf(0^+) - f'(0^+)$
2) Initial value theorem
   - $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$
3) Final value theorem
   - $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

   The final value theorem gives the final value ($t\to\infty$) of a time functions using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of sF(s) has any root having real part as Zero or positive, then the final value theorem is not valid.

1.4 TRANSFER FUNCTION

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

In control system the output (or response) is related to the input by a transfer function as defined earlier, i.e. $\frac{C(s)}{R(s)} = G(s)$ or

$C(s) = R(s)G(s)$

The output time response can be determined by taking inverse Laplace transform of relation of C(s).

Note:

- The open loop poles at origin in the transfer function determine the type of the system.

  e.g. If $G(s) = \frac{1}{s^3(s+2)(s+3)}$

  the number of open loop poles at origin is 3 hence the type of system is 3.
- The highest power of s in the denominator of the transfer function determines the order of the system.

  e.g. If $G(s) = \frac{1}{s^3(s+2)(s+3)}$

  the power of s in the denominator is 5. Hence, the order of the system is 5.
- **Impulse response of the system** is the system output when input is impulse. If the input is specified as unit impulse at t = 0, then R(s) = 1 and the transformed expression for the system output, is,

  $C(s) = G(s)$

  Thus the output time response is,

  $L^{-1}C(s) = L^{-1}G(s)$

  or $c(t) = g(t)$

  The inverse Laplace transform of G(s) is, therefore, called the impulse response of a system or the transfer function of a system is the Laplace transform of its impulse response.
- **Step response of the system** is the system output when input is unit step signal. The impulse response of the system can obtained differentiating the step response.

  e.g. If step response is $c_s(t) = e^{-2t}$ then the impulse response will be $c_i(t) = -2e^{-2t}$

1.4.1 PROPERTIES OF TRANSFER FUNCTION

1) The transfer function is defined only for a linear time-invariant system. It is not defined for nonlinear systems.
2) The transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response. Alternatively, the transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input.
3) All initial conditions of the system are set to zero.
4) The transfer function of a continuous – data system is expressed only as a function of the complex variables. It is not a function of the real variable, time, or any other variable that is used as the independent variable. For discrete – data systems modeled by difference equations, the transfer function is a function of z when the z-transform is used.

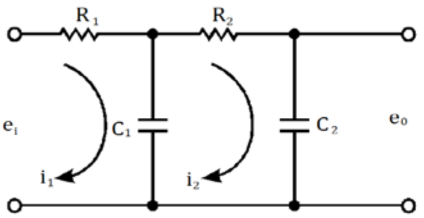
1.4.2 PROCEDURE TO DETERMINE TRANSFER FUNCTION

1) Formulate the equations for the system.
2) Take the Laplace transform of the system equations assuming initial conditions as zero.
3) Specify the system output and the input.
4) Take the ratio of the Laplace transform of the output and the Laplace transform

of the input which is nothing but the transfer function of system.

1.4.3 TRANSFER FUNCTIONS OF CASCADED ELEMENTS

Many feedback systems have components that load each other. Consider the system shown in Figure. Assume that e_i is the input and e_0 is the output. The equations for this system are



$$\frac{1}{C_1}\int\left(i_1 - i_2\right)dt + R_1 i_1 = e_i$$

$$\frac{1}{C_1}\int\left(i_2 - i_1\right)dt + R_2 i_2 + \frac{1}{C_1}\int i_2 dt = 0$$

$$\frac{1}{C_1}\int i_2 dt = e_0$$

Taking the Laplace transforms of Equations

$$\frac{1}{C_1 s}\left\{I_1(s) - I_2(s)\right\} + R_1 I_1(s) = E_i(s)$$

$$\frac{1}{C_1 s}\left\{I_2(s) - I_1(s)\right\} + R_2 I_2(s) + \frac{1}{C_1 S} I_2(s) = 0$$

$$\frac{1}{C_2 S} I_2(s) = E_0(s)$$

Eliminating $I_1(s)$ from above Equations and writing $E_i(s)$ in terms of $I_2(s)$, we find the transfer function between $E_0(s)$ and $E_i(s)$ to be

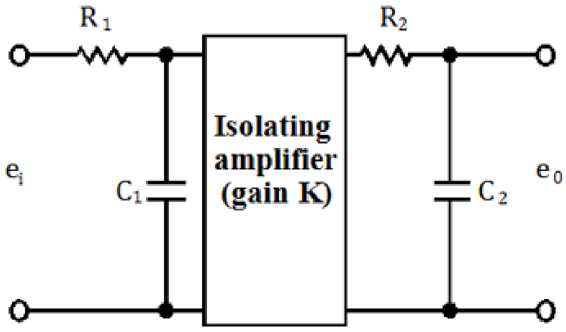
$$\frac{E_0(s)}{E_i(s)} = \frac{1}{\left(R_1C_1s+1\right)\left(R_2C_2s+1\right)+R_1C_2s}$$

$$= \frac{1}{\left(R_1C_1R_2C_2s^2\right)+\left(R_1C_1+R_2C_2+R_1C_2\right)s+1}$$

The overall transfer function is not the product of $1/\left(R_1C_1s+1\right)$ and $1/\left(R_2C_2s+1\right)$. The reason for this is that, when we derive the transfer function for an isolated circuit, we implicitly assume that the output is unloaded. In other words, the load impedance is assumed to be infinite, which means that no power is being withdrawn at the output.

1.4.4 TRANSFER FUNCTIONS OF NONLOADING CASCADED ELEMENTS

The two simple RC circuits, isolated by an amplifier as shown in Figure, have negligible loading effects, and the transfer function of the entire circuit equals the product of the individual transfer functions. Thus, in this case,

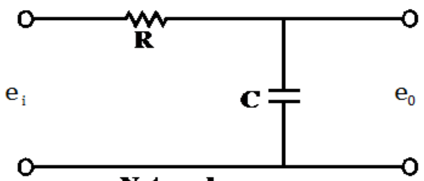


$$\frac{E_0(s)}{E_i(s)} = \frac{1}{\left(R_1C_1s+1\right)}(K)$$

$$\frac{1}{\left(R_2C_2s+1\right)}\frac{E_0(s)}{E_i(s)} = \frac{K}{\left(R_1C_1s+1\right)\left(R_2C_2s+1\right)}$$

Example:

Find the transfer function of the network given below



Network

Solution:

Transfer function of the network is

$$\frac{E_0(s)}{E_1(s)} = \frac{1}{RCs+1}$$

1.5 BLOCK DIAGRAMS

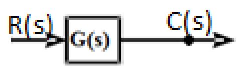
A block diagram of a system is a pictorial representation of the functions performed by each component and the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. In a block diagram all system variables are linked to each other through functional blocks.

The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals.

Note:

- In block diagrams the comparison of signals is indicated by **summing points.**
- The point from where signal is taken for feedback is called **take-off point**.
- The signal can travel only along the direction of the arrow.

1.5.1 BLOCK DIAGRAM FOR OPEN LOOP SYSTEM



Open-loop transfer function =

$$\frac{C(s)}{R(s)} = G(s)$$

1.5.2 BLOCK DIAGRAM FOR CLOSED LOOP SYSTEM

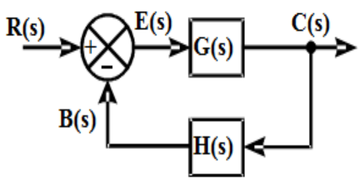


Figure: Closed-loop system

Feed forward transfer function

$$= \frac{C(s)}{E(s)} = G(s)$$

For the system shown in above figure, the output C(s) and input R(s) are related as follows:

Since C(s) = G(s) E(s)

E(s) = R(s) – B(s)

= R(s) – H(s) C(s)

Eliminating E(s) from these equations gives

C(s) = G(s)[R(s) – H(s)C(s)]

Or $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

The transfer function relating C(s) to R(s) is called the closed-loop transfer function. This transfer function relates the closed-loop system dynamics to the dynamics of the feed forward elements and feedback elements.

From Equation C(s) is given by

$$C(s) = \frac{G(s)}{1+G(s)H(s)} R(s)$$

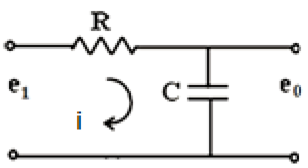
Thus the output of the closed-loop system clearly depends on both the closed-loop transfer function and the nature of the input.

1.5.3 BLOCK DIAGRAM REDUCTION TECHNIQUES

To draw a block diagram for a system, first write the equations that describe the dynamic behavior of each component. Then take the Laplace transforms of these equations, assuming zero initial conditions, and represent each Laplace- transformed equation individually in block form. Finally, assemble the elements into a complete block diagram.

Example

Draw the block diagram for the RC circuit shown in the figure.



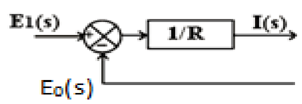
Solution

The equation for current in the circuit is

$$i = \frac{e_1 - e_0}{R} \quad \text{... (I)}$$

Its Laplace transform will be

$$I(s) = \frac{E_i(s) - E_0(s)}{R}$$



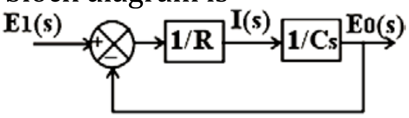
Now the equation for output voltage is

$$e_0 = \frac{\int idt}{C} \quad \text{... (II)}$$

Its Laplace transform will be

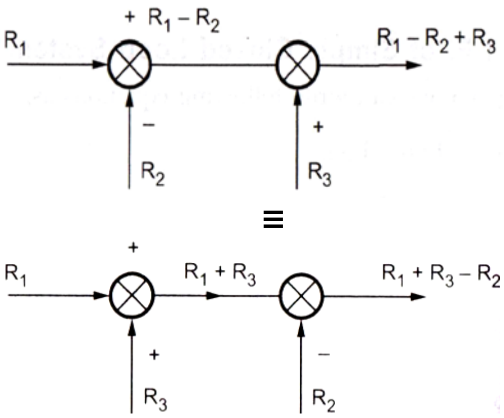
$$E_0(s) = \frac{I(s)}{Cs}$$

Now, assembling these two elements the overall block diagram is

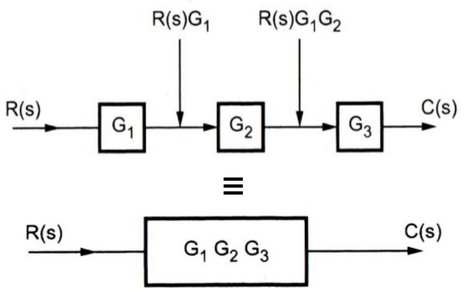


1.5.4 BLOCK DIAGRAM TRANSFORMATIONS

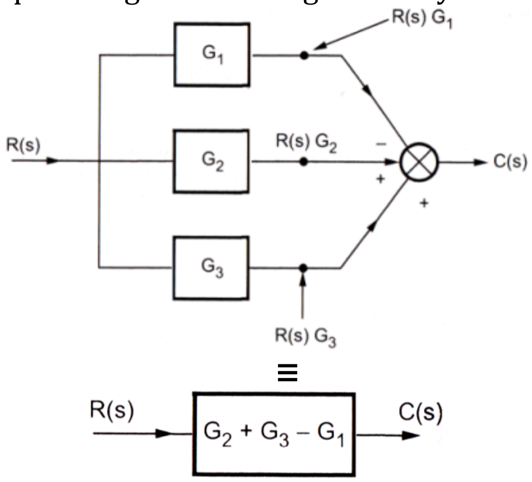
1) Interchanging the summing points: If there is no block or take-off point between 2 summing points, the summing points can be interchanged without affecting output.



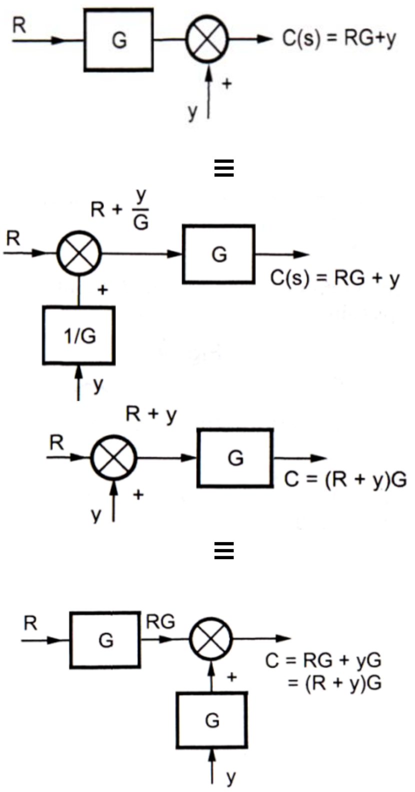
2) Merging the blocks in series: If there is no take-off or summing points in between blocks they can be merged into single block. The transfer function of resultant block will be product of individual transfer functions.



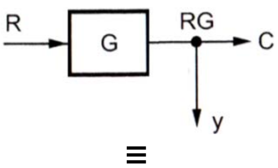
3) Merging the blocks in parallel: The transfer function of the blocks in parallel gets added algebraically.

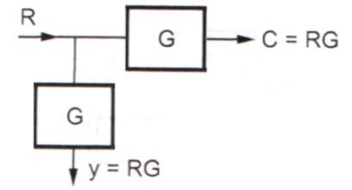


4) Shifting the summing points: The summing point can be shifted before the block or after the block using some additional blocks.

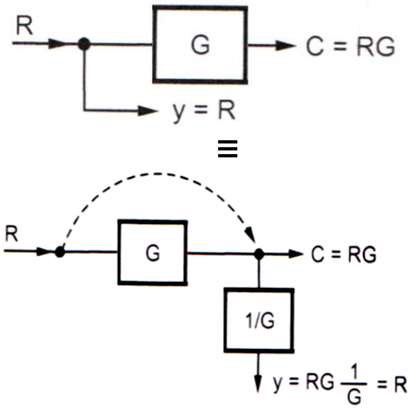


6) Shifting the take-off point before the block:

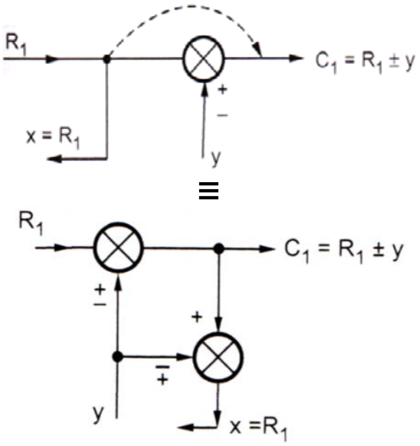




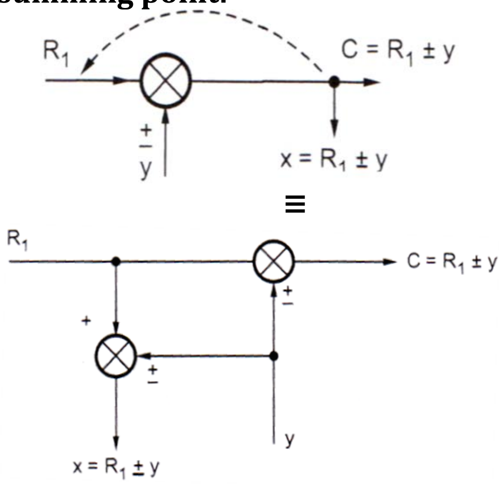
7) Shifting the take-off point after the block:



8) Shifting take-off point after summing point:

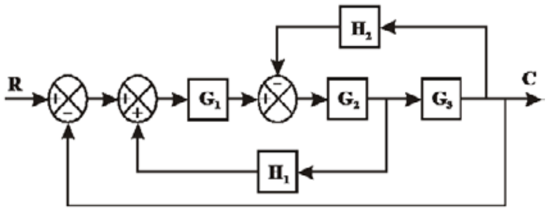


9) Shifting take-off point before summing point:



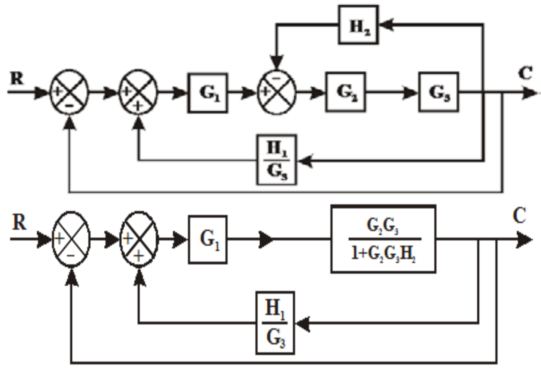
Example

Consider the system shown in Figure. Simplify this diagram.

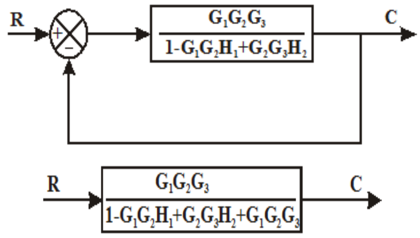


Solution:

By moving the summing point of the negative loop containing H2 outside the positive feedback loop containing H1, we obtain Figure.

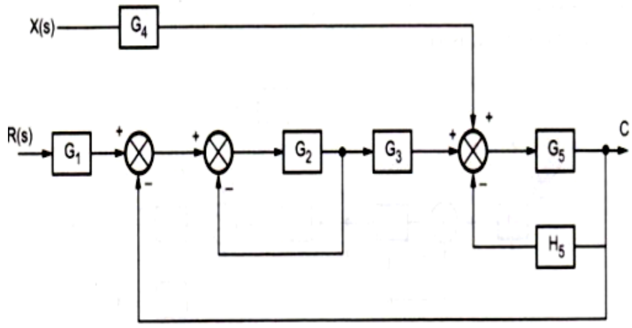


Eliminating the positive feedback loop



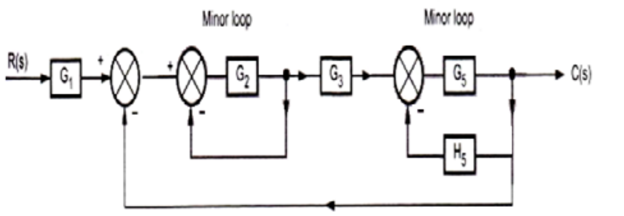
Example:

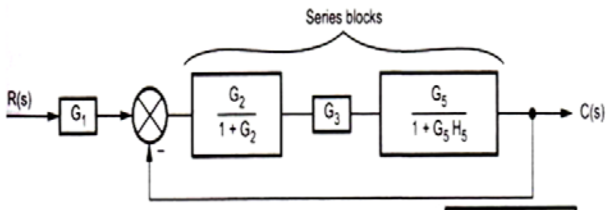
Find the transfer function from each input to the output C.

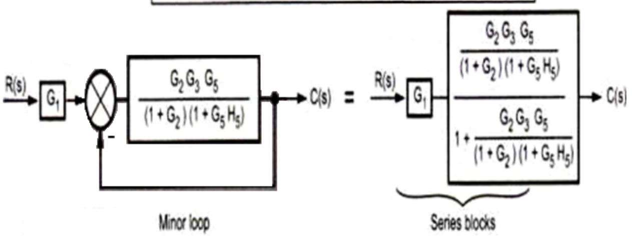


Solution:

Considering X(s)=0, the block diagram reduces to

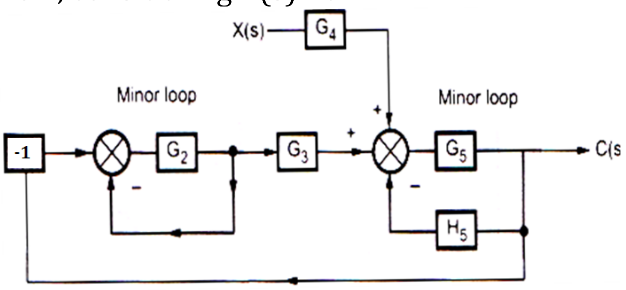


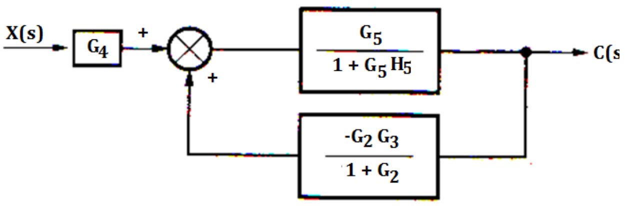




$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5}{1+G_2+G_5H_5+G_2G_5H_5+G_2G_3G_5}$$

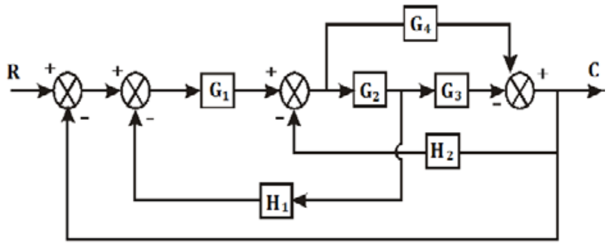
Now, considering R(s) =0



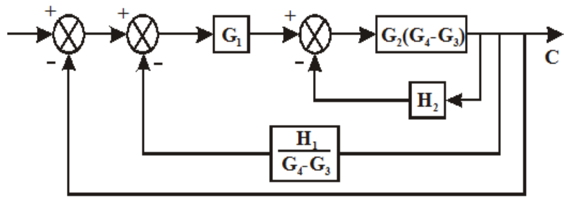


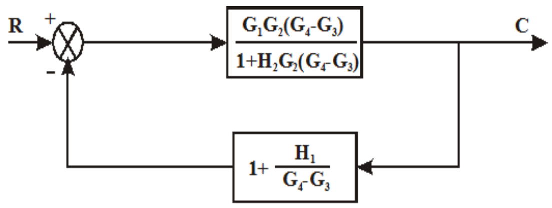
$$\therefore \frac{C(s)}{X(s)} = \frac{G_4G_5(1+G_2)}{1+G_2+G_5H_5+G_2G_5H_5+G_2G_3G_5}$$

Example:
Find the transfer function for the system whose block diagram representation is shown in Fig.



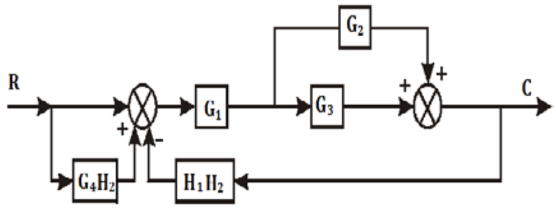
Solution:



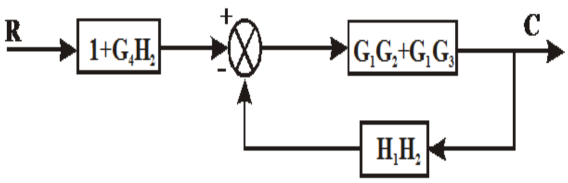


$$\frac{C}{R} = \frac{G_1G_2(G_4-G_3)}{1+H_5G_2(G_4-G_3)+G_1G_2H_1+G_1G_2(G_4-G_3)}$$

Example:
Find closed loop transfer function of system shown in Fig.



Solution:



$$\frac{C}{R} = \frac{G_1(G_2+G_3)(1+G_4H_2)}{1+G_1H_1H_2(G_2+G_3)}$$

1.6 SIGNAL FLOW GRAPHS

A SFG may be defined as a graphical means of portraying the input – output relationships between the variables of a set of linear algebraic equations. A linear system is described by a set of N algebraic equations:

$$y_i = \sum_{k=1}^{N} a_{kj} y_k$$

j=1, 2,………, N

These N equations are written in the form of cause – and – effect relations:

$$j^{th}\text{effect} = \sum_{k=1}^{N} (\text{gain from k to j}) \times (k^{th}\text{cause}) \text{ or}$$

simply

$$\text{output} = \sum \text{gain} \times \text{input}$$

Laplace transform equation

$$y_i(s) = \sum_{k=1}^{N} G_{kj}(s) Y_k(s) \qquad j=1,2,\ldots\ldots,N$$

Note:

- In a SFG signals can transmit through a branch only in the direction of the arrow.
- When constructing a SFG, junction points, or nodes, are used to represent variables.
- The modes are connected by line segments called branches, according to the cause – and – effect equations. The branches have associated branch gains and directions

1.6.1 DEFINITIONS OF SFG TERMS

1) **Input Node (Source):** An input node is a node that has only outgoing branches.
2) **Output Node (Sink):** An output node is node that has only incoming branches.
3) **Forward Path:** A forward path is a path that starts at an input node and ends at an output node, and along which no node is traversed more than once.
4) **Loop:** A loop is a path that originates and terminates on the same node and along which no other node is encountered more than once.
5) **Path Gain:** The product of the branch gains encountered in traversing a path is called the path gain.
6) **Forward-Path Gain:** The forward-path gain is the path gain of a forward path.
7) **Non touching Loops:** Two parts of a SFG are non touching if they do not share a common node.

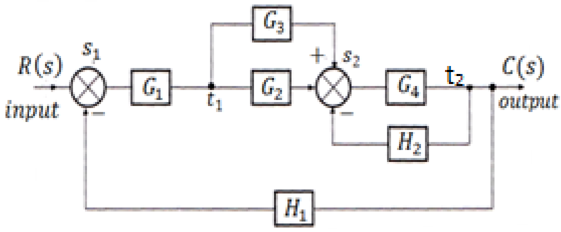
1.6.2 SFG FROM BLOCK DIAGRAM

An SFG can be drawn from given block diagram of a system following the steps:

1) Name all the summing points & take-off points.
2) Each summing point & take-off will point will be a node of SFG.
3) Connect all the nodes with branches instead of blocks & indicate the block transfer functions as the gains of the branches.

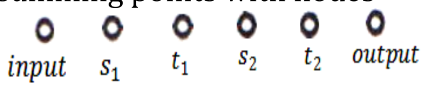
Example

Draw signal flow graph from the given block diagram.

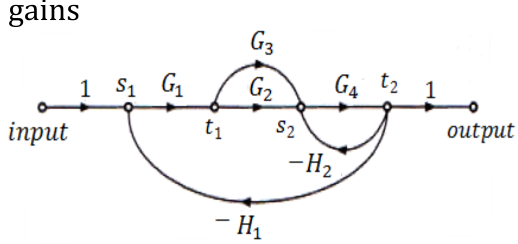


Solution

1) Represent the take off points & summing points with nodes



2) Now connect the nodes with the transfer functions of the blocks as the gains



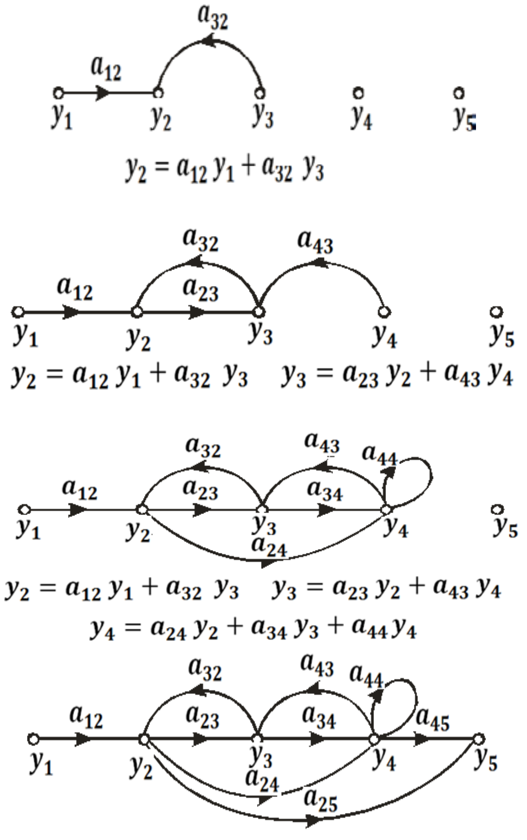
Example:

As an exp. on the construction of SFG, consider the following set of algebraic equations:

$$y_2 = a_{12}y_1 + a_{32}y_3$$

$$y_3 = a_{23}y_2 + a_{43}y_4$$
$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$
$$y_5 = a_{25}y_2 + a_{45}y_4$$

Solution:



Complete Graph

1.6.3 MASON'S GAIN FORMULA FOR SIGNAL FLOW GRAPHS

In many practical cases, we wish to determine the relationship between an input variable and an output variable of the signal flow graph. The transmittance between an input node and an output node is the overall gain, or overall transmittance, between these two nodes.

Mason's gain formula, which is applicable to the overall gain, is given by

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where, P_k = path gain or transmittance of k_{th} forward path

Δ = determinant of graph

= 1 – (sum of all individual loop gains) + (sum of gain products of all possible combinations of two non-touching loops) – (sum of gain products of all possible combinations of three non touching loops) +...

$$= 1 - \sum_a L_a + \sum_{b,c} L_b L_c - \sum_{d,e,f} L_d L_e L_f + \ldots.$$

$\sum_a L_a$ = sum of all individual loop gains

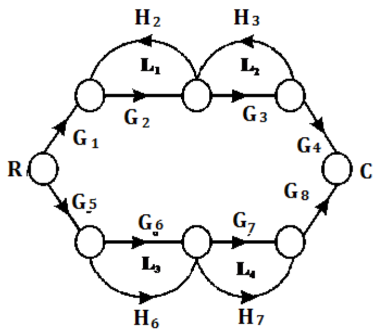
$\sum_{b,c} L_b L_C$ = sum of gain products of all possible combinations of two non touching loops.

$\sum_{d,e,f} L_d L_e L_f$ = sum of gain products of all possible combinations of three non touching loops.

Δ_k = Non –touching determinant to k^{th} forward path cofactor of the k^{th} forward path determinant of the graph with the loops touching the k^{th} forward path removed, that is, the cofactor Δ_k is obtained from Δ by removing the loops that touch path P_k

1.6.4 TRANSFER FUNCTION OF INTERACTING SYSTEM

A two-path signal-flow graph is shown in Fig. An example of a control system with multiple signal paths is a multi legged robot.



Two-path interacting system

The paths connecting the input R(s) and output Y(s) are

Path 1: $P_1 = G_1G_2G_3G_4$ and

Path 2: $P_2 = G_5G_6G_7G_8$.

There are four self-loops:

$L_1 = G_2H_2$, $L_2 = H_3G_3$,

$L_3 = G_6H_6$, $L_4 = G_7H_7$.

Loops L_1 and L_2 do not touch L_3 and L_4. Therefore the determinant is

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$.

The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from Δ. Therefore we have

$L_1 = L_2 = 0$ and $\Delta_1 = 1 - (L_3 + L_4)$

Similarly, the cofactor for path 2 is

$\Delta_2 = 1 - (L_1 + L_2)$.

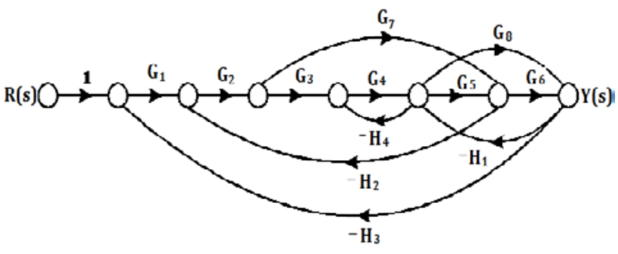
Therefore the transfer function of the system is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{G_1G_2G_3G_4(1-L_3-L_4)+G_5G_6G_7G_8(1-L_1-L_2)}{1-L_1-L_2-L_3-L_4+L_1L_3+L_1L_4+L_2L_3+L_2L_4}$$

1.6.5 TRANSFER FUNCTION OF COMPLEX SYSTEM

Finally, we shall consider a reasonably complex system that would be difficult to reduce by block diagram techniques. A system with several feedback loops and feed forward paths is shown in Fig.



Multiple loop system

The forward paths are

$P_1 = G_1G_2G_3G_4G_5G_6$,

$P_2 = G_1G_2G_7G_6$,

$P_3 = G_1G_2G_3G_4G_8$,

The feedback loops are

$L_1 = -G_2G_3G_4G_5H_2$,

$L_2 = -G_5G_6H_1$,

$L_3 = -G_8H_1$,

$L_4 = -G_7H_2G_2$,

$L_5 = -G_4H_4$,

$L_6 = -G_1G_2G_3G_4G_5G_6H_3$,

$L_7 = -G_1G_2G_7G_6H_3$,

$L_8 = -G_1G_2G_3G_4G_8H_3$,

Loop L_5 does not touch loop L_4 or loop L_7; loop L_3 does not touch loop L_4; and all other loops touch. Therefore the determinant is

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$

The cofactors are

$\Delta_1 = \Delta_3 = 1$ and $\Delta_2 = 1 - L_5 = 1 + G_4H_4$.

Finally, the transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$

Signal-flow graphs and Mason's signal-flow gain formula may be used profitably for the analysis of feedback control systems, electronic amplifier circuits, statistical systems, and mechanical systems, among many other examples.

1.6.6 OUTPUT NODES AND NON INPUT NODES

The gain formula can be applied only between a pair of input and output nodes. Often, it is of interest to find the relation between an output-node variable and a non input-node variable. For example, to find the relation y_7/y_2, which represents the dependence of y_7 on y_2; the latter is not an input.

Let y_{in} be an input and y_{out} be an output node of a SFG. The gain y_{out}/y_2, where y_2 is not an input, may be written as

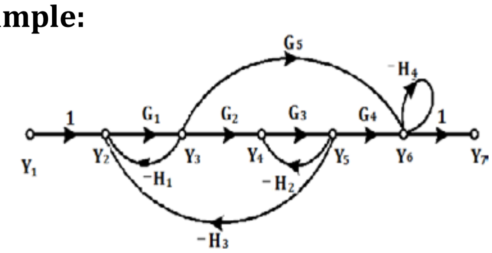
$$\frac{y_{out}}{y_2} = \frac{y_{out}/y_{in}}{y_2/y_{in}} = \frac{\dfrac{\sum P_k\Delta_k \,|_{\text{from } y_{in} \text{ to } y_{out}}}{\Delta}}{\dfrac{\sum P_k\Delta_k \,|_{\text{from } y_{in} \text{ to } y_2}}{\Delta}}$$

Since Δ is independent of the inputs and the outputs,

$$\frac{y_{out}}{y_2} = \frac{\sum P_k\Delta_k \,|_{\text{from } y_{in} \text{ to } y_{out}}}{\sum P_k\Delta_k \,|_{\text{from } y_{in} \text{ to } y_2}}$$

Notice that Δ does not appear in the last equation

Example:



From the SFG in Fig. The gain between y_2 and y_7 is written

$$\frac{y_7}{y_2}=\frac{y_7/y_1}{y_2/y_1}=\frac{G_1G_2G_3G_4+G_1G_5\left(1+G_3H_2\right)}{1.\left[1+G_3H_2+H_4+G_3H_2H_4\right]}P_{12}=1$$

Example:

Consider the SFG in Fig. above, the following input – output relations are obtained by use of the gain formula:

$$\frac{y_2}{y_1}=\frac{1+G_3H_2+H_4+G_3H_2H_4}{\Delta}$$

$$\frac{y_4}{y_1}=\frac{G_1G_2\left(1+H_4\right)}{\Delta}$$

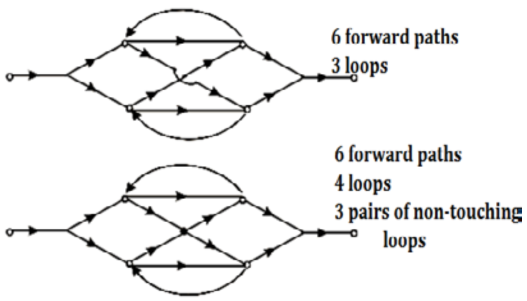
$$\frac{y_6}{y_1}=\frac{y_7}{y_1}=\frac{G_1G_2G_3G_4+G_1G_5\left(1+G_3H_2\right)}{\Delta}$$

Where

$$\Delta=1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1G_3H_1H_2$$
$$+G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4+G_1G_3H_1H_2H_4$$

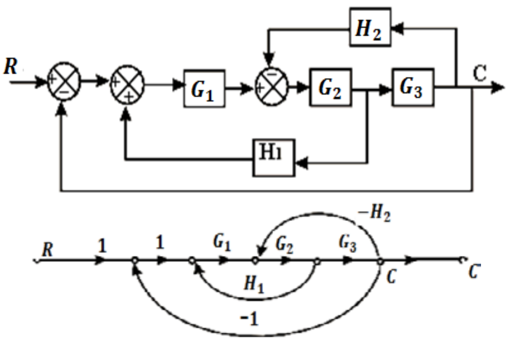
Example

Find the no. of forward paths, individual loops and non-touching pair in following SFGs



Example

Consider the system shown in Figure.



A signal flow graph for this system is shown in Figure. Let us obtain the closed loop transfer function C(s)/R(s) by use of Mason's gain formula.

In this system there is only one forward path between the input R(s) and the output C(s). The forward path gain is $P_1 = G_1G_2G_3$

From figure, we see that there are three individual loops. The gains of these loops are

$L_1 = G_1G_2H_1$

$L_2 = G_2G_3H_2$

$L_3 = G_1G_2G_3$

Note that since all three loops have a common branch, there are no non touching loops. Hence, the determinant Δ is given by

$\Delta = 1 – (L_1 + L_2 + L_3)$

$= 1 – G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$

The cofactor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained from Δ by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

$\Delta_1 = 1$

Therefore, the overall gain between the input R(s) and the output C(s), or the closed loop transfer function, is given by

$$\frac{C(s)}{R(s)}=P=\frac{P_1\Delta_1}{\Delta}$$

$$=\frac{G_1G_2G_3}{1+G_1G_2G_1+G_2G_3G_2+G_1G_2G_3}$$

Which is the same as the closed loop transfer function obtained by block diagram reduction. Mason's gain formula thus gives the overall gain C(s)/R(s) without a reduction of the graph.

Example:

Draw signal flow graphs for

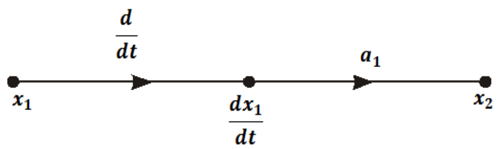
(a) $x_2=a_1\left(\frac{dx_1}{dt}\right)$

(b) $x_3=\frac{d^2x_3}{dt^2}+\frac{dx_1}{dt}-x_1$

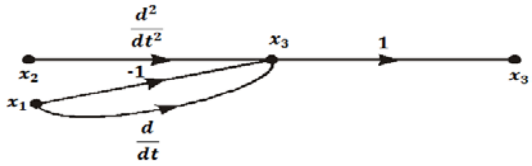
(c) $x_4=\int x_3dt$

Solution:

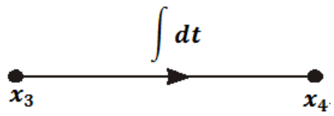
a)



b)

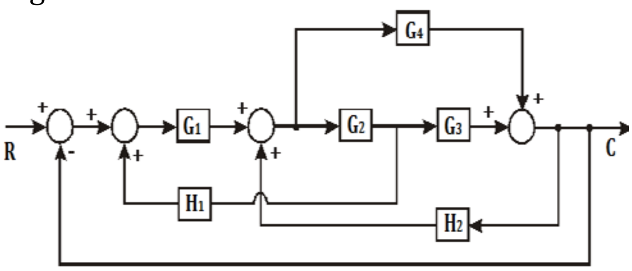


c)

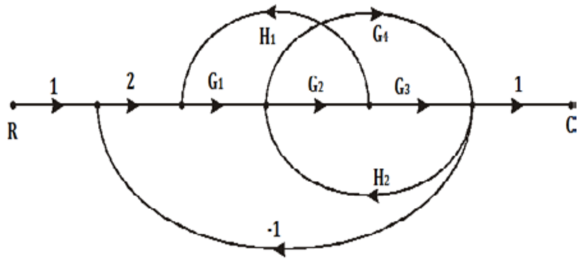


Example

Find C/R for the control system given in Fig.



Solution:



The signal flow graph is given in Fig.
The two forward path gains are
$P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$.
The five feedback loop gains are
$P_{11} = G_1G_2H_1$,
$P_{21} = G_2G_3H_2$,
$P_{31} = -G_1G_2G_3$,
$P_{41} = G_4H_2$,
and $P_{51} = -G_1G_4$.
Hence

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$

$$= 1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 - G_4H_2 + G_1G_4$$

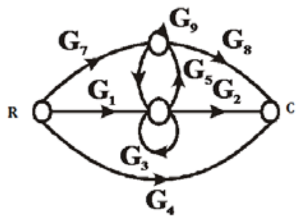
and $\Delta_1 = \Delta_2 = 1$

Finally,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$$

Example

Find C/R for the following system using Mason's gain rule



Solution

Forward Paths
$P_1 = G_1G_2$
$P_2 = G_4$
$P_3 = G_7G_8$
$P_4 = G_1G_5G_8$
$P_5 = G_7G_6G_2$
Loops
$L_1 = G_9$
$L_2 = G_3$
$L_3 = G_5G_6$

$$\Delta = (G_3 + G_9 + G_5G_6) + G_9G_3$$

$$\Delta_1 = 1 - G_9$$

$$\Delta_2 = 1 - (G_9 + G_3 + G_5G_6) + G_9G_3$$

$$\Delta_3 = 1 - G_3$$

$$\Delta_4 = 1$$

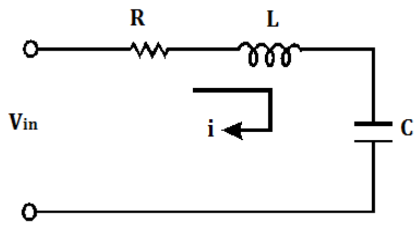
$$T \approx \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5}{\Delta}$$

$$= \frac{G_1G_2(1-G_9) + G_4(1-G_9-G_3-G_5G_6+G_9G_3) + G_7G_8(1-G_3)G_1G_5G_8}{1-G_3-G_9-G_5G_6+G_9G_3}$$

1.7 MATHEMATICAL MODELLING

A set of mathematical equations, describing the dynamic characteristics of a system is called mathematical model of the system.

1) Series RLC circuit



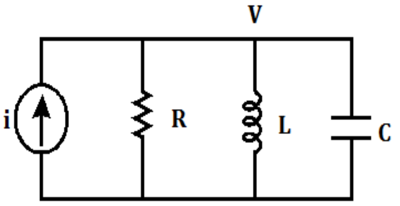
Applying KVL

$$iR + L\frac{di}{dt} + \frac{1}{c}\int idt = v_m$$

But $i = \frac{dq}{dt}$

$$\therefore \boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{v} = v_m}$$

2) Parallel RLC circuit



Applying KCL

$$\frac{v}{R} + \frac{1}{L}\int vdt + c\frac{dv}{dt} = i$$

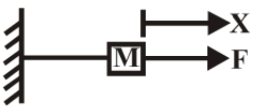
$$v = \frac{d\phi}{dt}$$

Where, ϕ = magnetic flux

$$c\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} = i$$

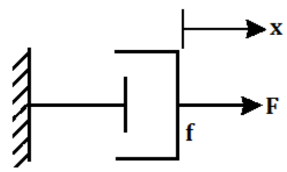
3) Translation system

a. Mass



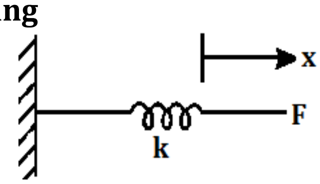
$$F = M\frac{d^2x}{dt^2}$$

b. Damper element

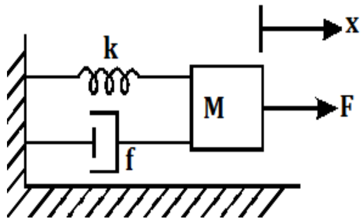


$$F = f\frac{dx}{dt}$$

c. Spring



F = Kx



At Balance

$$F - Kx - f\frac{dx}{dt} = M\frac{d^2x}{dt^2}$$

$$\therefore \boxed{M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx = F}$$

4) Rotational system

a. Inertia

$$T = J\frac{d^2\theta}{dt^2}$$

b. Damper element

$$T = f\frac{d\theta}{dt}$$

c. Spring twisted

$T = K\,\theta$

Mathematical model

$$T - f\frac{d\theta}{dt} - k\theta = J\frac{d^2\theta}{dt^2}:$$

$$\boxed{J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + k\theta = T}$$

1.7.1 ANALOGY

VoVoltage (SeriesRLC)	Current Parallel RLC	Force (Mass Damper)	Torque (Rotational)	Thermal	Liquid level	Pneumatic (air)
V	I	F	T	Temp	Head	Pressure
q	ϕ	X	θ	Heat Flow (in joule)	Liquid Flow (m^3)	Air flow Rate (m^3)
R	1/R	F	F	Thermal Resistance	Viscous Resistance	Air Resistanc e
I	V	Linear Velocity	Angular Velocity	Rate of flow of heat	Rate of flow of liquid	Rate of flow of air (m^3/s)
$\frac{1}{c}$	1/L	K	K	Heat Capacity (volume)	Volume of tank (volume)	Volume of container
L	C	M	J	-	-	-

Example

Consider the mechanical system shown in Fig. (a) and its electrical circuit analog shown in Fig.(b). The electrical circuit analogy is a force-current analogy as outlined in Table. The velocities, $v_1(t)$ and $v_2(t)$, of the mechanical system are directly analogous to the node voltages $v_1(t)$ and $v_2(t)$ of the electrical circuit. The simultaneous equations, assuming the initial conditions are zero, are

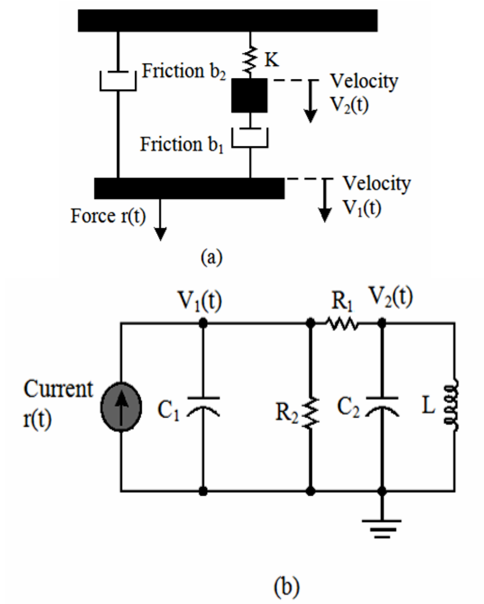


Figure: (a) Two-mass mechanical system (b) Two mode electric circuit analog $C_1 = M_1$, $C_2 = M_2$, $L = 1/K$, $R_1 = 1/b_1$, $R_2=1/b_2$

$$M_1sV_1(s)+(b_1+b_2)V_1(s)-b_1V_2(s)=R(s)$$

$$M_2sV_2(s)+b_1(V_2(s)-V_1(s))+k\frac{V_2(s)}{s}=0$$

These equations are obtained using the force equations for the mechanical system of Fig(a). Rearranging Eqs., we obtain

$$(M_1s+(b_1+b_2))V_1(s)+(-b_1)V_2(s)=R(s)$$

$$(-b_1)V_1(s)+\left(M_2s+b_1+\frac{k}{s}\right)V_2(s)=0$$

1.7.2 SENSITIVITY OF CONTROL SYSTEMS TO PARAMETER VARIATIONS

The first advantage of a feedback system is that the effect of the variation of the parameters of the process, G(s), is reduced. This illustrates the effect of parameter variations; let us consider a change in the process so that the new process is G(s) + ΔG(s). Then in the open-loop case, the change in transform of the output is

$$\Delta Y(s)=\Delta G(s)R(s)$$

In the closed-loop system, we have

$$Y(s)+\Delta Y(s)=\frac{G(s)+\Delta G(s)}{1+(G(s)+\Delta G(s))H(s)}R(s)$$

Then the change in the output is

$$\Delta Y(s)=\frac{\Delta G(s)}{(1+GH(s)+\Delta GH(s))(1+GH(s))}R(s)$$

When GH(s) >> ΔGH(s), as is often the case, we have

$$\Delta Y(s)=\frac{\Delta G(s)}{[1+GH(s)]}R(s)$$

The change in the output of the closed-loop system is reduced by the factor [1 + GH(s)], which is usually much greater than one over the range of complex frequencies of interest.

These are the greatest system complexity, need much larger forward path gain and possibility of system instability (it means undesired/ persistent oscillations of the output variable).

1.8 SENSITIVITY

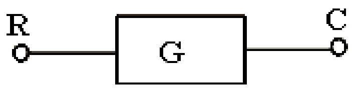
The sensitivity S_G^T is the ratio of percentage change in T to the percentage change in G.

Where, T – Transfer function

G- Forward path gain

i.e. $S_G^T=\frac{\partial T/T}{\partial G/G}=\frac{\partial T}{\partial G}\times\frac{G}{T}$

a) Open loop system



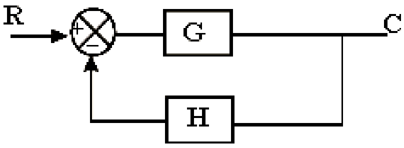
$$T=\frac{C}{R}=G$$

$$\frac{\partial T}{\partial G}=1$$

$$S_G^T=\frac{\partial T}{\partial G}\times\frac{G}{T}=1\times\frac{G}{G}=1$$

Hence open loop sensitivity is unity.

b) Closed loop system



$$T = C/R = \frac{G}{1+GH}$$

$$\frac{\partial T}{\partial G} = \frac{(1+GH)1 - GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

$$\Rightarrow S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T} = \frac{1}{(1+GH)^2}\frac{G}{G}(1+GH)$$

$$\therefore \quad S_G^T = \frac{1}{1+GH}$$

means $\left(S_G^T\right)_{closedloop} < \left(S_G^T\right)_{openloop}$

Hence closed loop system is lesser sensitive to parameter variation hence closed loop system is better.

GATE QUESTIONS(EC)(Basics of Control Systems)

Q.1 An electrical system and its signal-flow graph representations are shown in the figure (a) and (b) respectively. The values of G_2 and H, respectively are

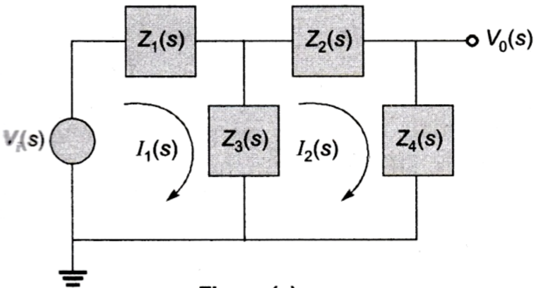


Figure (a)

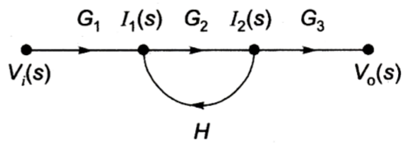


Figure (b)

a) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

b) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

c) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

d) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

[GATE -2001]

Q.2 The open loop DC gain of a unity negative feedback system with closed -loop transfer function $\dfrac{s+4}{s^2+7s+13}$ is

a) $\dfrac{4}{13}$
b) $\dfrac{4}{9}$
c) 4
d) 13

[GATE -2001]

Q.3 A system described by the following differential equation $\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y=x(t)$ is initially at rest. For input$(t)=2u(t)$, the output y(t) is

a) $\left(1-2e^{-t}+e^{-2t}\right)u(t)$
b) $\left(1+2e^{-t}-2e^{-2t}\right)u(t)$
c) $\left(0.5+e^{-t}+1.5e^{-2t}\right)u(t)$
d) $\left(0.5+2e^{-t}+2e^{-2t}\right)u(t)$

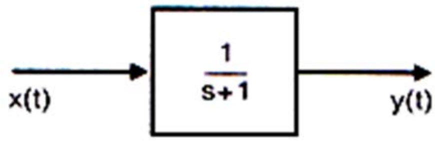
[GATE -2004]

Q.4 Despite the presence of negative feedback, control systems still have problems of instability because the

a) components used have nonlinearities
b) dynamic equations of the systems are not known exactly
c) mathematical analysis involves approximations.
d) system has large negative phase angle at high frequencies

[GATE -2005]

Q.5 In the system shown below, $x(t)=(\sin t)u(t)$. In steady -state, the response y(t) will be



a) $\dfrac{1}{\sqrt{2}}\sin\left(t-\dfrac{\pi}{4}\right)$
b) $\dfrac{1}{\sqrt{2}}\sin\left(t+\dfrac{\pi}{4}\right)$
c) $\dfrac{1}{\sqrt{2}}e^{-t}\sin t$
d) $\sin t-\cos t$

[GATE -2006]

Q.6 The unit-step response of a system starting from rest is given by $c(t)=1-e^{-2t}$ for $t\ge 0$. The transfer function of the system is

a) $\frac{1}{1+2s}$ b) $\frac{2}{2+s}$

c) $\frac{1}{2+s}$ d) $\frac{2s}{1+2s}$

[GATE -2006]

Q.7 The unit impulse response of a system is $h(t)=e^{-t}, t\geq 0$. For this system, the steady- state value of output for unit step input is equal to

a)-1 b)0

c)1 d)∞

[GATE -2006]

Q.8 The frequency response of a linear, time–invariant system is given by $H(f)=\frac{5}{1+j10\pi f}$. The step response of the system is

a) $5\left(1-e^{-5t}\right)u(t)$ b) $5\left(1-e^{-\frac{t}{5}}\right)u(t)$

c) $\frac{1}{5}\left(1-e^{-5t}\right)u(t)$) d) $\frac{1}{(s+5)(s+1)}$

[GATE -2007]

Q.9 A linear, time–invariant, causal continuous time system has a rational transfer function with simple poles at $s=-2$ and $s=-4$, and one simple zero at $s=-1$. A unit step u(t) is applied at the input of the system. At steady state, the output has constant value of 1 .The impulse response of this system is

a) $\left[\exp(-2t)+\exp(-4t)\right]u(t)$

b) $\left[-4\exp(-2t)+12\exp(-4t)-\exp(-t)\right]u(t)$

c) $\left[-4\exp(-2t)+12\exp(-4t)\right]u(t)$

d) $\left[-0.5\exp(-2t)+1.5\exp(-4t)\right]u(t)$

[GATE -2008]

Q.10 A system with the transfer function $\frac{Y(s)}{X(s)}=\frac{s}{s+p}$ has an output $y(t)=\cos\left(2t-\frac{\pi}{3}\right)$ for the input signal $x(t)=p\cos\left(2t-\frac{\pi}{3}\right)$ Then, the system parameter 'p' is

a) $\sqrt{3}$ b) $\frac{2}{\sqrt{3}}$

c)1 d) $\frac{\sqrt{3}}{2}$

[GATE -2010]

Q.11 A system with transfer function $G(s)=\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$ is excited sin (ωt).

The steady-state output of the system is zero at

a)ω =1 rad/s b)ω =2 rad/s

c)ω =3 rad/s d)ω =4 rad/s

[GATE-2012]

Q.12 Negative feedback in a closed-loop control system **DOES NOT**

a) reduce the overall gain

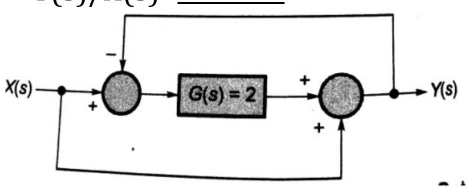
b) reduce bandwidth

c) improve disturbance rejection

d) reduce sensitivity to parameter variation

[GATE-2015-01]

Q.13 For the system shown in the figure, Y(s)/X(s)=________



[GATE-2017-02]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13
(c)	(b)	(a)	(d)	(a)	(b)	(c)	(b)	(c)	(b)	(c)	(b)	1

EXPLANATIONS

Q.1 **(c)**

From KVL in both loops In first loop.

$$V_i(s) = I_1(s)Z_1(s) + [I_1(s) - I_2(s)]Z_3(s)$$

$$V_i(s) = I_1(s)[Z_1(s) + Z_3(s)] - I_2(s)Z_3(s)$$

$$\frac{V_i(s)}{Z_1(s)+Z_3(s)} = I_1(s) - \frac{I_2(s)Z_3(s)}{Z_1(s)+Z_3(s)} \quad \text{...(i)}$$

In second loop

$$[I_2(s) - I_1(s)]Z_3(s) + I_2(s)Z_2(s)$$

$$+I_2(s)Z_4(s) = 0$$

$$I_2(s)(Z_2(s) + Z_3(s) + Z_4(s))$$

$$= I_1(s).Z_3(s)$$

$$G_2 = \frac{I_2(s)}{I_1(s)} = \frac{Z_3(s)}{Z_2(s) + Z_3 + (s) + Z_4(s)}$$

From SFG, $I_1(s)$

$$= V_iG_1(s) + I_2(s)H(s)$$

$$I_1(s) = V_i\frac{1}{Z_1+Z_3} + I_2\frac{Z_3}{Z_1+Z_3}$$

From (i)

$$\therefore H = \frac{Z_3}{Z_1+Z_3}$$

(comparing above two equations)

Q.2 **(b)**

$$CLTF = \frac{G(s)}{1+G(S)H(S)}$$

$$= \frac{s+4}{S^2+7s+13}$$

$$\frac{1+G(s)H(s)}{G(s)} = \frac{S^2+7s+13}{s+4}$$

H(s)=1 for unity feedback

$$\frac{1}{G(s)} = \frac{S^2+7s+13}{s+4} - 1$$

$$\frac{1}{G(s)} = \frac{S^2+6s+9}{s+4}$$

$$\therefore G(s) = \frac{s+4}{S^2+6s+9}$$

For D.C. s = 0

$$\therefore G(s) = \text{open loop gain} = \frac{4}{9}$$

Q.3 **(a)**

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$$

$$s^2Y(s) + 3sY(s) + 2Y(s) = X(s)$$

$$x(t) = 2u(t)$$

$$X(s) = \frac{2}{S}$$

$$\therefore (s^2+3s+2)Y(s) = \frac{2}{S}$$

$$Y(s) = \frac{2}{s(s+2)(s+1)}$$

$$\frac{2}{s(s+2)(s+1)} = \frac{A}{S} + \frac{B}{s+2} + \frac{C}{s+1}$$

$$2 = A(s+2)(s+1) + Bs(s+1)$$

$$+C(s+2)s$$

$$s = 0, 2 = 2A \Rightarrow A = 1$$

$$s = -1, 2 = -C \Rightarrow C = -2$$

$$s = 2, 2 = 2B \Rightarrow B = 1$$

$$Y(s) = \frac{1}{S} + \frac{1}{S+2} - \frac{2}{S+1}$$

$$y(t)[1 + e^{-2t} - 2e^{-t}]u(t)$$

Q.4 **(d)**

Q.5 **(a)**

$$y(t) = x(t)*h(t)$$

$$Y(s) = X(s)*H(s)$$

$$H(j\omega) = \frac{1}{s+1} = \frac{1}{\sqrt{2}}\angle -45°$$

$$= \frac{1}{\sqrt{2}}\angle -\frac{\pi}{4}$$

$$x(t)=(\sin t)u(t)$$
$$\therefore y(t)=\frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$$

Q.6 (b)

$$C(s)=\frac{1}{S}-\frac{1}{S+2}=\frac{2}{s(s+2)}$$
$$R(s)=\frac{1}{S}H(s)=\frac{C(s)}{R(s)}=\frac{2}{s+2}$$

Q.7 (c)

$$h(t)=e^{-t}$$
$$H(s)=\frac{1}{S+1}$$
$$R(s)=\frac{1}{S}$$
$$\text{Output }=H(s).R(s)=\frac{1}{(S+1)}.\frac{1}{S}$$
$$\frac{1}{S(S+1)}=\frac{A}{S}+\frac{B}{S+1}$$
$$1=A(s+1)+Bs$$
$$s=0,1=A$$
$$s=-1,1=-B$$
$$\therefore \text{Output}=\frac{1}{S}-\frac{1}{S+1}=(1-e^{-t})u(t)$$

When t=∞at steady state. Output =1

Q.8 (b)

$$H(f)=\frac{5}{1+j10\pi f}$$
$$H(s)=\frac{5}{1+5S}=\frac{5}{5\left(s+\frac{1}{5}\right)}=\frac{1}{s+\frac{1}{5}}$$
$$\text{Step response }=\frac{1}{S}\frac{1}{\left(s+\frac{1}{5}\right)}$$
$$\frac{1}{S}*\frac{1}{\left(s+\frac{1}{5}\right)}=\frac{A}{S}+\frac{B}{s+\frac{1}{5}}$$
$$\Rightarrow A\left(s+\frac{1}{5}\right)+Bs=1$$

When $s=0, A=5$

When $s=\frac{-1}{5}, B=-5$

$$Y(s)=\frac{5}{S}-\frac{5}{s+\frac{1}{5}}$$
$$\Rightarrow y(t)=5\left[1-e^{-\frac{t}{5}}\right]u(t)$$

Q.9 (c)

Transfer functions,

$$H(s)=\frac{K(s+1)}{(s+2)(s+4)}$$

Input, $R(s)=\frac{1}{s}$

Output $C(s)=R(s)H(s)$

Given: $\lim_{s\to 0} s\,C(s)=1$

Or $\lim_{s\to 0}\frac{s.K(s+1)}{s(s+2)(s+4)}=1$

Or $\frac{K}{8}=1$

$$\Rightarrow K=8$$
$$H(s)=\frac{8(s+1)}{(s+2)(s+4)}$$
$$=\frac{4}{S+2}+\frac{12}{S+4}$$
$$h(t)=(-4e^{-2t}+12e^{-4t})u(t)$$

Which is also the required impulse response of the system.

Q.10 (b)

Phase difference between input and output,

$$\Phi=-\frac{\pi}{3}-\left(-\frac{\pi}{2}\right)=\frac{\pi}{6}=30^\circ$$

And $\omega=2\text{rad/sec}$

From the transfer function,

$$\Phi=90^\circ-\tan^{-1}\frac{\omega}{p}$$
$$90^\circ-\tan^{-1}\frac{2}{p}=30^\circ$$

Q.11 (c)

For sinusoidal excitation

$s = j\omega$

$\therefore G(j\omega)$

$$= \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+1)(j\omega+3)(j\omega+4)}$$

For zero steady –state output

$|G(j\omega)| = 0$

$$= \frac{(-w^2+9)\sqrt{w^2+4}}{(\sqrt{w^2+1})(\sqrt{w^2+9})(\sqrt{w^2+16})}$$

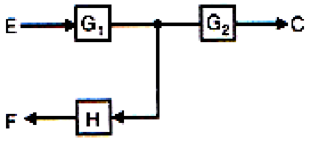
For zero steady-state output

$\Rightarrow \quad \omega^n = 9$

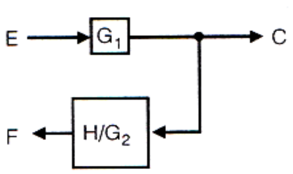
$\Rightarrow \quad \omega = 3\text{rad/sec}$

GATE QUESTIONS(EC)(Block Diagram & SFG)

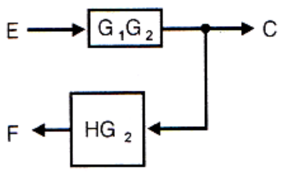
Q.1 The equivalent of the block diagram in the figure is given as



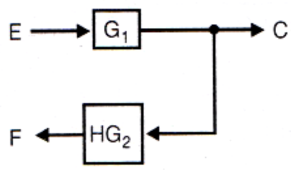
a)



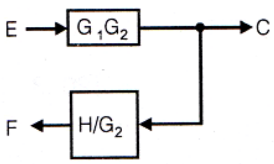
b)



c)

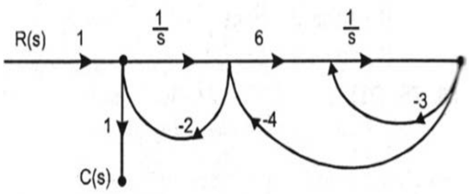


d)



[GATE -2001]

Q.2 The signal flow graph of a system is shown in the figure. The transfer function $\frac{C(s)}{R(s)}$ of the system

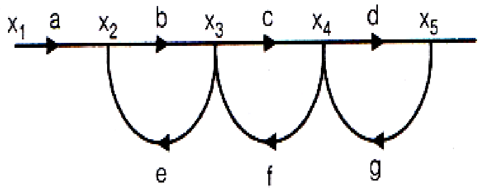


a) $\frac{6}{S^2+29s+6}$ b) $\frac{6s}{S^2+29s+6}$

c) $\frac{s(s+2)}{S^2+29s+6}$ d) $\frac{s(s+27)}{S^2+29s+6}$

[GATE -2003]

Q.3 Consider the signal flow graph shown in the figure. The gain x_5/x_1 is



a) $\frac{1-(be+cf+dg)}{abc}$

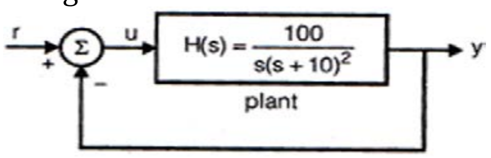
b) $\frac{bedg}{1-(be+cf+dg)}$

c) $\frac{abcd}{1-(be+cf+dg)+bedg}$

d) $\frac{1-(be+cf+dg)+bedg}{abcd}$

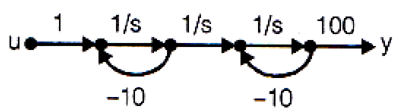
[GATE -2004]

Q.4 The input –output transfer function of a plant $H(s)=\frac{100}{s(s+10)^2}$ .The plant is placed in a unity negative feedback configuration as shown in the figure below.

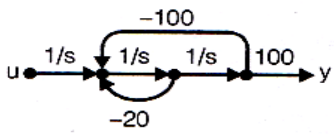


The signal flow graph that DOES NOT model the plant transfer function H(s) is

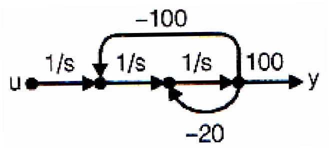
a)



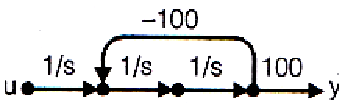
b)



c)

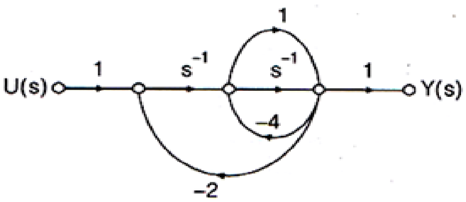


d)



[GATE -2011]

Q.5 The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is given as

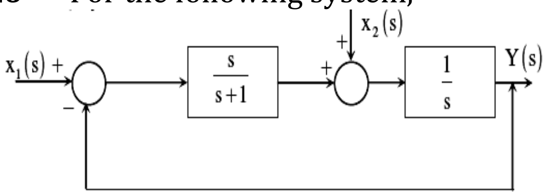


a) $\frac{s+1}{5s^2+6s+2}$ b) $\frac{s+1}{s^2+6s+2}$

c) $\frac{s+1}{5s^2+4s+2}$ d) $\frac{1}{5s^2+6s+2}$

[GATE-2013]

Q.6 For the following system,



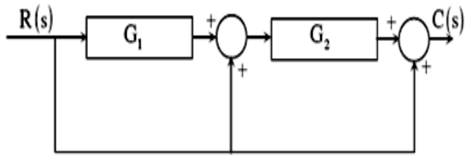
When $X_1(s)=0$, the transfer function $\frac{y(s)}{x_2(S)}$ is

a) $\frac{s+1}{s^2}$ b) $\frac{1}{s+1}$

c) $\frac{s+2}{s(s+1)}$ d) $\frac{s+1}{s(s+2)}$

[GATE-2014]

Q.7 Consider the following block diagram in the figure.



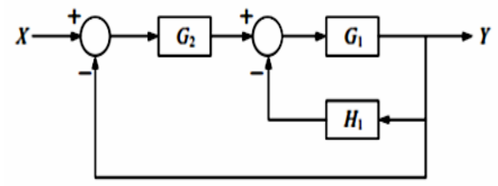
The transfer function $\frac{C(S)}{R(S)}$ is

a) $\frac{G_1G_2}{1+G_1G_2}$ b) $G_1G_2+G_1+1$

c) $G_1G_2+G_2+1$ d) $\frac{G_1}{1+G_1G_2}$

[GATE-2014]

Q.8 The block diagram of a feedback control system is shown in the figure. The overall closed-loop gain G of the system is



a) $G=\frac{G_1G_2}{1+G_1H_1}$

b) $G=\frac{G_1G_2}{1+G_1G_2+G_1H_1}$

c) $G=\frac{G_1G_2}{1+G_1G_2H_1}$

d) $G=\frac{G_1G_2}{1+G_1G_2+G_1G_2H_1}$

[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6	7	8
(d)	(d)	(c)	(d)	(a)	(d)	(c)	(b)

EXPLANATIONS

Q.1 (d)

Take off point is moved after G_2 so $/G_2$.

Q.2 (d)

$P_1 = 1$

$\Delta_1 = 1 + \frac{3}{S} + \frac{24}{S} = \frac{S+27}{S}$

$G(s) = \frac{P_1\Delta_1}{1-(\text{loopgain}) + \text{pairofnon} - \text{touchingloops}}$

$L_1 = \frac{-3}{S}, L_2 = \frac{-24}{S}, L_3 = \frac{-2}{S}$

L_1 and L_3 are non –touching .

$$\therefore G(s) = \frac{\frac{(s+27)}{S}}{1-\left(\frac{-3}{S}-\frac{24}{S}-\frac{2}{S}\right)+\frac{-2}{S}\times\frac{-3}{S}}$$

$$= \frac{\frac{(s+27)}{S}}{1+\frac{29}{S}\times\frac{6}{S^2}} \frac{C(s)}{R(s)} = \frac{s(s+27)}{S^2+29s+6}$$

Q.3 (c)

$P_1 = abcd, \Delta_1 = 1$

$L_1 = be, L_2 = cf, L_3 = dg$

Non-touching loops are

$L_1 \& L_3 = bedg$

$$\therefore \frac{x_5}{x_1} = \frac{abcd}{1-(be+cf+dg)+bedg}$$

Q.4 (d)

For option (d),

$$\frac{Y(s)}{U(s)} = \frac{100/S^3}{1+\frac{100}{S^2}}$$

Which is not transfer function of H(s).

Q.5 (a)

No of forward path =2

$P_1 = \frac{1}{S^2}, P_2 = \frac{1}{S^2}$

$\Delta_1 = 1; \nabla_2 = 1$

$L_1 = \frac{-4}{5}, L_2 = \frac{-2}{S^2}$

$L_3 = -4, L_4 = \frac{-2}{S}$

No non touching loops

$\Delta k = 1[L_1 + L_2 + L_3 + L_4]$

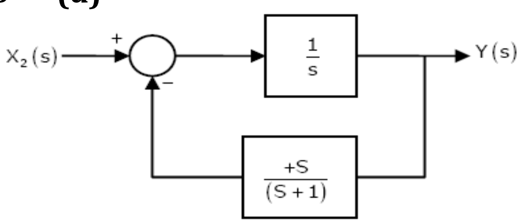
$= 1 + \frac{4}{S} + \frac{2}{S^2} + 4 + \frac{5}{S}$

$= \frac{S^2 + 4s + 2 + 4S^2 + 4s}{S^2}$

$= \frac{5s^2 + 6s + 2}{S^2}$

$= \frac{\frac{1}{S^2} + \frac{1}{S^2}}{\frac{5s^2+6s+2}{S^2}} = \frac{s+1}{5s^2+6s+2}$

Q.6 (d)



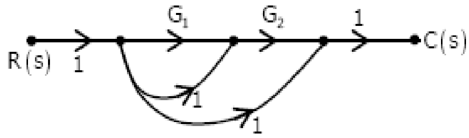
If $X_1(s) = 0$

$\frac{Y(s)}{X_2(s)}$; The block diagram becomes

$$\frac{Y(s)}{X_2(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s}.\frac{s}{(s+1)}} = \frac{\frac{1}{s}}{\frac{s+2}{s+1}} \Rightarrow \frac{(s+1)}{s(s+2)}$$

Q.7 **(c)**

By drawing the signal flow graph for the given block diagram



Number of parallel paths are three

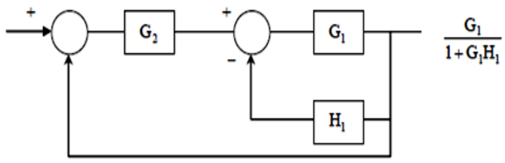
Gains $P_1 = G_1G_2$, $P_2 = G_2$, $P_3 = 1$

By mason's gain formula,

$$\frac{C(S)}{R(S)} = P_1 + P_2 + P_3$$

$$\Rightarrow G_1G_2 + G_2 + 1$$

Q. 8 **(b)**



$$\frac{Y}{X} = \frac{G_1G_2}{1+G_1G_2+G_1H_1}$$

GATE QUESTIONS(EE)(Basics of Control Systems)

Q.1 The transfer function of the system described by $\frac{d^2y}{dt^2}+\frac{dy}{dt}=\frac{du}{dt}+2u$ with u is input and y as output is

a) $\frac{(s+2)}{(s^2+s)}$ b) $\frac{(s+1)}{(s^2+s)}$

c) $\frac{2}{(s^2+s)}$ d) $\frac{2s}{(s^2+s)}$

[GATE-2002]

Q.2 A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12\left(1-e^{-2t}\right)$$

The response of the system as $t\to\infty$ is

a) $x=6$ b) $x=2$

c) $x=2.4$ d) $x=-2$

[GATE-2003]

Q.3 A control system with certain excitation is governed by the following mathematical equation

$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+5e^{-4t}+2e^{-5t}$$

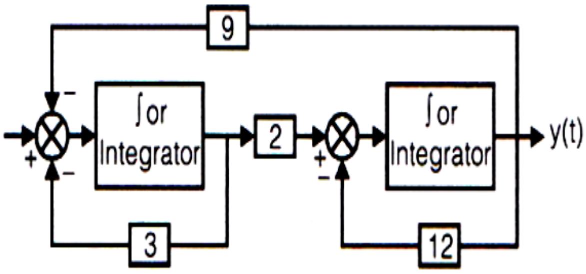
The natural time constants of the response of the system are

a) 2s and 5s b)3s and 6s

c) 4s and 5s d)1/3s and 1/6s

[GATE-2003]

Q.4 The block diagram of a control system is shown in figure. The transfer function G(s) =Y(s)/U(s) of the system is



a) $\frac{1}{18\left(1+\frac{s}{12}\right)\left(1+\frac{s}{3}\right)}$

b) $\frac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$

c) $\frac{1}{27\left(1+\frac{s}{12}\right)\left(1+\frac{s}{9}\right)}$

d) $\frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{3}\right)}$

[GATE-2003]

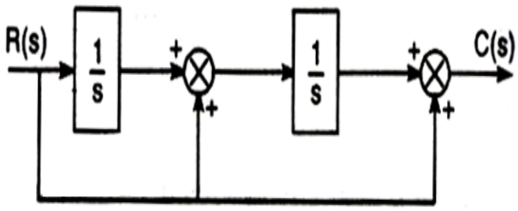
Q.5 For a tachometer, if $\theta(t)$ is the rotor displacement in radians, e(t) is the output voltage and K_t is the tachometer constant in V/rad/sec, then the transfer function, $\frac{E(s)}{Q(s)}$ will be

a) $K_t s^2$ b) $\frac{K_t}{s}$

c) $K_t s$ d) K_t

[GATE-2004]

Q.6 For the block diagram shown in figure, the transfer function $\frac{C(s)}{R(s)}$ is equal to



a) $\frac{s^2+1}{s^2}$ b) $\frac{s^2+s+1}{s^2}$

c) $\frac{s^2+s+1}{s^2}$ d) $\frac{1}{s^2+s+1}$

[GATE-2004]

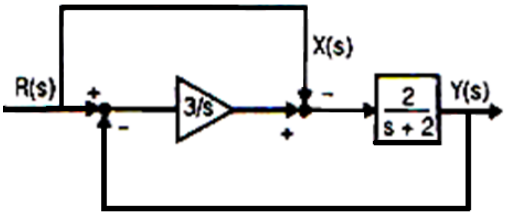
Q.7 The unit impulse response of a second order under damped system starting from rest is given by $c(t)=12.5\,e^{-6t}\sin 8t, t\ge 0$ The steady-state value of the unit step response of the system is equal to

a) 0 b) 0.25

c) 0.5 d) 1.0

[GATE-2004]

Q.8 When subjected to a unit step input, the closed loop control system shown in the figure



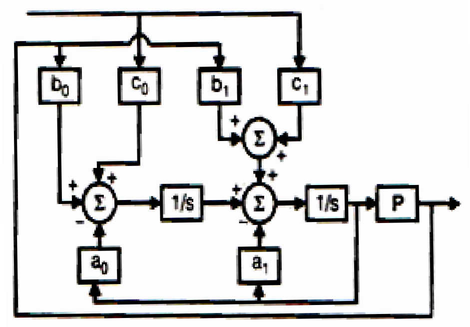
will have a steady state error of

a) -1.0 b) -0.5

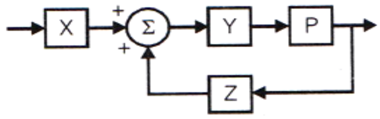
c) 0 d) 0.5

[GATE-2005]

Q.9 The system shown in figure below



Can be reduced to the form



With

a) $X=c_0s+c_1, Y=1\big/\left(s^2+a_0s+a_1\right) Z=b_0s+b_1$

b) $X=1, Y=\frac{c_0s+c_1}{\left(s^2+a_0s+a_1\right)}, Z=b_0s+b_1$

c) $X=c_1s+c_0, Y=\frac{\left(b_1s+b_0\right)}{\left(s^2+a_1s+a_0\right)}, Z=1$

d) $X=c_1s+c_0, Y=\frac{1}{\left(s^2+a_1s+a_0\right)}, Z=b_1s+b_0$

[GATE-2007]

Q.10 A function y(t) satisfies the following differential equation:

$$\frac{dy(t)}{dt}+y\left(t\right)=\delta\left(t\right)$$

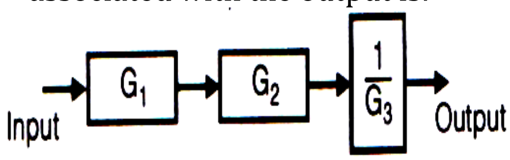
Where $\delta\left(t\right)$ is the delta function. Assuming zero initial condition, and denoting the unit step function by u(t), y(t) can be of the form

a) e^{t} b) e^{-t}

c) $e^{t}u(t)$ d) $e^{-t}u(t)$

[GATE-2008]

Q.11 The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as G_1, G_2 and $\frac{1}{G_3}$. The relative small errors associated with each respective subsystem G_1, G_2 and G_3 are $\varepsilon_1, \varepsilon_2$ and ε_3 .The error associated with the output is:



a) $\varepsilon_1+\varepsilon_2+\frac{1}{\varepsilon_3}$ b) $\frac{\varepsilon_1.\varepsilon_2}{\varepsilon_3}$

c) $\varepsilon_1+\varepsilon_2-\varepsilon_3$ d) $\varepsilon_1+\varepsilon_2-\varepsilon_3$

[GATE-2009]

Q.12 The response h(t) of a linear time invariant system to an impulse δ(t), under initially relaxed condition is $h(t)=e^{-t}+e^{-2t}$. The response of this system for a unit step input u (t) is

a) $u(t)+e^{-t}+e^{-2t}$

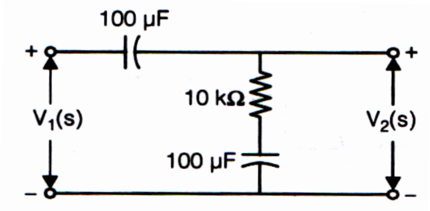
b) $(e^{-t}+e^{-2t})u(t)$

c) $(1.5-e^{-t}-0.5e^{-2t})u(t)$

d) $e^{-t}\delta(t)+e^{-2t}u(t)$

[GATE-2011]

Q.13 The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is

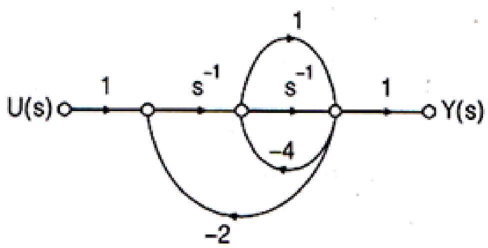


a) $\frac{0.5s+1}{s+1}$ b) $\frac{3s+6}{s+2}$

c) $\frac{s+2}{s+1}$ d) $\frac{s+1}{s+2}$

[GATE-2013]

Q.14 The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is given as

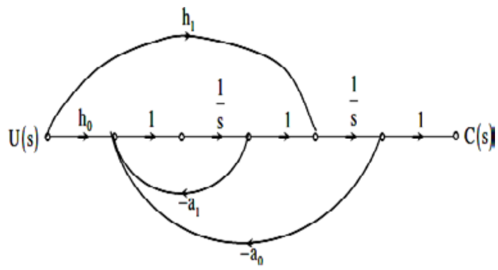


a) $\frac{s+1}{5s^2+6s+2}$ b) $\frac{s+1}{s^2+6s+2}$

c) $\frac{s+1}{5s^2+4s+2}$ d) $\frac{1}{5s^2+6s+2}$

[GATE-2013]

Q.15 The signal flow graph of a system is shown below. U(s) is the input and C(s) is the output



Assuming, $h_1=b_1$ and $h_0=b_0-b_1a_1$, the input-output transfer function, $G(s)=\frac{C(s)}{U(s)}$ of the system is given by

a) $G(s)\frac{b_0s+b_1}{s^2+a_0s+a_1}$

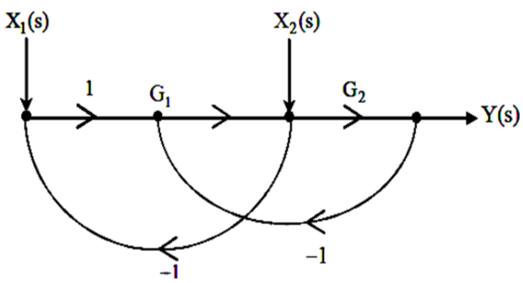
b) $G(s)\frac{a_1s+a_0}{a^2+b_1s+b_0}$

c) $G(s)\frac{b_1s+b_0}{a^2+a_1s+a_0}$

d) $G(s)\frac{a_0s+a_1}{a^2+b_0s+b_1}$

[GATE-2014]

Q.16 For the signal flow graph shown in the figure, which one of the following expressions is equal to the transfer function $\left.\frac{Y(s)}{X_2(s)}\right|_{X_1(s)=0}$?

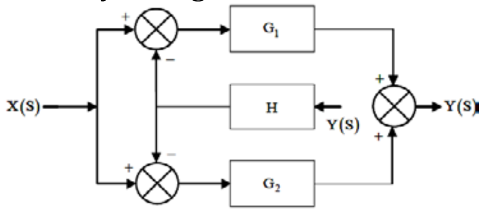


a) $\frac{G_1}{1+G_2(1+G_1)}$ b) $\frac{G_2}{1+G_1(1+G_2)}$

c) $\frac{G_1}{1+G_1G_2}$ d) $\frac{G_2}{1+G_1G_2}$

[GATE-2015]

Q.17 Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below:



a) $\frac{G_1}{1-HG_1}+\frac{G_2}{1-HG_2}$

b) $\frac{G_1}{1+HG_1}+\frac{G_2}{1+HG_2}$

c) $\frac{G_1+G_2}{1+H(G_1+G_2)}$

d) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

[GATE-2015]

Q.18 For the system governed by the set of equations:

$dx_1/dt = 2x_1 + x_2 + u$

$dx_2/dt = -2x_1 + u$

$y = 3x_1$

the transfer function Y(s)/U(s) is given by

a) $3(s+1)/(s^2-2s+2)$

b) $3(2s+1)/(s^2-2s+1)$

c) $(s+1)/(s^2-2s+1)$

d) $3(2s+1)/(s^2-2s+2)$

[GATE-2015]

Q.19 For a system having transfer function $G(s) = \frac{-s+1}{s+1}$, a unit step input is applied at time t=0. The value of the response of the system at t=1.5 sec (round off to three decimal places) is _____.

[GATE-2017-01]

Q.20 Match the transfer functions of the second-order systems with the nature of the systems given below.

P. $\frac{15}{s^2+5s+15}$

Q. $\frac{25}{s^2+10s+25}$

R. $\frac{35}{s^2+18s+35}$

Nature of system

I. Over damped

II. Critically damped

III. Critically damped

a) P-I, Q-II, R-III
b) P-II, Q-I, R-III
c) P-III, Q-II, R-I
d) P-III, Q-I, R-II

[GATE-2018]

Q.21 The number of roots of the polynomial $s^7+s^6+7s^5+14s^4+31s^3+73s^2+25s+200$ in the open left half of the complex plane is

a) 3 b) 4 c) 5 d) 6

[GATE-2018]

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
(a)	(c)	(b)	(b)	(c)	(b)	(d)	(c)	(d)	(d)	(c)	(c)	(d)	(a)
15	**16**	**17**	**18**	**19**	**20**	**21**							
(c)	(b)	(c)	(a)	0.554	(c)	(a)							

EXPLANATIONS

Q.1 (a)

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}=\frac{du}{dt}+2u$$

$$\Rightarrow s^2Y(s)+sY(s)=sU(s)+2U(s)$$

$$\therefore \frac{Y(s)}{U(s)}=\frac{s+2}{(s^2+s)}$$

Q.2 (c)

Taking (LT) on both sides

$$(s^2+6s+5)\times(s)=12\left(\frac{1}{s}-\frac{1}{s+2}\right)$$

$$=\frac{24}{s(s+2)}$$

$$X(s)=\frac{24}{s(s+2)(s+1)(s+5)}$$

Response at $t\rightarrow\infty$

Using final value theorem

$$\underset{t\rightarrow\infty}{\text{Lt}}\, x(t)=\underset{s\rightarrow 0}{\text{Lt}}\, sX(s)$$

$$=\underset{s\rightarrow 0}{\text{Lt}}\frac{s\times 24}{s(s+1)(s+2)(s+5)}=2.4$$

Q.3 (b)

Natural time constant of the response depends only on poles of the system.

$$T(s)=\frac{C(s)}{R(s)}=\frac{1}{s^2+s/2+1/18}$$

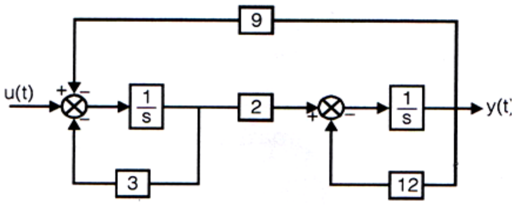
$$=\frac{18}{18s^2+9s+1}=\frac{1}{(6s+1)(3s+1)}$$

This is in the form $=\frac{1}{(1+sT_1)(1+sT_2)}$

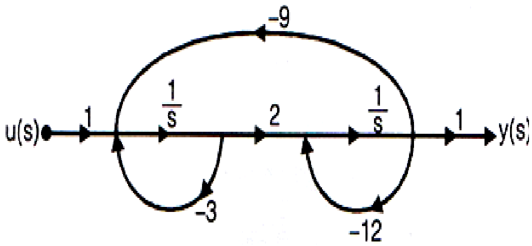
T_0, T_2 = 6sec, 3sec

Q.4 (b)

Integrator are represented as 1/s in S-domain



As per the block diagram, the corresponding signal flow graph is drawn



One forward path $P_1=2/s^2$

The individual loops are,

$$L_1=-\frac{3}{s},L_2=-\frac{12}{s}\text{ and }L_3=-\frac{18}{s^2}$$

L_1 and L_1 are non-touching loops

$$L_1L_2=\frac{36}{s^2}$$

The loops touches the forward path

$\Delta_1=1$

The graph determinant is

$$\Delta=1-(L_1+L_2+L_3)+L_1L_2$$

$$=1+\frac{3}{s}+\frac{12}{s}+\frac{18}{s^2}+\frac{36}{s^2}$$

Applying mason's gain formula

$$G(s)=\frac{Y(s)}{U(s)}=\frac{P_1\Delta_1}{\Delta}$$

$$=\frac{2/s^2}{1+\frac{3}{s}+\frac{12}{s}+\frac{18}{s^2}+\frac{36}{s^2}}=\frac{2}{s^2+15s+54}$$

$$=\frac{2}{(s+9)(s+6)}=\frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{6}\right)}$$

Q.5 **(c)**

$\theta(t)$ = rotor displacement in radians

$\omega(t) = \frac{d\theta}{dt}$ = angular speed in rad/sec

Output voltage; $e(t) = K_t \omega(t)$

$= K_t \frac{d\theta}{dt}$.

Taking Laplace transform on both sides $E(s) = K_t s\theta(s) \Rightarrow \frac{E(s)}{\theta(s)} = K_t s$

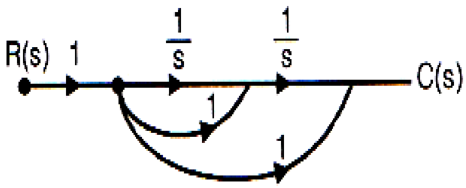
Q.6 **(b)**

Method-1: Using block-diagram reduction technique.

So, transfer function

$= \frac{C(s)}{R(s)} = \frac{s^2+s+1}{s^2}$

Method-2: Using signal flow graph



Three forward paths.

$P_1 = \frac{1}{s}\frac{1}{s} = \frac{1}{s^2}, P_2 = 1.\frac{1}{s} = \frac{1}{s}$ & $P_3 = 1$

The no. of individual loop=0

So graph determinant $= \Delta = 1$

and $\Delta_1 = \Delta_2 = \Delta_3 = 1$

Applying Mason's gain formula

$G(s) = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$

$= \frac{\frac{1}{s^2}.1 + \frac{1}{s}.1 + 1.1}{1} \qquad = \frac{s^2+s+1}{s^2}$

Q.7 **(d)**

Transfer function of a system is the unit impulse response of the system.

Transfer function $= \frac{C(s)}{R(s)}$

$= \mathcal{L}\left[12.5e^{-6t}\sin 8t\right]$

$= 12.5 \times \frac{8}{(s+6)^2 + 8^2}$

$= \frac{100}{(s+6)^2 + 8^2}$

When input is unit step, $R(s) = \frac{1}{s}$

$C(s) = \frac{1}{s}.\frac{100}{(s+6)^2+8^2}$

Steady-state value of response, using final value theorem

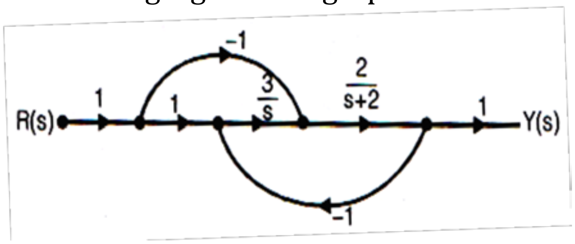
$C_{steady-state} = \underset{t\to\infty}{Lt}\, C(t)$

$= \underset{s\to 0}{Lt}\, s(s)$

$= \underset{s\to 0}{Lt}\, s\left[\frac{1}{s}.\frac{100}{(s+6)^2+8^2}\right] = 1$

Q.8 **(c)**

Using signal flow grap



Forward path gains

$P_1 = (-1)\times\frac{2}{s+2} = \frac{-2}{s+2}$

And $P_2 = \frac{3}{s}.\frac{2}{s+2} = \frac{2}{s(s+2)}$

Individual loop

$L_1 = -1\times\frac{3}{s}\times\frac{2}{s+2} = \frac{-6}{s(s+2)}$

Loop touches forward paths, therefore,

$V_1 = 1$ and $V_2 = 1$

$D = 1 - L_1$

$= 1 + \frac{6}{s(s+2)} = \frac{s(s+2)+6}{s(s+2)}$

Using Mason's gain formula,

$$\frac{Y(s)}{R(s)}=\frac{P_1\Delta_1+P_2\Delta_2}{\Delta}$$

$$=\frac{-\frac{2}{s+2}\times 1+\frac{6}{s(s+2)}\times 1}{\frac{s(s+2)+6}{s(s+2)}}$$

$$\frac{Y(s)}{R(s)}=\frac{6-2s}{s^2+2s+6}$$

For unit step input, R(s)=1/s

$$Y(s)=R(s).\left(\frac{6-2s}{s^2+2s+6}\right)$$

Error =E(s)=R(s)-Y(s)

$$=R(s)-R(s)\left(\frac{6-2s}{s^2+2s+6}\right)$$

$$=R(s)\left[1-\frac{6-2s}{s^2+2s+6}\right]$$

$$E(s)=\frac{1}{s}\left[\frac{s^2+4s}{s^2+2s+6}\right]$$

Steady state value of error, using final value theorem

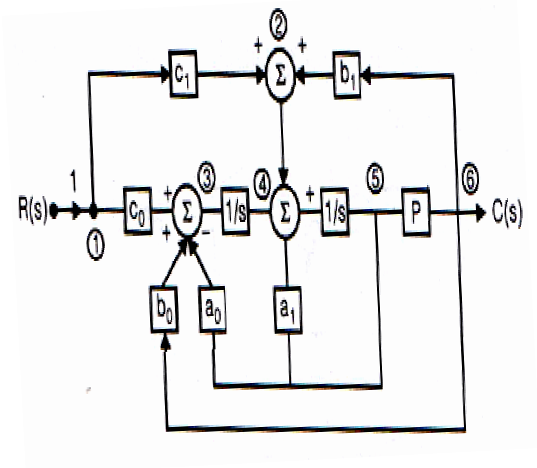
$$e_{ss}=\lim_{s\to 0} sE(s)$$

$$=\lim_{s\to 0} s\left[\frac{1}{s}.\frac{s^2+4s}{s^2+2s+6}\right]$$

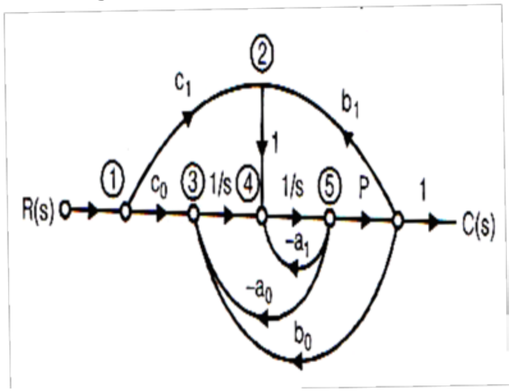
$$=\lim_{s\to 0}\left[\frac{s^2+4s}{s^2+2s+6}\right]=0$$

Q.9 **(d)**

The block-diagram can be redrawn as



Single flow graph of the block-diagram

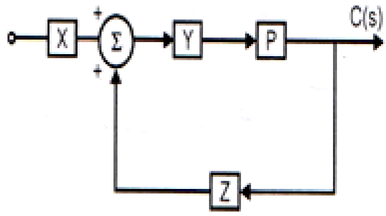


There are two forward paths.

$$P_1=1\times c_0\times\frac{1}{s}\times\frac{1}{s}\times P=\frac{C_0P}{s^2}$$

$$P_2=1\times c_1\times 1\times\frac{1}{s}\times P=\frac{C_1P}{s}$$

These are four individual loops



$$L_1=-a_1\times\frac{1}{s}=-\frac{a_1}{s}$$

$$L_2=\frac{1}{s}\times\frac{1}{s}\times -a_0=-\frac{a_0}{s^2}$$

$$L_3=\frac{1}{s}\times\frac{1}{s}\times P\times b_0=\frac{b_0P}{s^2}$$

$$L_4=b_1\times P\times\frac{1}{s}=\frac{b_1P}{s^2}$$

All the loops touch forward paths

$$\Delta_1=\Delta_2=1$$

$$\Delta=1-(L_1+L_2+L_3+L_4)$$

$$=1+\frac{a_0}{s^2}+\frac{a_1}{s}-\frac{b_0P}{s^2}-\frac{b_1P}{s}$$

Using Mason's gain formula

$$\frac{C(s)}{R(s)}=\frac{P_1\Delta_1+P_2\Delta_2}{s}$$

$$=\frac{\frac{c_0P}{s^2}+\frac{c_1P}{s}}{1+\frac{a_1}{s}+\frac{a_0}{s^2}-\frac{b_0P}{s^2}-\frac{b_1P}{s}}$$

$$\frac{C(s)}{R(s)}=\frac{c_0P+c_1p_s}{S^2+(a_0-b_1)S+(a_0-b_0P)}$$

$$\frac{C(s)}{R(s)}=\frac{P(c_0+c_1s)/(s^2+a_1s+a_0)}{1-(b_0+b_1s)P/(s^2+a_1s+a_0)}$$

$$=\frac{P(c_0+c_1s)/(s^2+a_1s+a_0)}{1-(b_0+b_1s)P/(s^2+a_1s+a_0)} \quad \text{... (i)}$$

$$\frac{C(s)}{R(s)}=\frac{xyP}{1-yzP} \quad \text{... (ii)}$$

Comparing eq.(i) and (ii), we get

$$xy=\frac{c_0+c_1s}{S^2+a_1s+a_0}$$

$$yz=\frac{b_0+b_1s}{S^2+a_1s+a_0}$$

Hence option (d) is correct.

Q.10 (d)

Taking (L.T.) on both sides

$$Y(s)(s+1)=1$$

$$\therefore Y(s)=\frac{1}{s+1}$$

Taking inverse laplace transform

$$Y(t)=e^{-t}u(t)$$

Q.11 (c)

$$\frac{dG_1}{G_1}=\epsilon_1,\frac{dG_2}{G_2}=\epsilon_2\ \&\frac{dG_3}{G_3}\ \epsilon_3$$

$$\text{Output }(y)_0=\frac{G_1G_3}{G_3}x$$

where X=input in

$$y=InG_1+InG_2-InG_3+Inx$$

Differentiating both sides

$$\frac{dy}{y}=\frac{dG_1}{G_1}+\frac{dG_2}{G_2}-\frac{dG_3}{G_3}+\frac{dx}{x}$$

No error is specified in input so

$$\frac{dx}{x}=0$$

$$\frac{dy}{y}=\epsilon_1+\epsilon_2-\epsilon_3.$$

Q.12 (c)

Transfer function of system is impulse response of the system with zero initial conditions.

Transfer function

$$=H(s)=\mathcal{L}\left(e^{-1}+e^{-2t}\right)$$

$$=\frac{1}{s+1}+\frac{1}{s+2}$$

$$H(s)=\frac{C(s)}{R(s)}=\left(\frac{1}{s+1}+\frac{1}{s+2}\right)$$

$$R(s)=\frac{1}{s}=(\text{step input})$$

$$C(s)=R(s).H(s)$$

$$=\frac{1}{s}\left(\frac{1}{s+1}+\frac{1}{s+2}\right)=\frac{1}{S(S+1)}+\frac{1}{S(S+2)}$$

$$C(s)=\left(\frac{1}{s}-\frac{1}{s+1}\right)+\frac{1}{2}\left(\frac{1}{s}-\frac{1}{s+2}\right)$$

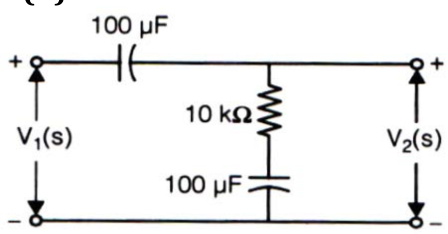
$$=\frac{1.5}{S}-\frac{1}{S+1}-\frac{0.5}{S+2}$$

$$\text{Response }=C(t)=\mathcal{L}^{-1}\left[C(s)\right]$$

$$=\mathcal{L}^{-1}\left[\frac{1.5}{s}-\frac{1}{s+1}-\frac{0.5}{s+2}\right]$$

$$C(t)=(1.5-e^{-t}-0.5e^{-2t})u(t)$$

Q.13 (d)



$$\frac{V_2(s)}{V_1(s)}=\frac{R+\frac{1}{Cs}}{\frac{1}{Cs}+R+\frac{1}{Cs}}=\frac{1+RCs}{2+RCs}$$

$$=\frac{1+10\times10^3\times100\times10^{-6}s}{2+10\times10^3\times100\times10^{-6}s}=\frac{1+s}{2+s}$$

Q.14 (a)

No. of forward path =2

$$P_1=\frac{1}{S^2},P_2=\frac{1}{S^2}$$

$\Delta_1 = 1; \nabla_2 = 1$

$L_1 = \frac{-4}{5}, L_2 = \frac{-2}{S^2}$

$L_3 = -4, L_4 = \frac{-2}{S}$

No non touching loops

$\Delta k = 1[L_1 + L_2 + L_3 + L_4]$

$= 1 + \frac{4}{S} + \frac{2}{S^2} + 4 + \frac{5}{S}$

$= \frac{S^2 + 4s + 2 + 4S^2 + 4s}{S^2}$

$= \frac{5s^2 + 6s + 2}{S^2}$

$= \frac{\frac{1}{S^2} + \frac{1}{S^2}}{\frac{5s^2 + 6s + 2}{S^2}} = \frac{s+1}{5s^2 + 6s + 2}$

Q.15 (c)

From the signal flow graph,

$G(s) = \frac{C(s)}{U(s)}$

By mason's gain relation,

Transfer function =

$= \frac{P_1\Delta_1 + P_2\Delta_2 + \ldots}{\Delta}$

$P_1 = \frac{h_1}{S}; P_2 = \frac{h_0}{S_2}$

$\Delta_1 = \left[1 + \frac{a_1}{s}\right]; \Delta_2 = 1 \qquad \Delta = 1 + \frac{a_1}{s} + \frac{a_0}{s^2}$

Transfer function =

$\frac{\frac{h_1}{s}\left[1 + \frac{a_1}{s}\right] + \frac{b_0}{s^2}}{1 + \frac{a_1}{s} + \frac{a_0}{s^2}} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$

Q.16 (b)

$P_1 = G_2$

$\Delta = 1 - [-G_1G_2 - G_1] = 1 + G_1(1 + G_2)$

$TF = \frac{P_1\Delta_1}{\Delta} = \frac{G_2}{1 + G_1[1 + G_2]}$

Q.18 (a)

$\frac{dx_1}{dt} = 2x_1 + x_2 + 4$

$\frac{dx_2}{dt} = -2x_1 + 4$

$y = 3x_1$

Considering the standard equation

$x_i = AX + BU$

$y = Cx + DU$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}[4]$

$y = [30]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Transform function $C(SI - A)^{-1}B$

$G(s) = [3 \quad 0]\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}\right]^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$[3 \quad 0]\begin{bmatrix} s-2 & -1 \\ 0 & s \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$[3 \quad 0]\begin{bmatrix} s & 1 \\ -2 & s-2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$s^2 - 2s + 2$

$= \frac{1}{s^2 - s + 2}[3 \quad 0]\begin{bmatrix} s+1 \\ -2+s-2 \end{bmatrix}$

$= \frac{1}{s^2 - 2s + 2}[3 \quad 0]\begin{bmatrix} s+1 \\ s-4 \end{bmatrix}$

$= \frac{3(s+1)}{s^2 - 2s + 2}$

Q.19 0.554

Q.20 (c)

Option	Characteristic Equation	Damping Ratio (ξ)	Damping
P	$s^2 + 5s + 15$	$\xi = 0.645$	Under damping
Q	$s^2 + 10s + 25$	$\xi = 1$	Critical damped
R	$s^2 + 18s + 35$	$\xi = 1.52$	Over damped

Hence, the correct option is (C).

Q.21 (a)

Given: Characteristic equation,
$s^7 + s^6 + 7s^5 + 14s^4 + 31s^3 + 73s^2 + 25s + 200$
The R-H table is given by,

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	32	96	0	
s^2	24	200	0	
s^1	-170.67	0		
s^0	200			

From the above table number of sign change in the first column is 4. So, the number of left hand pole are
7 - 4 = 3.

$8s^4 + 48s^2 + 200 = 0$

$s^2 = x$

$8x^2 + 48x + 200 = 0$

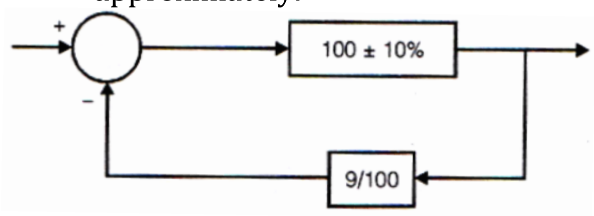
$x^2 + 6x + 25 = 0$

$x = -3 \pm j4$

$s^2 = -3 \pm j4$

GATE QUESTIONS(EE)(Block Diagram & SFG)

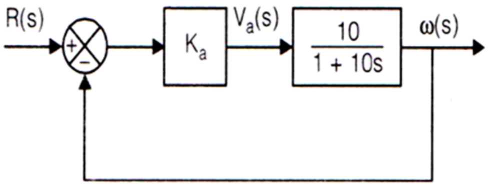
Q.1 As shown in the figure, a negative feedback system has an amplifier of gain 100 with ±10% tolerance in the forward path, and an attenuator of value 9/100 in the feedback path. The overall system gain in approximately:



a)10±1% b)10 ± 2%
c) 10 ±5% d) 10±10%

[GATE-2010]

Q.2 The open- loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)}=\frac{10}{1+10s}$. When connected in feedback as shown below,



the approximate value of K_a that will reduce the time constant of closed loop system by one hundred times as compared to that of the open- loop system is

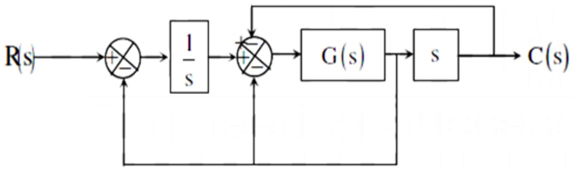
a) 1 b) 5
c) 10 d) 100

[GATE-2013]

Q.3 The closed-loop transfer function of a system is $(s)=\frac{4}{(s^2+0.4s+4)}$. The steady state error due to unit step input is

[GATE-2014]

Q.4 The block diagram of a system is shown in the figure



If the desired transfer function of the system is $\frac{C(s)}{R(s)}=\frac{s}{s^2+s+1}$ then G(s) is

a)1 b)s
c)1/s d) $\frac{-s}{s^3+s^2-s-2}$

[GATE-2014]

ANSWER KEY:

1	2	3	4
(a)	(c)	(0)	(b)

EXPLANATIONS

Q.1 (a)

$G = 100 \pm 10\%$

$\frac{\Delta G}{G} = 10\% or 0.1$

$H = 9/100$

Overall gain

$T = \frac{G}{1+GH}$ ...(i)

$T = \frac{100}{1+100\times\frac{9}{100}} = 10$

$\frac{dT}{dG} = \frac{(1+GH)-GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$

$dT = \frac{dG}{(1+GH)^2}$ ...(ii)

$\frac{\Delta T}{T} = \frac{\Delta G}{G}\times\frac{1}{(1+GH)}$

$\frac{\Delta T}{T} = 10\times\frac{1}{1+100\times\frac{9}{100}}\%$

So, overall system gain $= 10 \pm 1\%$

Q.2 (c)

$G(s)H(S) = \frac{10K}{1+10s}$ O.L.T.F

$\tau = 10 sec$

C.L.T.F $\tau = \frac{10}{100} = 0.1$

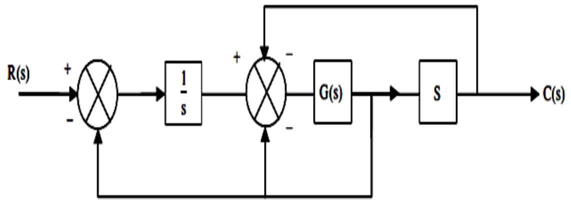
C.L.T.F $= \frac{10K}{10K+1+10s}$

$\tau = 0.1 = \frac{10K}{10K+1} \Rightarrow K \cong 10$

Q.3 (0)

Steady state error for Type-1 for unit step input is 0.

Q.4 (b)

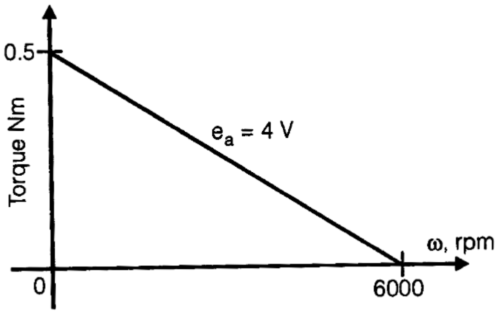


If G(s) = S

$\frac{C(s)}{R(s)} = \frac{S}{s^2+s+2}$

GATE QUESTIONS(IN)(Basics of Control Systems)

Q.1 The torque-speed curve of a constant field armature controlled DC servomotor is shown in the figure. The armature resistance in Ω and torque constant in Nm/A of the motor respectively are



a) (1.76,0.68)   b) (1.76,0.85)
c) (2.00,0.25)   d) (0.01, 0.81)
[GATE-2004]

Q.2 The rotor of the control transformer of a synchro pair gives a maximum voltage of 1.0 V at a particular position of the rotor of the control transmitter. The transmitter rotor is now rotated by 30° anticlockwise keeping the transformer rotor stationary. The transformer rotor voltage for this position is
a) 1.0 V   b) 0.566 V
c) 0.5 V   d) 0 V
[GATE-2010]

Q.3 The transfer function of a Zero – Order-Hold system with sampling interval T is
a) $\frac{1}{s}\left(1-e^{-Ts}\right)$ b) $\frac{1}{s}\left(1-e^{-Ts}\right)^2$
c) $\frac{1}{s}e^{-Ts}$ d) $\frac{1}{s^2}e^{-Ts}$
[GATE-2012]

Q.4 Unit step response of a linear time invariant (LT) system is given by $y(t) = (1 - e^{-at})u(t)$. Assuming zero initial condition, the transfer function of the system is
[GATE-2012]

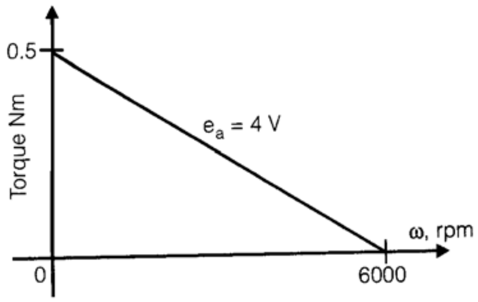
a) $\frac{1}{s+1}$ b) $\frac{2}{(s+1)(s+2)}$
c) $\frac{1}{(s+2)}$ d) $\frac{2}{(s+2)}$

ANSWER KEY:

1	2	3	4
(c)	(b)	(a)	(d)

EXPLANATIONS

Q.1 (c)



We know

$T_m(t) = K_m(t)\phi i_a(t)$

$T_m(t) = K_i i_a(t)$ ($\because \phi$ = constant)

Among the options only the armature resistance $R_a = 2.00\ \Omega$ will satisfy the torque equation.

$i_a(t) = \frac{e_a}{R_a} = \frac{4}{2} = 2.0\text{Amp}$

$K_i = \frac{T_m(t)}{i_a(t)} = \frac{0.5}{2.0}\text{N}-\text{m/A} = 0.25\text{N-m/A}$

$\therefore K_i = 0.25\text{N}-\text{m/A}$

Q.2 (b)

$v_r = v_m \cos\phi$

$= 1.0\ \cos 30$

$v_r = 0.866\text{v}$

Q.3 (a)

The transfer function of a zero-order hold system having a sampling interval T is $\frac{1}{s}\left(1-e^{-Ts}\right)$.

Q.4 (d)

Given unit step response

$y(t) = (1-e^{-2t})u(t)$

impulse response $h(t) = \frac{d}{dt}y(t)$

$h(t) = \delta(t) - e^{-2t}\delta(t) + 2e^{-2t}u(t)$

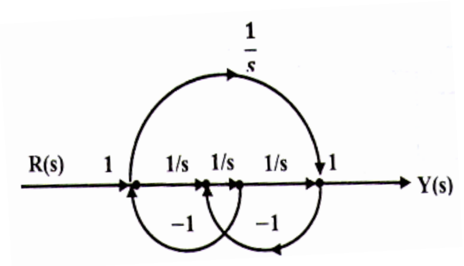
$= 2e^{-2t}u(t)$

$L\{h(t)\} = \text{Transfer function}$

$\text{T.F.} = L\{2e^{-2t}4(t)\} = \frac{2}{s+2}$

GATE QUESTIONS(IN)(Block Diagram & SFG)

Q.1 The signal flow graph representation of a control system is shown below. The transfer function $\frac{Y(s)}{R(s)}$ is computed as



a) $\frac{1}{S}$

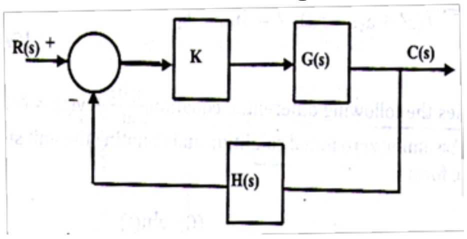
b) $\frac{S^2+1}{S(S^2+2)}$

c) $\frac{S(S^2+1)}{S^2+2}$

d) $1-\frac{1}{S}$

[GATE-2006]

Q.2 A feedback control system with high K, is shown in the figure below:



Then the closed loop transfer function is.

a) Sensitive to perturbations in G(s) and H(s)

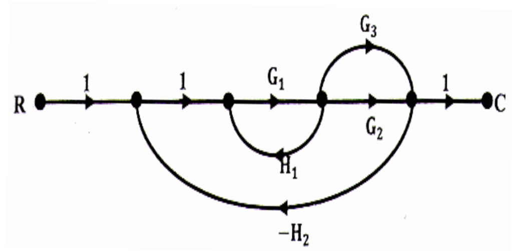
b) Sensitive to perturbations in G(s) and but not perturbations H(s)

c) Sensitive to perturbations in H(s) and but not to perturbations G(s)

d) Insensitive to perturbations in G(s) and H(s)

[GATE-2007]

Q.3 The signal flow graph of a system is given below.



The transfer function (C/R) of the system is

a) $\frac{(G_1G_2+G_1G_3)}{(1+G_1G_2H_2)}$

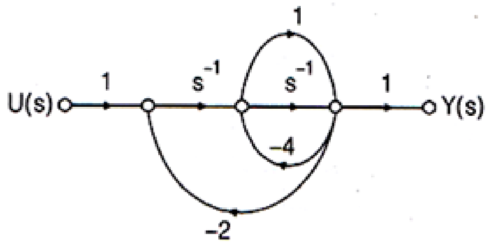
b) $\frac{(G_1G_2+G_1G_3)}{(1-G_1H_1+G_1G_2H_2)}$

c) $\frac{(G_1G_2+G_1G_3)}{(1-G_1H_1+G_1G_2H_2+G_1G_3H_2)}$

d) $\frac{(G_1G_2+G_1G_3)}{(1-G_1H_1+G_1G_2H_2+G_1G_3H_2+G_1G_2G_3H_1)}$

[GATE-2011]

Q.4 The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is given as



a) $\frac{s+1}{5s^2+6s+2}$

b) $\frac{s+1}{s^2+6s+2}$

c) $\frac{s+1}{5s^2+4s+2}$

d) $\frac{1}{5s^2+6s+2}$

[GATE-2013]

ANSWER KEY:

1	2	3	4
(a)	(c)	(c)	(a)

EXPLANATIONS

Q.1 (a)

Q.2 (c)

$S_G^T = \frac{1}{(1+KG(s).H(s))} \to 0$ as k is high

$$S_H^T = \frac{-kG(S).H(s)}{(1+KG(s).H(s))} \to -1$$

Q.3 (c)

$P_1 = G_1G_2; P_2 = G_1G_3$

$L_1 = G_1H_1; L_2 = -G_1G_2H_2; L_3$

$= -G_1G_3H_2$

$\Delta_1 = 1; \Delta_2 = 1.$

$\Delta = 1-(L_1 + L_2 + L_3)$

$= 1 - G_1H_1 + G_1G_2H_2 + G_1G_3H_2$

$\therefore \frac{C}{R} = \frac{(P_1\Delta_1 + P_2\Delta_2)}{\Delta}$

$= \frac{(G_1G_2 + G_1G_3)}{(1 - G_1H_1 + G_1G_2H_2 + G_1G_3H_2)}$

Q.4 (a)

No. of forward path =2

$P_1 = \frac{1}{S^2}, P_2 = \frac{1}{S^2}$

$\Delta_1 = 1; \nabla_2 = 1$

$L_1 = \frac{-4}{5}, L_2 = \frac{-2}{S^2}$

$L_3 = -4, L_4 = \frac{-2}{S}$

No non touching loops

$\Delta k = 1[L_1 + L_2 + L_3 + L_4]$

$= 1 + \frac{4}{S} + \frac{2}{S^2} + 4 + \frac{5}{S}$

$= \frac{S^2 + 4s + 2 + 4S^2 + 4s}{S^2}$

$= \frac{5s^2 + 6s + 2}{S^2}$

$= \frac{\frac{1}{S^2} + \frac{1}{S^2}}{\frac{5s^2 + 6s + 2}{S^2}} = \frac{s+1}{5s^2 + 6s + 2}$

2 TIME DOMAIN ANALYSIS

2.1 INTRODUCTION

Time response of the output means behavior of the response with respect to the time. In a practical system, output of the system takes some time to reach its final value. The final state achieved by the system response (output) is called **steady state**.

Following are the characteristics of the Steady State Response of a control system.

- The part of the time response that remains even after the transients have died out, is said to be steady state response.
- The steady state part of time response reveals the accuracy of a control system.
- Steady state error is observed if actual output does not exactly match with the input.

The state of output between application of input & steady state is called **transient state**.

Following are the characteristics of the Transient State Response of a control system.

- The part of the time response which goes to zero after a large interval of time, is known as transient response.
- It reveals the nature of response.
- It gives an indication about the speed of response.

Hence the total time response of the system can be written as

$c(t) = C_{ss} + C_t(t)$

Note:

The difference in the desired & actual output is called steady state error e_{ss} .

2.1.1 STANDARD TEST SIGNALS:

The various inputs affecting the performance of the system are mathematically represented as standard Test signal.

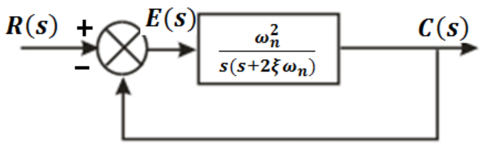
I. Sudden input → Step Signal
II. Velocity input → Ramp signal
III. Acceleration input → Parabolic signal
IV. Sudden shock → Impulse signal – Stability analysis

2.2 TIME RESPONSE OF A SECOND ORDER CONTROL SYSTEM FOR UNIT STEP INPUT:

A second order control system is one wherein the highest power of s in the denominator of its transfer function equals 2. Transfer function of a second order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \ldots \text{(I)}$$

The parameter ξ and ω_n will be explained later. The block diagram representation of the transfer function given by above expression



Block Diagram of a Second order Control System

The response of the system is given by

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left[\left(\omega_n\sqrt{1-\zeta^2} + \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]\right)\right]$$

Where $\omega_d = \omega_n\sqrt{1-\zeta^2}$

and $\boxed{\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}$

The error is given by

$e(t) = r(t) - c(t)$ And $r(t) = 1$ (unit step)

$\therefore e(t) =$

$$1-\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}+\tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]\right)\right]$$

Or

$$e(t)=\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left[\omega_n\sqrt{1-\zeta^2}+\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

The steady state error is

$$e_{ss}=\lim_{t\to\infty}\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}.\sin\left[\omega_n\sqrt{1-\zeta^2}t+\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

= 0

- The time response expression indicates that for values of $\xi < 1$the response presents exponentially decaying oscillations having a frequency $\omega_n\sqrt{1-\xi^2}$ and the time constant of exponential decay is $1/\xi\omega_n$.
- The term ω_n is called natural frequency of oscillations.
- The term $\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped frequency of oscillations.

2.2.1 DAMPING RATIO AND DAMPING FACTOR

The two roots can be expressed as

$s_1, s_2 = \zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$= -\alpha \pm j\omega_d$

Where

$\alpha = \zeta\omega_n$ is called damping factor and

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

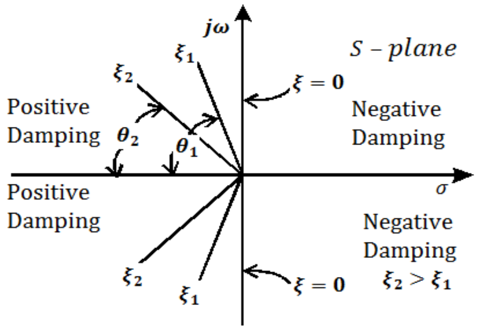
Note:

α appears as the constant that is multiplied to t in the exponential term, therefore α controls the rate of rise or decay of the unit-step response.

Now, ξ is called as damping ratio & it is given by

$$\xi = \frac{\alpha}{\omega_n}$$

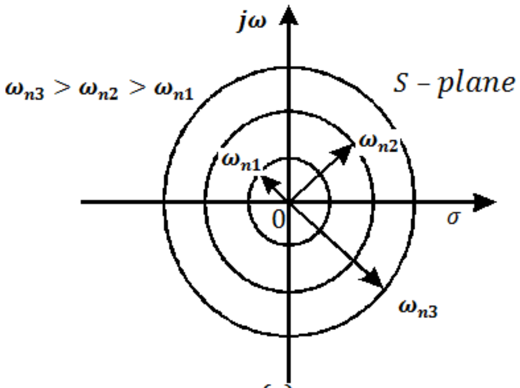
$$\frac{\text{actual damping factor}}{\text{damping factor at critical damping(i.e. when } \xi = 1)}$$



The above figure shows loci of damping ratio ξ. ξ is the cosine of the angle between the radial line to the roots and the negative axis when the roots and the negative axis when the roots are in the left-half s-plane, or $\xi = \cos\theta$.

2.2.2 NATURAL UNDAMPED FREQUENCY

When $\xi = 0$, the damping is zero, the roots of the characteristic equation are imaginary, shows that the unit-step response is purely sinusoidal. Therefore, ω_n corresponds to the frequency of the undamped sinusoidal response.



(a)

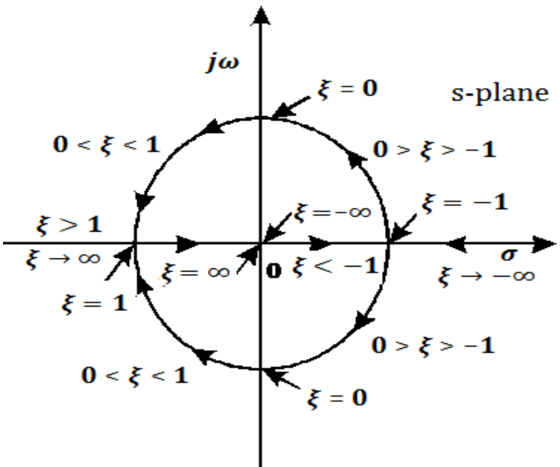
The above figure shows the loci of ω_n. ω_n is the radial distance from the roots to the origin of the s-plane

Note:

1) The left-half s-plane corresponds to positive damping (i.e., the damping factor or damping ratio is positive). Positive damping causes the unit-step response to settle to a constant final

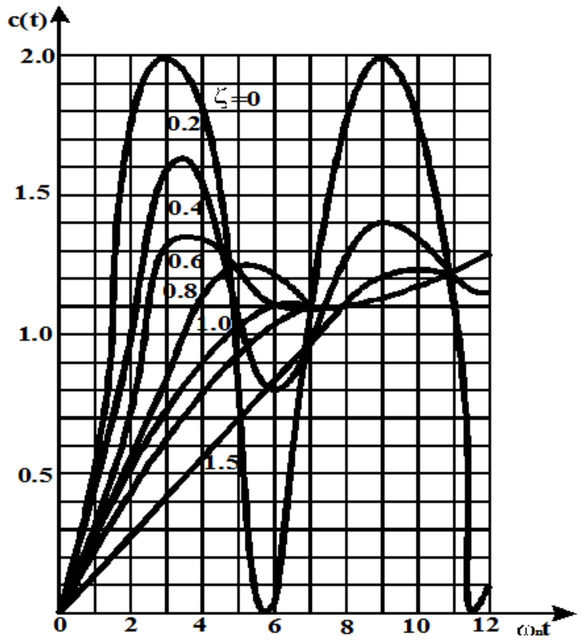
value in steady state due to the negative exponent of exp ($-\xi \omega_n t$). The system is stable.

2) The right-half s-plane corresponds to negative damping. Negative damping gives a response that grows in magnitude without bound with time, and the system is unstable.
3) The imaginary axis corresponds to zero damping ($\alpha = 0$ or $\xi = 0$). Zero damping results in a sustained oscillation response, and the system is marginally stable or marginally unstable.
4) $0 < \zeta < 1 : s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}\ (-\zeta\omega_n < 0)$
   Under damped
5) $\zeta = 1$: $s_1, s_2 = -\omega_n$
   Critically damped
6) $\zeta > 1 : s_1, s_2 = -\zeta \pm \omega_n\sqrt{\zeta^2 - 1}$ Over damped
7) $\zeta = 0 : s_1, s_2 = \pm j\omega_n$ undamped
8) $\zeta < 0 : s_1, s_2 = -\zeta\ j\omega_n + j\omega_n\sqrt{1-\zeta^2}$
   negatively damped

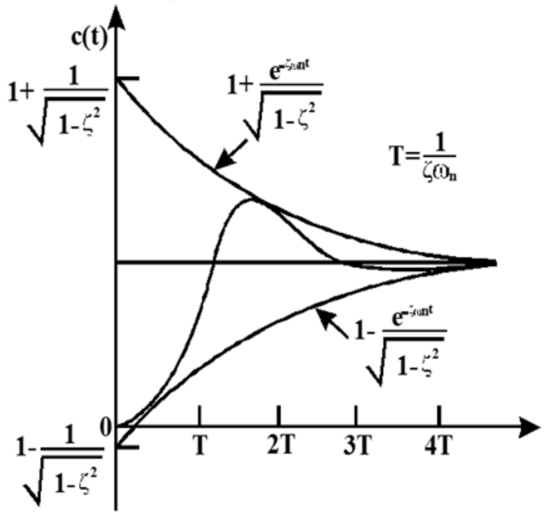


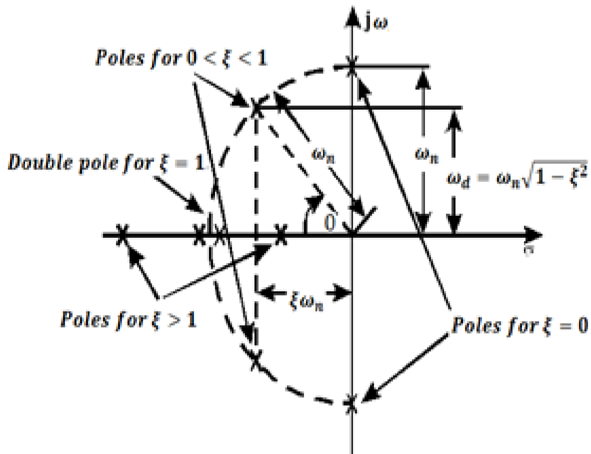
Locus of roots of characteristic equation of second order system

2.2.3 TIME RESPONSE FOR DIFFERENT VALUES OF ξ



2.2.4 POLE LOCATIONS FOR DIFFERENT VALUES OF ξ





Pole locations for second order system

2.3 TIME RESPONSE SPECIFICATION

In specifying the transient response characteristics of a control system to a unit-step input, it is common to specify the following:

1. Delay time, t_d
2. Rise time, t_r
3. Peak time, t_p
4. Maximum overshoot, M_p
5. Setting time, t_s

These specifications are defined in what follows and are shown graphically in figure.

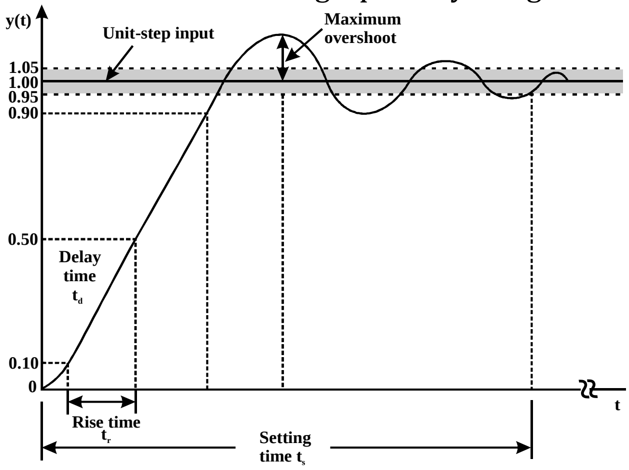


Figure: Typical unit-step response of a control system

1) Delay time t_d : The delay time is the time required for the response to reach half the final value in the first attempt.

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

2) Rise time t_r: The rise time is the time required for the response to rise from 10% to 90% of its final value for over damped system. For under damped second-order systems, the 0% to 100% rise time is normally used.

$$t_r = \frac{\pi-\theta}{\omega_d}$$

Where, $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

3) Peak time t_p: The peak time is the time required for the response to reach the first peak of the overshoot.

$$t_p = \frac{n\pi}{\omega_d} = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$$

Where $n = 1, 2, 3, 4, 5, 6\ldots\ldots\ldots$

Note:

- n = 1 for 1st over shoot (+ve peak)
- n = 2 for 1st undershoot (-ve peak)
- n = 3 for 2nd over shoot
- n = 4 for 2nd undershoot

1) Peak Overshoot M_P: The maximum overshoot is the maximum peak value of the response curve measured form unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined as

$$\%M_p = \frac{c(t_p)-c(\infty)}{c(\infty)}\times 100\%$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}}\times 100\%$$

The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

2) Settling time t_s: The setting time is the time required for the response curve to reach and stay within a range about the final value of size specified in percentage of the final value (usually 2% or 5%).

a) Settling time for 2% transition band: It is the time taken by the oscillations to decrease and stay within a limit of 2 % of the final value & it is given by

$$T_s = 4T \text{ where } T = \frac{1}{\xi\omega_n}$$

$$\therefore \quad T_s = \frac{4}{\xi\omega_n}$$

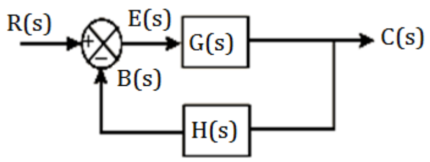
b) Settling time for 5% transition band: It is the time taken by the oscillations to decrease and stay within a limit of 5% of the final value & it is given by

$$T_s = 3T \text{ where } T = \frac{1}{\xi\omega_n}$$

$$\therefore \quad T_s = \frac{3}{\xi\omega_n}$$

2.4 STEADY STATE ERROR

It is the difference between the actual output and the desired output. The steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input specified as either step or ramp or parabolic.



The error signal generated after comparing input & feedback signal is given by

$E(s) = R(s) - B(s)$

Where, $B(s)$ is the feedback signal & it is given by

$B(s) = H(s)C(s)$

Now, the output $C(s) = G(s)E(s)$

$\therefore \; E(s) = R(s) - H(s)G(s)E(s)$

$$\Rightarrow E(s) = \frac{1}{1+G(s)H(s)}R(S)$$

This $E(s)$ is the error in Laplace domain & the corresponding error in time domain is e(t). Now the steady state error is the error when $t \to \infty$.

i.e. $e_{ss} = \lim_{t\to\infty} e(t)$

from the final value theorem

$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

$$\therefore e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

2.4.1 STEADY STATE ERROR FOR UNIT STEP INPUT

For unit step input, $R(s) = \frac{1}{s}$

We know that,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} = \frac{s \times \frac{1}{s}}{1+G(s)H(s)}$$

$$= \frac{1}{1+\lim_{s\to 0} G(s)H(s)} = \frac{1}{1+K_P}$$

Where,

$K_P = \lim_{s\to 0} G(s)H(s)$ is called positional error constant.

Case 'a': For type '0'

$K_P = \text{constant}$

$\therefore \; e_{ss} = \text{constant}$

Case 'b': For type '1':

$K_P \to \infty$

$$\therefore e_{ss} = \frac{1}{1+\infty} = 0$$

Case 'c': For type '2':

$K_P \to \infty$

$$\therefore \; e_{ss} = \frac{1}{1+\infty} = 0$$

Note:

For the same type of input, as the system type increases the steady state error decreases.

2.4.2 STEADY STATE ERROR FOR RAMP INPUT

For ramp input, $R(s) = \frac{1}{s^2}$

We know that,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)} \lim_{s\to 0} = \frac{s \times \frac{1}{s^2}}{1+G(s)H(s)}$$

$$= \frac{1}{\lim_{s\to 0} s + \lim_{s\to 0} sG(s)H(s)} = \frac{1}{K_v}$$

Where, $K_V = \lim_{s\to 0} sG(s)H(s)$ is called velocity error constant.

Case 'a': For type '0'

$K_v = \lim_{s\to 0} sG(s)H(s) = 0$

$$\therefore \; e_{ss} = \frac{1}{K_v} = \infty$$

Case 'b': For type '1':

$K_v = \text{constant}$

$\therefore \quad e_{ss} = \frac{1}{K_v} = \text{constant}$

Case 'c': For type '2':

$K_v \to \infty$

$\therefore \quad e_{ss} = \frac{1}{\infty} = 0$

Note:

As the unit input changes from unit step to ramp and ramp to parabola the steady state error increases for the same type.

2.4.3 STEADY STATE ERROR FOR PARABOLIC INPUT

For parabolic input, $R(s) = \frac{1}{s^3}$

We know that,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)} \lim_{s\to 0} = \frac{s \times \frac{1}{s^3}}{1+G(s)H(s)}$$

$$= \frac{1}{\lim_{s\to 0} s^2 + \lim_{s\to 0} s^2 G(s)H(s)} = \frac{1}{K_a}$$

Where, $K_a = \lim_{s\to 0} s^2 G(s)H(s)$ is called acceleration error constant.

Case 'a': For type '0'

$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$

$\therefore \quad e_{ss} = \frac{1}{K_a} = \infty$

Case 'b': For type '1':

$K_a = 0$

$\therefore \quad e_{ss} = \frac{1}{K_a} = \infty$

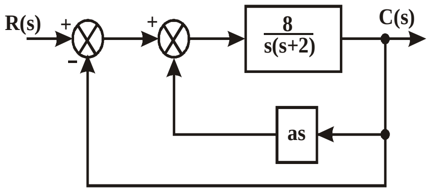
Case 'c': For type '2':

$K_v = \text{constant}$

$\therefore \quad e_{ss} = \text{constant}$

Example:

The system illustrated in Fig. is a unity feedback control system with a minor feedback loop (output derivative feedback).



a) In the absence of derivative feedback (a = 0), determine the damping factor and natural frequency. Also determine the steady-state error resulting from a unit-ramp input.

b) Determine the derivative feedback constant which will increase the damping factor the system to 0.7. What is the steady-state error to unit-ramp input with this setting of the derivative feedback constant?

Solution

a) With a = 0, the characteristic equation when derivative feedback is zero, the system will be a unity feedback system with transfer function

$$\frac{C(s)}{R(s)} = \frac{8}{s^2 + 2s + 8}$$

$$\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 150E, \text{Where}(r-\theta)\text{is the}$$

The characteristics equation is

$s^2 + 2s + 8 = 0$

Equating with the standard form

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

We get $\omega_n = \sqrt{8} = 2\sqrt{2}\text{rad/sec}$

And $2\xi\omega_n = 2$

$\therefore \quad \xi = \frac{1}{2\sqrt{2}} = 0.353$

Now for ramp input the velocity error constant is

$$K_V = \lim_{s\to 0} sG(s)H(s) = s \times \frac{8}{s(s+2)} = \frac{8}{2} = 4$$

$$\text{ess}(\text{to unit} - \text{ramp}) = \frac{1}{4} = 0.25$$

b) With derivative feedback, the transfer function is

$$\frac{C(s)}{R(s)} = \frac{8}{s^2 + (2+8a)s + 8}$$

The characteristics equation is

$s^2 + (2+8a)s + 8 = 0$

Equating with the standard form

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

We get

$\omega_n = \sqrt{8} = 2\sqrt{2}$rad / sec And

$2\xi\omega_n = 2 + 8a$

$2 \times 0.7 \times 2\sqrt{2} = 2 + 8a$

$\Rightarrow \; a = 0.245$

Now for ramp input the velocity error constant is

$K_V = \lim_{s \to 0} sG(s)H(s)$

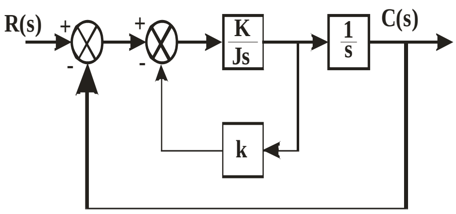
$= s \times \frac{8}{s(s+2+8a)} = \frac{8}{2+8a}$

$\therefore \; ess(\text{to unit} - \text{ramp}) = \frac{2+8a}{8}$

$= 0.495$

Example:

Determine the values of K and k of the closed-loop shown in fig. so that the maximum overshoot in unit-step response is 25% and the peak time is 2 sec. Assume that J = 1 kg-m^2.



Solution:

The closed-loop transfer function is

$\frac{C(S)}{R(S)} = \frac{K}{Js^2 + Kks + K}$

By substituting J = 1 kg-m^2 into this last equation, we have

$\frac{C(S)}{R(S)} = \frac{K}{s^2 + Kks + K}$

Now equating with the standard transfer function

$\omega_n = \sqrt{K}$

The maximum overshoot M_p is

$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.25$

From which

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.25$ or

$\xi = 0.404$

The peak time t_p is specified as 2 sec. And so

$t_p = \frac{\pi}{\omega_d} = 2$ or $\omega_d = 1.57$

Then the undamped natural frequency ω_n is

$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{1.57}{\sqrt{1-0.404^2}} = 1.72$

There, we obtain

$K = \omega_n^2 = 1.72^2 = 2.95$ N − m

$k = \frac{2\zeta\omega_n}{K} = \frac{2 \times 0.404 \times 1.72}{2.95} = 0.471$sec

Example:

A servomechanism is represented by the equation actuating signal. Calculate the value of damping ratio, undamped and damped frequency of oscillations

Solution:

$\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 150E$ Or

$\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 150(r - \theta)$

The equation in Laplace domain is

$s^2\theta(s) + 10s\theta(s) = 150(R(s) - \theta(s))$

$\frac{\theta(s)}{R(s)} = \frac{150}{s^2 + 10s + 150}$

Comparing this with

$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n^2 = 150$

$\therefore \; \zeta = \frac{10}{2 \times 12.25} = 0.41$

$\omega_n^2 = 150$

$\therefore \; \omega_n = 1225$rad / sec

$2\zeta\omega_n = 10 \qquad \therefore \; \zeta = \frac{10}{2 \times 12.25} = 0.41$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$= 12.25\sqrt{1-0.41^2}$

$= 11.17$rad / sec

Example

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{s(1+sT)}$$

Where, T and K are constants having positive values. By what factor amplifier gain be reduced so that

a) The peak overshoot of unit step response of the system is reduced from 75% to 25%

b) The damping ratio increases from 0.1 to 0.6

Solution

$$G(s) = \frac{K}{s(1+sT)}$$

Let the value of damping ration is ζ_1 when the peak overshoot is 75% and ζ_2 when peak overshoot in 25%

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

For $M_p = 75\%$

$\xi = \xi_1 = 0.091$

and for $M_P = 25\%$ $\zeta = \zeta_2 = 0.4037$

Transfer function

$$= \frac{G(s)}{1+G(s)H(s)} = \frac{K}{Ts^2+s+K} = \frac{C(s)}{R(s)}$$

or $\dfrac{C(s)}{R(s)} = \dfrac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$

Therefore

$$\omega_n = \sqrt{K/T} \text{ and } 2\zeta\omega_n = \frac{1}{T}$$

Example

The open loop transfer function of a unity feedback system is $G(s) = \dfrac{K}{s(1+Ts)}$. Find by what factor the gain K be reduced so that the overshoot is reduced from 60% to 15%

(b) Find by what factor the gain K should be reduced so that the damping ratio is increased from 0.1 to 0.6

Solution

$$\frac{C(s)}{R(s)} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

Comparing with

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n^2 = \frac{K}{T} \text{ and } 2\zeta\omega_n = \frac{1}{T}$$

Therefore $\zeta = \dfrac{1}{2\sqrt{KT}}$

Let

$\zeta_1 = 0.1$ when gain is K_1 and

$\zeta_2 = 0.6$ when gain is K_2

$$\therefore\ 0.1 = \frac{1}{2\sqrt{K_1T}} \text{ and } 0.6 = \frac{1}{2\sqrt{K_2T}}$$

or $K_2 = \dfrac{K_1}{36}$

Therefore the gain should be reduced by a factor 36

Let ζ_1 be the damping ratio when the percentage overshoot is 60%

$$\therefore\ 0.60 = e^{\frac{-\zeta_1\times 3.14}{\sqrt{1-\zeta_1^2}}} \text{ or } \zeta_1 = 0.1604$$

Similarly

$$0.15 = e^{\frac{-\zeta_2\times 3.14}{\sqrt{1-\zeta_2^2}}} \text{ or } \zeta_2 = 0.52 \text{ or}$$

$$\frac{\zeta_1}{\zeta_2} = \sqrt{\frac{K_1}{K_2}}\ \frac{0.1604}{0.52} = \sqrt{\frac{K_2}{K_1}} \quad \text{or}$$

$$K_2 = \frac{K_1}{10.51}$$

Therefore gain should be reduced by a factor 10.51

GATE QUESTIONS(EC)

Q.1 If the characteristic equation of a closed –loop system is $S^2 + 2s + 2 = 0$, then the system is

a)overdamped

b)critically damped

c)underdamped

d)undamped

[GATE -2001]

Q.2 Considers a system with the transfer function $(s) = \frac{s+6}{Ks^2 + s + 6}$. Its damping ratio will be 0.5 when the value of K is

a)2/6 b)3

c)1/6 d)6

[GATE -2002]

Q.3 The transfer function of system is $G(s) = \frac{100}{(s+1)(s+100)}$. For a unit-step input to the system the approximate settling time for 2% criterion is

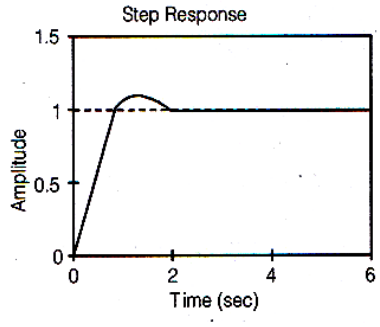
a)100 sec b)4 sec

c)1 sec d)0.01 sec

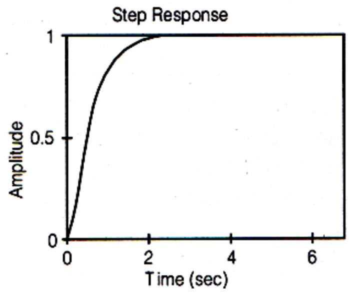
[GATE -2002]

Q.4 A second-order system has the transfer function $\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4}$. With r(t) as the unit-step function, the response c(t) of the system is represented by

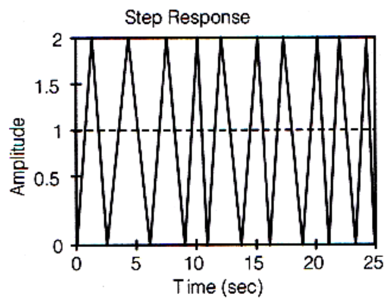
a)



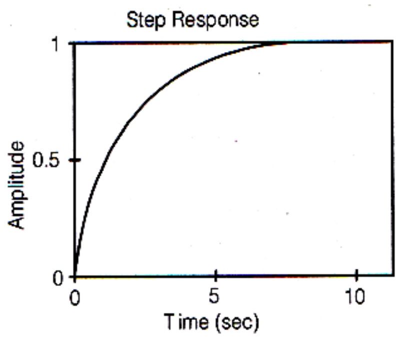
b)



c)



d)



[GATE -2003]

Q.5 A causal system having the transfer function $H(s)=\frac{1}{s+2}$ is excited with $10u(t)$. The time at which the output reaches 99% of its steady state value is

a) 2.7sec b) 2.5 sec
c) 2.3 sec d) 2.1 sec

[GATE -2004]

Q.6 In the derivation of expression for peak percent overshoot, $M_p = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)\times 100$ Which one of the following conditions is NOT required?

a) System is linear and time invariant
b) The system transfer function has a pair of complex conjugate poles and no zeros
c) There is no transportation delay in the system
d) The system has zero initial conditions.

[GATE -2005]

Q.7 A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively

a) 1 and 20 b) 0 and 20
c) 0 and 1/20 d) 1 and 1/20

[GATE -2005]

Q.8 The transfer function of a plant is $(s)=\frac{5}{(s+5)(s^2+s+1)}$. The second-order approximation of $T(s)$ using dominant pole concept is

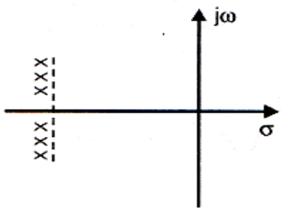
a) $\frac{1}{(s+5)(s+1)}$ b) $\frac{5}{(s+5)(s+1)}$

c) $\frac{5}{s^2+s+1}$ d) $\frac{1}{s^2+s+1}$

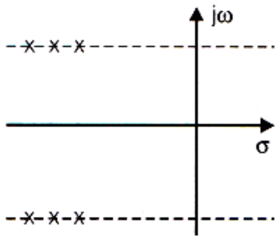
[GATE -2007]

Q.9 Step response of a set of three second-order under damped systems all have the same percentage overshoot. Which of the following diagrams represents the poles of three systems?

a)



b)



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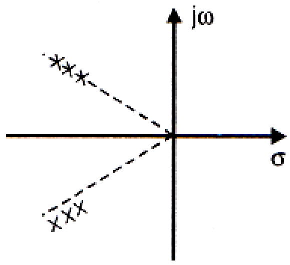

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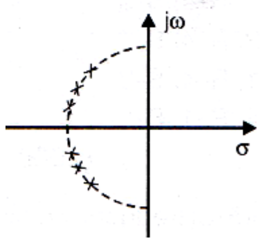


[GATE -2008]

Q.10 Group I lists a set of four transfer functions. Group II gives a list of possible step responses (t). Match the step responses with the corresponding transfer functions.

Group I

$P = \frac{25}{S^2+25}$

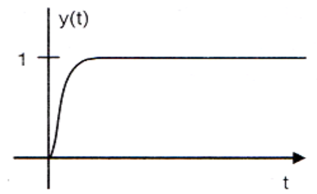
$Q = \frac{36}{S^2+20s+36}$

$R = \frac{36}{S^2+12s+36}$

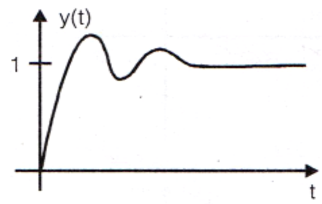
$S = \frac{36}{S^2+7s+49}$

Group II

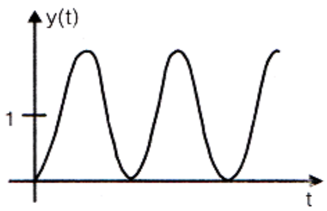
1)



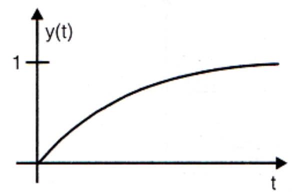
2)



3)



4)



a)P-3,Q-1,R-4,S-2 b)P-3,Q-2,R-4,S-1
c)P-2,Q-1,R-4,S-3 d)P-3, Q-4,R-1,S-2

[GATE -2008]

Q.11 The unit step response of an under-damped second order system has steady state value of -2 .Which one of the following transfer functions has these properties?

a) $\frac{-2.24}{s^2+2.59s+1.12}$

b) $\frac{-3.82}{s^2+1.91s+1.91}$

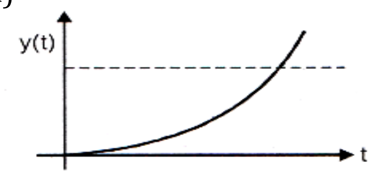
c) $\frac{-2.24}{s^2-2.59s+1.12}$

d) $\frac{-3.82}{s^2-1.91s+1.91}$

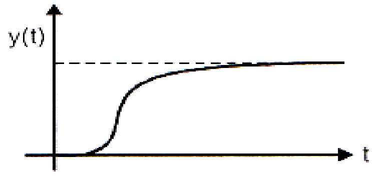
[GATE -2009]

Q.12 The differential equation $100\frac{d^2y}{dt^2}-20\frac{dy}{dt}+y=x(t)$ describes a system with an input x(t) and an output y(t) .The system, which is initially relaxed, is excited by a unit step input. The output y(t) can be represented by the waveform

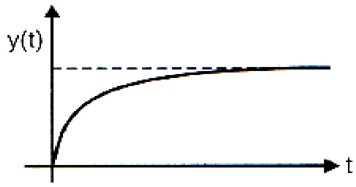
a)



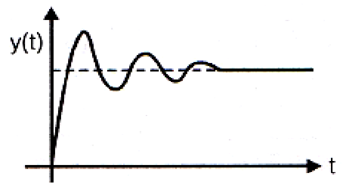
b)



c)

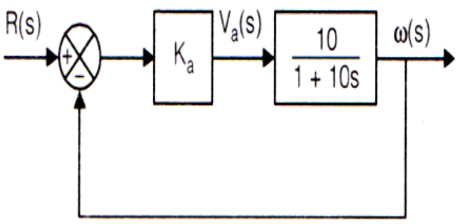


d)



[GATE -2011]

Q.13 The open- loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)}=\frac{10}{1+10s}$. When connected in feedback as shown below,

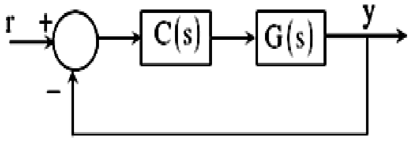


the approximate value of K_a that will reduce the time constant of closed loop system by one hundred times as compared to that of the open- loop system is

a)1 b)5
c)10 d)100

[GATE-2013]

Q.14 For the following feedback system $G(s)=\frac{1}{(S+1)+(S+2)}$. The 2% settling time of the step response is required to be less than 2 seconds.



Which one of the following compensators C(s) achieves this?

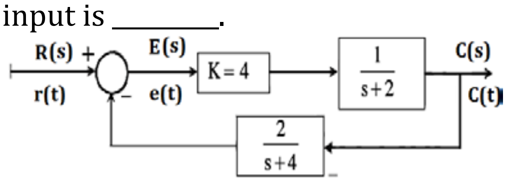
a) $3\left(\frac{1}{s+5}\right)$ b) $5\left(\frac{0.03}{s}+1\right)$
c) $2(s+4)$ d) $4\left(\frac{s+8}{s+3}\right)$

[GATE-2014]

Q.15 The natural frequency of an undamped second-order system is 40 rad/s. If the system is damped with a damping ratio 0.3, the damped natural frequency in rad/s is _______.

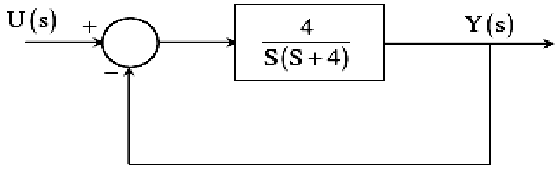
[GATE-2014]

Q.16 The steady state error of the system shown in the figure for a unit step input is ________.



[GATE-2014]

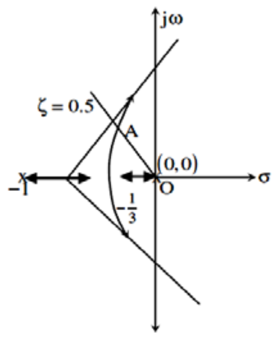
Q.17 For the second order closed-loop system shown in the figure, the natural frequency (in rad/s) is



a) 16 b) 4
c) 2 d) 1

[GATE-2014]

Q.18 The characteristic equation of a unity negative feedback system 1 + KG(s) = 0. The open loop transfer function G(s) has one pole at 0 and two poles at -1. The root locus of the system for varying K is shown in the figure.



The constant damping ratio line, for ζ = 0.5, intersects the root locus at point A. The distance from the origin to point A is given as 0.5. The value of K at point A is__________.

[GATE-2014]

Q.19 The output of a standard second-order system for a unit step input is given as $y(t)=1-\frac{2}{\sqrt{3}}e^{-t}\cos\left(\sqrt{3}t-\frac{\pi}{6}\right)$. The transfer function of the system is

a) $\frac{2}{(s+2)(s+\sqrt{3})}$ b) $\frac{1}{s^2+2s+1}$
c) $\frac{3}{s^2+2s+3}$ d) $\frac{4}{s^2+2s+4}$

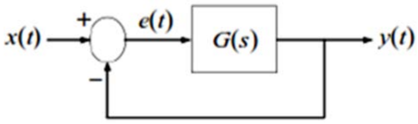
[GATE-2015]

Q.20 The open-loop transfer function of a unity-feedback control system is given by $G(s)=\frac{K}{s(s+2)}$.

For the peak overshoot of the closed-loop system to a unit step input to be 10%, the value of K is ________.

[GATE-2016]

Q.21 For the unity feedback control system shown in the figure, the open-loop transfer function G(s) is given as $G(s)=\frac{2}{s(s+1)}$



The steady state error e_{ss} due to a unit step input is

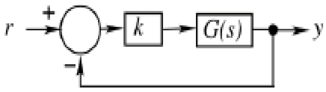
a) 0 b) 0.5

c) 1.0 d) ∞

[GATE-2016]

Q.22 In the feedback system shown below $G(s)=\frac{1}{(s^2+2s)}$.

The step response of the closed-loop system should have minimum setting time and have no overshoot.



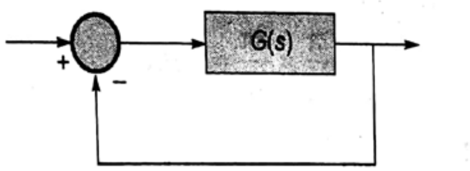
The required value of gain k to achieve this is

[GATE-2016]

Q.23 The open loop transfer function

$G(s)=\frac{(s+1)}{s^p(s+2)(s+3)}$

Where p is an integer, is connected in unity feedback configuration as shown in the figure.



Given that the steady state error is zero for unit step input and is 6 for unit ramp input, the value of the parameter p is ________

[GATE-2017-01]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(c)	(c)	(b)	(b)	(c)	(c)	(a)	(d)	(c)	(d)	(b)	(a)	(c)	(c)
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**					
38.15	0.5	(c)	0.375	(d)	2.86	(a)	1	1					

EXPLANATIONS

Q.1 (c)

$S^2 + 2\xi\omega_n + \omega_n^2 = 0$

$2\xi\omega_n = 2, \xi = \frac{1}{\omega_n}$

$\omega_n = \sqrt{2}$

$= \frac{1}{\sqrt{2}} \xi < 1 (\text{underdamped})$

Q.2 (c)

$G(s) = \frac{s+6}{K\left(s^2 + \frac{s}{K} + \frac{6}{K}\right)}$

Comparing with $S^2 + 2\xi\omega_n + \omega_n^2$

$\omega_n = \sqrt{\frac{6}{K}}$

$2\xi\omega_n = \frac{1}{K}$

$2 \times 0.5 \times \sqrt{\frac{6}{K}} = \frac{1}{K}$

$\Rightarrow \frac{6}{K} = \frac{1}{K^2}$

$K = \frac{1}{6}$

Q.3 (b)

$G(s) = \frac{100}{(s+1)(s+100)}$

Taking dominant pole consideration,

$s = -100$ pole is not taken.

$\therefore G(s) = \frac{100}{s+1}$

Now it is 1st order system

$\therefore t_s = 4T = 4 \times 1 = 4s$

Q.4 (b)

$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4}$

$2\xi\omega_n = 4, \omega_n = 2$

$\therefore \xi = 1 (\text{Critical damping})$

$t_s = \frac{4}{\xi\omega_n} = \frac{4}{1 \times 2} = 2$

Q.5 (c)

$H(s) = \frac{1}{s+2}$

$r(t) = 10u(t)$.

$R(s) = \frac{10}{s}$

$C(s) = H(s).R(s) = \frac{1}{s+2}.\frac{10}{s}$

$\Rightarrow \frac{10}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} 10 =$

$A(s+2) + Bs$

s=0,10=2A

$\Rightarrow A = 5$

$s = -2, 10 = -2B$

$\Rightarrow B = -5$

$\therefore C(s) = \frac{5}{s} - \frac{5}{s+2}$

$c(t) = 5\left[1 - e^{-2t}\right]$

Steady state value when t=0 is 5.99% of steady state value reaches at

$5\left[1 - e^{-2t}\right] = 0.99 \times 5$

$\Rightarrow 1 - e^{-2t} = 0.99$

$e^{-2t} = 0.1$

$\Rightarrow -2t = \ln 0.1$

$\Rightarrow t = 2.3 \sec$

Q.6 (c)

Q.7 (a)

$e_{SS} = \lim_{s \to 0} sE(s), R(s) = \frac{1}{S^2}$

$$= \lim_{s\to 0} s\frac{R(s)}{1+G(s)}$$

$$= \lim_{s\to 0}\frac{1}{S+sG(s)} = \text{finite(given)}$$

$$K_v = \lim_{s\to 0} sG(s)$$

$$e_{ss} = \lim_{s\to 0}\frac{1}{sG(s)} = \text{finite} = 5\% = \frac{1}{20}$$

$\therefore K = 20$

K_v is finite for type 1 system having ramp input.

Q.8 (d)

In dominant pole concept, the factor that has to be eliminated should be in time constant form.

$$\frac{5}{(s+5)(S^2+s+1)} = \frac{5}{5\left(1+\frac{s}{5}\right)(S^2+s+1)}$$

$$= \frac{1}{S^2+s+1}$$

Q.9 (c)

Peak overshoot depends on ξas

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

Where $\xi = \cos^{-1}\theta$

Where θ is the angle made by pole from negative real axis .To make M_p same, θ should be the same.

Q.10 (d)

Comparing the given transfer function with

$$\frac{\omega_n^2}{s^2+2\xi\omega_n S+\omega_n^2}$$

In $P = \frac{25}{s^2+25}\xi = 0$

Therefore P is undamped

$$Q = \frac{36}{s^2+20s+36}$$

$$\Rightarrow \xi = \frac{20}{2\times 6} = 1.67$$

$\Rightarrow$ Qis over damped

$$R = \frac{36}{s^2+12s+36}$$

$$\Rightarrow \xi = \frac{12}{2\times 6} = 1$$

$\Rightarrow$ Ris critically damped

$$S = \frac{49}{s^2+7s+49}$$

$$\Rightarrow \xi = \frac{7}{2\times 7} = 0.5$$

$\Rightarrow$ S is under damped

Q.11 (b)

Steady state Value = -2

Denominator:

$2\xi\omega_n = 1.91, \omega_n^2 = 1.91$

$\Rightarrow \omega_n \cong 1.4$

$$\xi = \frac{1.91}{2\omega_n} = \frac{1.91}{2.8} < 1 \text{ ...under damped}$$

Q.12 (a)

$$100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$$

Taking Laplace transform of both sides

$$100s^2Y(s) - 20sY(s) + Y(s) = X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{100s^2-20s+1} = \frac{1}{(10s-1)^2}$$

Poles are at $s = \frac{1}{10}, \frac{1}{10}$

As poles are on the right –hand side of s-plane so given system is unstable system. Only option (a) represents unstable system.

Q.13 (c)

$$G(s)H(S) = \frac{10K}{1+10s}\text{O.L.T.F}$$

$\tau = 10\,\text{sec}$

$$\text{C.L.T.F } \tau = \frac{10}{100} = 0.1$$

$$\text{C.L.T.F} = \frac{10K}{10K+1+10s}$$

$$\tau = 0.1 = \frac{10K}{10K+1} \Rightarrow K \cong 10$$

Q.14 (c)

By observing the options, if we place other options, characteristic equation will have 3rd order one, where we cannot describe the settling time.

If C(s) = 2(s+4) is considered

The characteristic equation, is

$s^2 + 3s + 2 + 2s + 8 = 0$

$s^2 + 5s + 10 = 0$

Standard character equation

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n^2 = \sqrt{10}; \xi\omega_n = 2.5$

Given, 2% settling time,

$\Rightarrow \frac{4}{\xi\omega_n} < 2 \Rightarrow \xi\omega_n > 2$

Q.15 (38.15 r/ sec)

Given $\omega_n = 40$ r / sec

$\xi = 0.3$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\omega_d = 40\sqrt{1-(0.3)^2}$

$\omega_d = 38.15 r/sec$

Q.16 (0.5)

Given $G(s) = \frac{4}{s+2} H(s) = \frac{2}{s+4}$

For unit step input,

$k_p = \lim_{s\to 0} G(s)H(s)$

$k_p = \lim_{s\to 0}\left(\frac{4}{s+2}\right)\left(\frac{2}{s+4}\right)$

$k_p = 1$

Steady state error = $e_{ss} = \frac{A}{1+k_p}$

$e_{ss} = \frac{1}{1+1}$

$e_{ss} = \frac{1}{2} \Rightarrow 0.50$

Q.17 (c)

Transfer function $\frac{Y(s)}{U(S)} = \frac{4}{S^2 + 4s + 4}$

If we compare with standard 2nd order system transfer function

i.e., $\frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

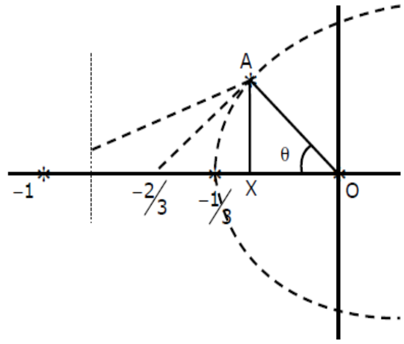
$w_n^2 = 4 \Rightarrow w_n = 2 rad/sec$

Q.18 (0.375)

We know that the co-ordinate of point A of the given root locus i.e. magnitude condition

$|G(s)H(s)| = 1$

Here, the damping factor ξ = 0.5 and the length of 0A = 5 ξ = 0.5



Then in the right angle triangle

$\cos\theta = \frac{OX}{OA} \Rightarrow \cos 60 = \frac{OX}{0.5} \Rightarrow OX = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{AX}{OA} \Rightarrow \sin 60 = \frac{AX}{0.5} \Rightarrow AX = \frac{\sqrt{3}}{4}$

So, the co-ordinate of point A is

$-1/4 + j\sqrt{3}/4$

Substituting the above value of A in the transfer function and equating to 1

i.e. by magnitude condition,

$\left|\frac{k}{s(s+1)^2}\right|_{s=-1/4+j\sqrt{3}/4} = 1$

$k = \sqrt{\frac{1}{16}+\frac{3}{16}} \cdot \left(\sqrt{\frac{9}{16}+\frac{3}{16}}\right)^2$

$k = 0375$

Q.19 (d)

Here $\xi\omega_n = 1$

$\sqrt{1-\xi^2} = \frac{\sqrt{3}}{2}$

$\xi = \frac{1}{2}$

$\omega_n = 2$

Q.20 (2.86)

K = 2.86

Peak over shoot 10%

$\Rightarrow e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1$

$\Rightarrow \left[\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right]^2 = [1n0.1]^2$

$\Rightarrow \frac{1-\xi^2}{\xi^2} = \left[\frac{\pi}{1n0.1}\right]^2$

$\Rightarrow \frac{1}{\xi^2} = 1 + \left[\frac{\pi}{1n0.1}\right]^2$

$\Rightarrow \frac{1}{\xi^2} = 2.86$

$\Rightarrow \xi^2 = \frac{1}{2.86} = 0.34$

$\Rightarrow \xi = 0.59$

$\rightarrow$ The characteristic equation of above transfer function is

$s^2 + 2s + k = 0$

Comparing with standard equation

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\Rightarrow 2\xi\sqrt{k} = 2$

$\Rightarrow \sqrt{k} = \frac{2}{2\xi}$

$\Rightarrow k\left[\frac{2}{2\xi}\right]^2 = \frac{1}{\xi^2} = 2.86$

$k = 2.86$

Q.21 (a)

For unit step input ess $= \frac{1}{1+k_p}$

$k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{2}{s(s+1)} = \infty$

So, $e_{ss} = \frac{1}{1+\infty} = 0$

Q.22 (1)

Minimum setting time and no overshoot implies case of critical damping.

At critical damping $\zeta = 1$.

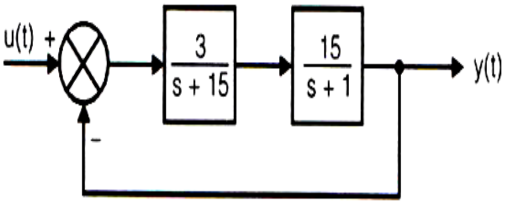
$H(s) = \frac{k}{s^2 + 2s + k}$

$\omega_n = \sqrt{k}$

$2\zeta\omega_k = 2 \Rightarrow 2.1 \times \sqrt{k} = 2 \Rightarrow k = 1$

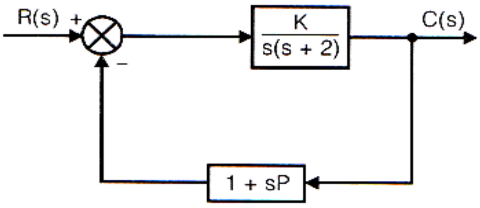
GATE QUESTIONS(EE)

Q.1 The block diagram shown in figure gives a unity feedback closed loop control system. The steady state error in the response of the above system to unit step input is



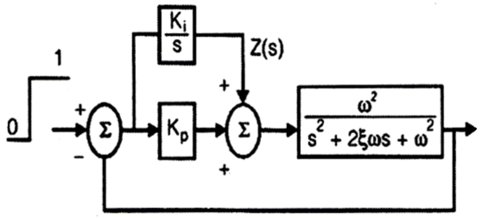
a) 25% b) 0.75%
c) 6% d) 33%
[GATE-2003]

Q.2 The block diagram of a closed loop control system is given by figure. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_n of 5rad/sec, are respectively equal to



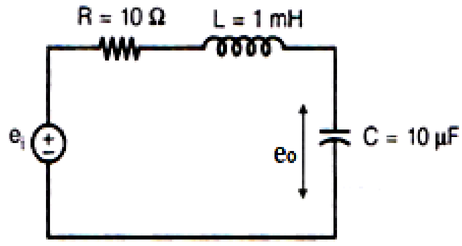
a) 20 and 0.3 b) 20 and 0.2
c) 25 and 0.3 d) 25 and 0.2
[GATE-2004]

Q.3 Consider the feedback system shown below which is subjected to a unit step input. The system is stable and has following parameters $k_p = 4$, $k_i = 10$, $\omega = 500$ and $\xi = 0.7$. The steady state value of Z is



a) 1 b) 0.25
c) 0.1 d) 0
[GATE-2007]

Statement for common data Questions Q.4 and Q.5:
R-L-C circuit shown in figure



Q.4 For a step-input e_i, the overshoot in the output e_0 will be
a) 0, since the system is not under damped
b) 5%
c) 16%
d) 48%
[GATE-2007]

Q.5 If the above step response is to be observed on a non-storage CRO, then it would be best have the e_i as a
a) step function
b) square wave of 50Hz
c) square wave of 300Hz
d) square wave of 2.0KHz
[GATE-2007]

Q.6 The transfer function of a linear time invariant system is given as

$$G(s)=\frac{1}{s^2+3s+2}$$

The steady state value of the output of the system for a unit impulse input applied at time instant t=1 will be

a) 0 b) 0.5
c) 1 d) 2

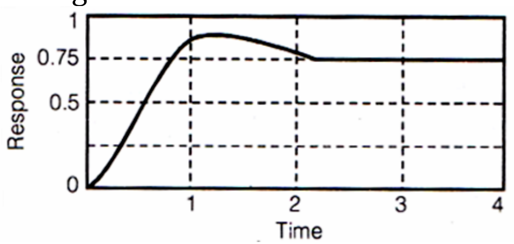
[GATE-2008]

Q.7 The transfer function of a system is given as $\frac{100}{s^2+20s+100}$. The system is

a) an overdamped system
b) an underdamped system
c) a critically damped system
d) an unstable system

[GATE-2008]

Q.8 The unit-step response of a unity feedback system with open loop transfer function G(s)=K/((s+1)(s+2)) is shown in the figure. The value of K is



a) 0.5 b) 2
c) 4 d) 6

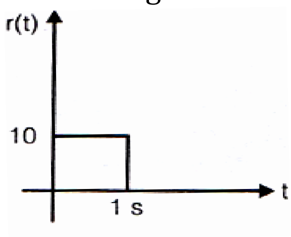
[GATE-2009]

Q.9 For the system $\frac{2}{(s+1)}$, the approximate time taken for a step response to reach 98% of its final value is

a) 1s b) 2s
c) 4s d) 8s

[GATE-2010]

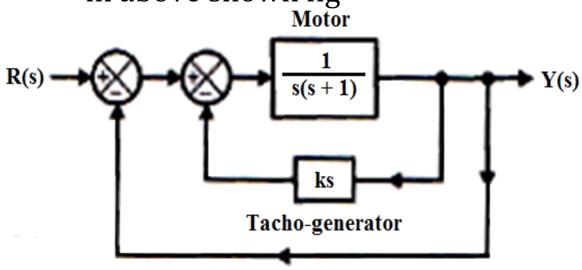
Q.10 The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input r(t) having a magnitude of 10 and a duration of one second, as shown in the figure is



a) 0 b) 0.1
c) 1 d) 10

[GATE-2011]

Q.11 A two-loop position control system in above shown fig



The gain k of the Tacho-generator influences mainly the

a) peak overshoot
b) natural frequency of oscillation
c) phase shift of the closed loop transfer function at very low frequencies ($\omega\to0$)
d) phase shift of the closed loop transfer function at very high frequencies($\omega\to\infty$)

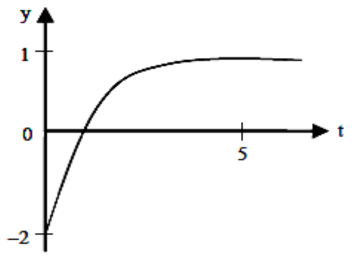
[GATE-2011]

Q.12 An open loop control system results in a response of e^{-2t} (sin5t + cos5t) for a unit impulse input. The DC gain of the control system is ________.

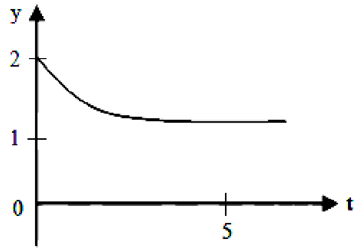
[GATE-2015]

Q.13 The unit step response of a system with the transfer function $G(s)=\frac{1-2s}{1+s}$ is given by which one of the following waveforms?

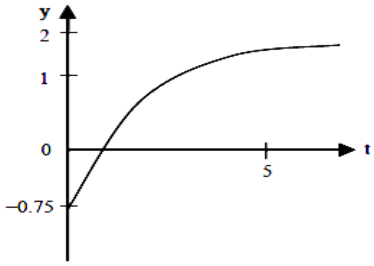
a)



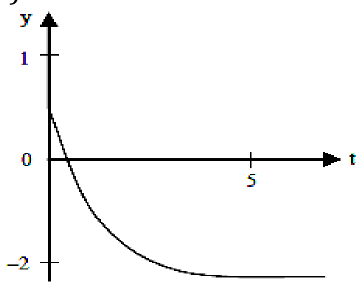
b)



c)



d)



[GATE-2015]

Q.14 A second-order real system has the following properties:

a) the damping ratio ζ = 0.5 and undamped natural frequency $\omega_n = 10 \text{rad/s}$

b) the steady state value of the output, to a unit step input, is 1.02.

The transfer function of the system is

a) $\dfrac{102}{s^2 + 5s + 100}$

b) $\dfrac{102}{s^2 + 10s + 100}$

c) $\dfrac{100}{s^2 + 10s + 100}$

d) $\dfrac{102}{s^2 + 5s + 100}$

[GATE-2016]

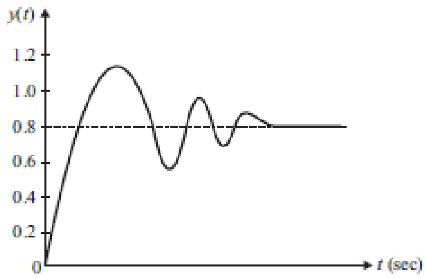
Q.15 Consider a unity feedback system with forward transfer function is given by,

$$G(s) = \frac{1}{(s+1)(s+2)}$$

The steady state error in the output of the system for a unit-step input is ____ (up to 2 decimal places).

[GATE-2018]

Q.16 The unit step response $y(t)$ of a unity feedback system with open loop transfer function $G(s)H(s) = \dfrac{K}{(s+1)^2 (s+2)}$ is shown in the figure. The value of K is ________ (up to 2 decimal places).



[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(a)	(d)	(a)	(c)	(c)	(a)	(c)	(d)	(c)	(a)	(a)	0.241	(a)	(b)
15	**16**												
0.67	8												

EXPLANATIONS

Q.1 (a)

$$G(s)=\left(\frac{3}{s+15}\right)\times\left(\frac{15}{s+1}\right)$$

$$=\frac{45}{(s+1)(s+15)}\ \&\ H(s)=1$$

Open loop transfer function

$$=G(s).H(s)=\frac{45}{(s+1)(s+15)}$$

The system is type-0

Steady state error to unit-step input where

k_p = Position error constant

$$=\underset{s\to 0}{LT}\,G(s)H(s)$$

$$\Rightarrow\quad =\underset{s\to 0}{LT}\,G(s)H(s)$$

$$k_p=\underset{s\to 0}{LT}\frac{45}{(s+1)(s+15)}=3$$

$$e_{ss}=\frac{1}{1+k_p}=\frac{1}{1+3}$$

$$=0.25 or 25\%$$

Q.2 (d)

$$G(s)=\frac{k}{s(s+2)}\ and\ H(s)=1+sP$$

Closed-loop transfer function

$$T(s)=\frac{G(s)}{1+G(s)H(s)}$$

$$=\frac{k/s(s+2)}{1+\frac{k}{s(s+2)}.(1+sP)}$$

$$=\frac{k}{s(s+2)+k(1+sP)}$$

$$\Rightarrow\quad T(s)=\frac{k}{s^2+(2+kP)s+k}$$

So, characteristic equation

$$=s^2+(2+kP)s+P$$

Comparing with standard equation

$$=s^2+2\xi\omega_n s+\omega^2_n$$

$$k=\omega^2_n=5^2=25$$

(where ω_n = undamped natural frequency)

$$2\xi\omega_n=2+kP$$

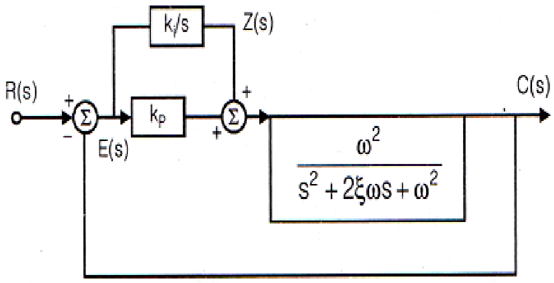
$$\Rightarrow\ 2\times 0.7\times 5=2+25P$$

(where ξ = damping ratio)

P=0.2

Q.3 (a)

Step input $\Rightarrow R(s)=1/s$



$$G(s)=\left(k_p+\frac{k_i}{s}\right)\left(\frac{\omega^2}{s^2+2\xi\omega s+\omega^2}\right)$$

and H(s)=1

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{G(s)}{1+G(s)}$$

$$E(s)=R(s)-C(s)$$

$$=R(s)\left[1-\frac{C(s)}{R(s)}\right]$$

$$=R(s)\left[1-\frac{G(s)}{1+G(s)}\right]=\frac{R(s)}{1+G(s)}$$

$$Z(s)=\frac{k_i}{s}.E(s)=\frac{\frac{k_i}{s}.R(s)}{1+G(s)}$$

Steady state value of Z

$$Z_{ss}=\underset{s\to 0}{LT}\,s\,Z(s)$$

$$= \underset{s \to 0}{LT} \frac{s.\frac{k_i}{s}.\frac{1}{s}}{1+\left(k_p+\frac{k_i}{s}\right)\left(\frac{\omega^2}{s^2+2\xi\omega s+\omega^2}\right)}$$

$$= \frac{k_i}{k_i.\frac{\omega^2}{\omega^2}} = 1$$

$$2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{L} \times \frac{1}{2\omega_n} = \frac{R}{L} \times \frac{\sqrt{LC}}{2} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

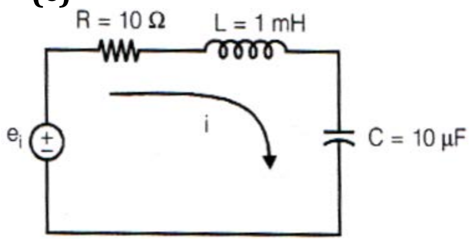
$$\xi = \frac{10}{2}\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}} = 0.5$$

Overshoot $= P_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$= e^{-\pi\times0.5\sqrt{1-0.5^2}}$

$= 0.163$ or $16.3\%M$

Q.4 (c)



$$e_i = R_i + L\frac{di}{dt} + \frac{1}{c}\int idt$$

Taking Laplace transform

$$E_i(s) = \left(R + Ls + \frac{1}{Cs}\right)I(s)$$

$$I(s) = \frac{E_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$e_0 = \frac{1}{c}\int idt$$

$$\Rightarrow E_0(s) = \frac{1}{Cs}I(s) = \frac{1}{Cs}\left[\frac{E_i(s)}{R + Ls + \frac{1}{Cs}}\right]$$

$$t_0(s) = \frac{E_i(s)}{RCs + LSC^2 + 1}\frac{E_0(s)}{E_i(s)}$$

$$= \frac{1}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Characteristic eq.

$$\Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Comparing with $s^2 + 2\xi\omega_n s + \omega_n{}^2$

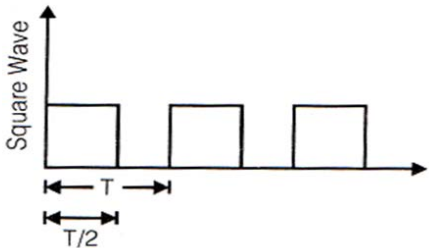
$$\omega_n = \frac{1}{\sqrt{Lc}}$$

Q.5 (c)

$$\omega_n = \frac{1}{\sqrt{Lc}}$$

$$= \frac{1}{\sqrt{1\times10^{-3}\times10\times10^{-6}}}$$

$$= 10^4 \text{ rad/ sec}$$



Settling time $(t_s) = \frac{4}{\xi\omega_n}$

$$= \frac{4}{10^4\times0.5} = 0.8\text{msec}$$

For a square wave T/2 should be greater than t_s

For $f_1 = 50\text{Hz}$

$$\Rightarrow \frac{T_1}{2} = \frac{1}{2\times50} = 10\text{ms} >> t_s$$

For $f_2 = 300\text{Hz}$

$$\Rightarrow \frac{T_2}{2} = \frac{1}{2\times300} = 1.67\text{ms} > t_s$$

For $f_3 = 2\text{kHz}$

$$\Rightarrow \frac{T_3}{2} = \frac{1}{2\times2\times10^3} = 0.25\text{ms} > t_s$$

Therefore, it would be best to have e_i as a square wave of 300Hz.

Q.6 (a)

r(t) unit impulse applied at t

$= 1 = \delta(t-1) R(s) = 1[r(t)] = e^{-s}$

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2+3s+2}$$

$$C(s) = R(s)G(s) = \frac{e^{-s}}{s^2+3s+2}$$

Steady state value of output, using final value theorem

$$c_{ss} = \underset{s\to 0}{LT} s\,C(s)$$

$$= \underset{s\to 0}{LT} \frac{se^{-s}}{s^2+3s+2} = 0$$

Q.7 (c)

$$M(s) = \frac{100}{s^2+20s+100}$$

Comparing with standard form,

$$M(s) = \frac{\omega_n^{\,2}}{s^2+2\xi\omega_n s+\omega_n^{\,2}}$$

$\therefore 2\xi\omega_n = 20$

$\omega_n = 100$

$\xi = 1$

$\therefore \omega_n = 10$

$\therefore$ The system is critically damped.

Q.8 (d)

Steady state value of response=0.75

Input is unit-step so steady state error

$e_{ss} = 1-0.75 = 0.25$

$$\text{Error} = E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$\text{Where } G(s) = \frac{k}{(s+1)(s+2)}$$

$$H(s) = 1 \text{ and } R(s) = \frac{1}{s}$$

Steady state error using find value theorem

$$e_{ss} = \underset{s\to 0}{LT} sE(s)$$

$$= \underset{s\to 0}{LT} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{LT} \frac{s.\frac{1}{s}}{1+\frac{k}{(s+1)(s+2)}} = \frac{1}{1+k/2}$$

$$\Rightarrow 0.25 = \frac{1}{1+k/2}$$

$$\Rightarrow 1+\frac{k}{2} = 4$$

$\Rightarrow k = 6$

Q.9 (c)

$$\frac{C(s)}{R(s)} = \frac{2}{s+1}$$

$$R(s) = \frac{1}{s}(\text{step input})$$

$$C(s) = R(s)\left(\frac{2}{s+1}\right) = \frac{2}{s(s+1)}$$

$$= 2\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$C(t) = \mathcal{L}^{-1}[C(s)] = 2[1-e^{-t}]$

Final value of $C(t) = C_{ss} = 2$

98% of $C_{ss} = 0.98\times 2 = 1.96$

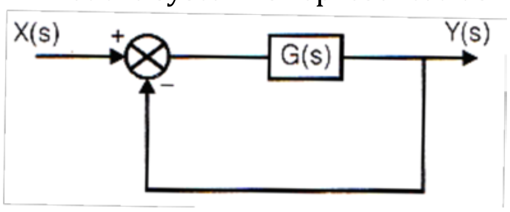
Let t=T, the response reaches 98% of its final values.

$1.96 = 2[1-e^{-t}]$

$T \approx 4\text{sec}.$

Q.10 (a)

Let the system is represented as



$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)}$$

H(s) =1(unity feedback)

$$\text{Error} = E(s) = \frac{X(s)}{1+G(s)}$$

Steady-state error for

$$x(s)=\frac{1}{s}, e_{ss}=0.1$$

$$e_{ss}=\underset{s\to 0}{LT}\, sE(s)$$

$$=\underset{s\to 0}{LT}\frac{sX(s)}{1+G(s)}$$

$$\Rightarrow 0.1$$

$$=\underset{s\to 0}{LT}\frac{s\times\frac{1}{s}}{1+G(s)}$$

$$\Rightarrow \underset{s\to 0}{LT}\frac{1}{1+G(s)}0.1$$

When input

$$x(t)=10\left[u(t)-u(t\text{-}1)\right]$$

$$x(s)=10\left[\frac{1}{s}-\frac{e^{-s}}{s}\right]$$

$$=\frac{10}{s}\left[1-e^{-s}\right]$$

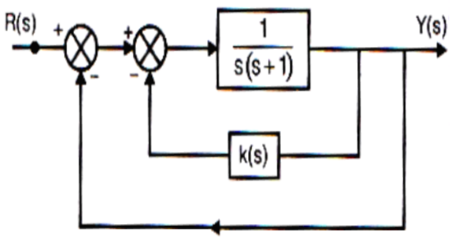
$$e_{ss}=\underset{s\to 0}{LT}\, sE(s)$$

$$=\underset{s\to 0}{LT}\frac{s\times(s)}{1+G(s)}$$

$$=\underset{s\Rightarrow 0}{LT}\frac{s\times\frac{10}{s}\left[1-e^{-s}\right]}{1+G(s)}$$

$$\Rightarrow e_{ss}=\underset{s\to 0}{LT}\frac{10(1-e^{-s})}{1+G(s)}=0$$

Q.11 (a)



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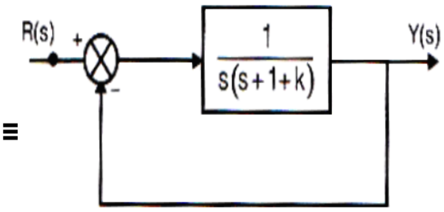


$$G(s)=\frac{1}{s(s+1+k)}\text{ and } H(s)=1$$

Characteristic equation

$1+G(s)H(s)=0$

$$\Rightarrow 1+\frac{1}{s(s+1+k)}=0$$

$$\Rightarrow s(s+1+k)+1=0$$

$$\Rightarrow s^2(R+1)s+1=0$$

Comparing with

$$s^2+2\xi\omega_n s+\omega_n^2$$

natural frequency $\omega_n=1$ remains constant and does not depend on k

$$2\xi\omega_n=k+1$$

$$\xi=\frac{k+1}{2}$$

damping ratio depends on k

peak overshoot $=m_p=e^{-\xi\pi/\sqrt{1-\xi^2}}$

Since m_p depends on ξ which depends on k. Hence peak overshoot is influenced by gain (k) of the techo-generator.

Q.12 (0.241)

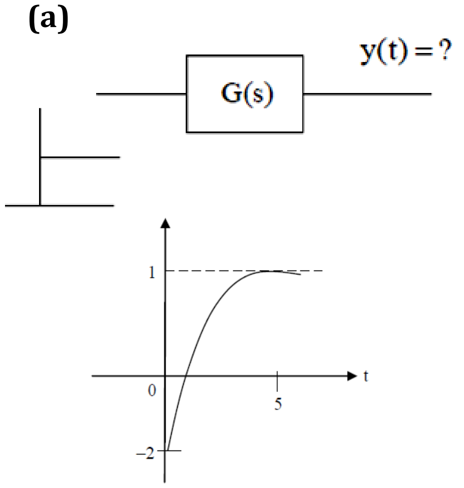
$$g(t)=e^{-2}[\sin 5t+\cos 5t]$$

$$G(s)=\frac{5}{(s+2)^2+5^2}+\frac{s+2}{\{s+2\}+5^2}$$

DC gain means $\left|G(s)\right|_s=0$

$$G(0)=\frac{5}{2^2+5^2}+\frac{2}{2^2+5^2}=\frac{7}{29}$$

Q.13 (a)



$$Y(s)=G(s)\times U(s)$$

$$Y(s)=\frac{(1-2s)}{(1+s)}.\frac{1}{s}$$

$$Y(s)=\frac{A}{(s)}+\frac{B}{(s+1)}$$

$$A=1, B=-3$$

$$y(t)=u(t)-3e^{-t}u(t)$$

$$y(t)=\left(1-3e^{-t}\right)u(t)$$

Q.14 (b)

The standard 2nd order T/F is

$$\left[K\frac{\omega_n^2}{s^2+2\xi\omega_n^2+\omega_n^2}\right]$$

It is given that $\xi=0.5$ & $\omega_n=10$

$$G(s)=K\frac{100}{s^2+10s+100}$$

Now to satisfy the steady state O/P 1.02

$$y(\infty)=\lim_{s\to 0}\frac{1}{s}\left(\frac{100}{s^2+10s+100}\right)K=1.02\Rightarrow K=1.$$

$$G(s)=\frac{1.02\times 100}{s^2+10s+100}=\frac{102}{s^2+10s+100}$$

Q.15 0.67

Given: The open loop transfer function for given unity feedback system is

$$G(s)H(s)=\frac{1}{(s+1)(s+2)}$$

$$\left[H(s)=1\right]$$

Steady state error for unit step input

$$(e_{ss})=\frac{1}{1+K_p}$$

Where, K_p (Position error coefficient)

$$=\lim_{s=0}G(s)H(s)$$

$$K_p=\lim_{s=0}\frac{1}{(s+1)(s+2)}=\frac{1}{2}$$

Steady state error

$$(e_{ss})=\frac{1}{1+\frac{1}{2}}=\frac{1}{1.5}=0.67$$

Hence, steady state error in the output of the system for a unit-step input is **0.67.**

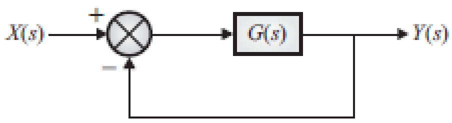
Q.16 8

Given $G(s)=\dfrac{K}{(s+1)^2(s+2)}$

CLTF is given by,

$$\frac{Y(s)}{X(s)}=\frac{G(s)H(s)}{1+G(s)H(s)}=\frac{G(s)}{1+G(s)}$$

[Unit feedback system]



$$\frac{Y(s)}{X(s)}=\frac{\frac{K}{(s+1)^2(s+2)}}{1+\frac{K}{(s+1)^2(s+2)}}$$

$$\frac{Y(s)}{X(s)}=\frac{K}{(s+1)^2(s+2)+K}$$

$$Y(s)=\frac{1}{s}\times\frac{K}{(s+1)^2(s+2)+K}$$

From final value theorem,

$$y(\infty)=\lim_{s\to 0}sY(s)=0.8$$

[From time response shown in the figure steady state value in time domain is 0.8]

$$\frac{K}{2+K}=0.8$$

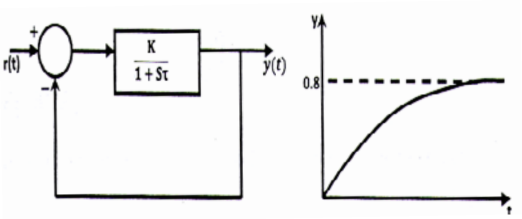
$$K=1.6+0.8K$$

$$K=8$$

Hence, the value of K is **8.**

GATE QUESTIONS(IN)

Q.1 If a first order system and its time response to a unit step input are as shown below. The gain K is



a) 0.25 b) 0.8
c) 1 d) 4

[GATE-2008]

Statement for linked Answer Questions Q.2 & Q.3:
A unity feedback system has open loop transfer function $G(s)=\frac{100}{s(s+p)}$. The time at which the response to a unit step input reaches its peak is $\frac{\pi}{8}$ seconds.

Q.2 The damping coefficient for the closed loop system is
a) 0.4 b) 0.6
c) 0.8 d) 1

[GATE-2008]

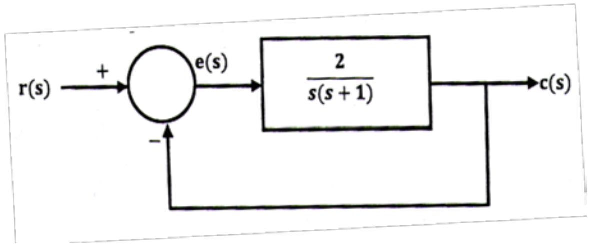
Q.3 The value of p is
a) 6 b) 12
c) 14 d) 16

[GATE-2008]

Q.4 A unity feedback system has the transfer function $\frac{K(s+b)}{S^2(s+20)}$. The value of b for which the loci of all the three roots of the closed loop characteristic equation meet at a single point is
a) 10/9 b)20/9
c) 30/9 d)40/9

[GATE-2009]

Q.5 A unit ramp input is applied to the system shown in the shown in the adjoining figure. The steady state error in its output is



a)0 b)0.5
c)1 d)2

[GATE-2010]

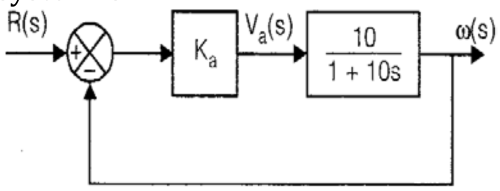
Q.6 A unity feedback system has an open loop transfer function $G(s)\frac{k}{s(s+3)}$ .The value of k that yields a damping ratio of 0.5 for the closed loop system is
a)1 b)3
c)5 d)9

[GATE-2010]

Q.7 The unit-step response of a negative unity feedback system with the open- loop transfer function is $G(s)=\frac{6}{s+5}$
a) $1-e^{-5t}$ b) $6-6e^{-5t}$
c) $\frac{6}{5}-\frac{6}{5}e^{-5t}$ d) $\frac{6}{11}-\frac{6}{11}e^{-11t}$

[GATE-2011]

Q.8 The open- loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)}=\frac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of closed loop system by one hundred times as compared to that of the open- loop system is



a) 1
b) 5
c) 10
d) 100

[GATE-2013]

Q.9 A plant has an open-loop transfer function,

$$G_P(s)\frac{20}{(S+0.1)(S+2)(S+100)}$$

The approximate model obtained by retaining only one of the above poles, which is closest to the frequency response of the original transfer function at low frequency is

a) $\frac{0.1}{s+0.1}$
b) $\frac{2}{s+2}$
c) $\frac{100}{s+100}$
d) $\frac{20}{s+0.1}$

[GATE-2014]

Q.10 A system with transfer function $G(s)=\frac{1}{s^2+1}$ has zero initial conditions. The percentage overshoot in its step response is ________ %

[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
(d)	(b)	(b)	(b)	(b)	(d)	(d)	(c)	(a)	100

EXPLANATIONS

Q.1 (d)

$$T(s)=\frac{k}{(ST+1+K)} \text{ and}$$

$$C(s)=\frac{k}{S(ST+k+1)}$$

$$C_{ss}=\lim_{S\to 0} S.C(s)=\frac{k}{k+1}$$

$$=0.8=\frac{4}{5}\Rightarrow 5k$$

$$=4k+4\Rightarrow k=4$$

Q.2 (b)

$$T(s)=\frac{100}{(s^2+Ps+100)}$$

$$\omega_n=10 \text{ and } t_p=\frac{\pi}{8}=\frac{\pi}{\omega_n\sqrt{1-\delta^2}}$$

$$\Rightarrow \xi=3/5$$

Q.3 (b)

$$0.6=\frac{P}{20}\Rightarrow P=12$$

Q.4 (b)

$$G(s)=\frac{k(s+b)}{s^2(s+20)}$$

For unity feedback, characteristic equation is $1+G(s)=0$

$$\Rightarrow s^3+20s^2+ks+kb=0$$

$$\Rightarrow k=\frac{(s^3+20s^2)}{(s+b)}$$

We need to find the breakaway point.

So, $\frac{dk}{ds}=0$

Or, $-\frac{dk}{ds}=0$

$$(s+b)(3s^2+40s)-(s^3+20s^2)=0$$

$$\Rightarrow 3s^2+40s^2+3bs^2+40bs-s^3$$
$$+20s^2=0$$

$$\Rightarrow 2s^3+(3b+20)s^2+40bs=0$$

Now, s = 0 is not the breakaway point

So, $2s^3+(3b+20)s+40b=0$

For all the three root loci to meet at a single point, we need that this equation has equal roots.

So, $(3b+20)^2=4\times 2\times 40b$

$$\Rightarrow 9b^2+120b+400=320b$$

$$\Rightarrow 9b^2-200b+400=0$$

$$\Rightarrow 9b(b-20)-10(b-20)=0$$

$$\Rightarrow (9b-20)(b-20)=0$$

So, $b=20$ or $\frac{20}{9}$

But b=20is not the required value of b because it will cancel out an open –loop pole so, $b=\frac{20}{9}$ is the required value

Q.5 (b)

$$G(s)\frac{2}{s(s+1)}$$

For unit Ramp input

$r(t)=R.t.u(t) R=1$

Velocity error constant,

$$K_v=\underset{s\to 0}{Lt} S.G(s)=2$$

$$\therefore e_{ss}=\frac{R}{K_v}=\frac{1}{2}=0.5$$

Q.6 (d)

$$T(s)=\frac{K}{(s^2+3s+K)}$$

$$\omega_n\sqrt{K} \text{ and } \xi=\frac{3}{2\omega_n}=\frac{3}{2\sqrt{K}} \text{ or}$$

$K=\frac{9}{4\xi^2}$ For $\xi=0.5 \Rightarrow K=9$.

$\%overshoot = e^{\frac{-r\xi}{\sqrt{1-\xi^2}}} \times 100 = 100$

Q.7 (d)

$$CLTF=\frac{C(s)}{R(s)}=\frac{6}{s+11}$$

The unit step response is given by

$$C(s)=\frac{6}{s(s+11)}=\frac{6/11}{s}-\frac{6/11}{s+11}$$

$$C(t)=\left(\frac{6}{11}-\frac{6}{11}e^{-11t}\right)u(t)$$

Q.8 (c)

$$G(s)H(S)=\frac{10K}{1+10s} \text{ O.L.T.F}$$

$$\tau=10\,\text{sec}$$

$$\text{C.L.T.F } \tau=\frac{10}{100}=0.1$$

$$\text{C.L.T.F}=\frac{10K}{10K+1+10s}$$

$$\tau=0.1=\frac{10K}{10K+1} \Rightarrow K \cong 10$$

Q.9 (a)

Given

$$G(s)=\frac{20}{(s+0.1)(s+2)(s+100)}$$

$$=\frac{20}{(0.1)(2)(100)\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}$$

$$G(s)=\frac{1}{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}$$

Approximate model at low frequency

then $G(s)=\frac{1}{\left(1+\frac{s}{0.1}\right)}$

$$\Rightarrow G(s)=\frac{0.1}{(s+0.1)}$$

Q.10 (100)

Comparing the denominator, $\xi=0$

3 TIME DOMAIN STABILITY

3.1 INTRODUCTION

The stability of a system relates to its response to inputs or disturbances. A system which remains in a constant state unless affected by an external action and which returns to a constant state when the external action is removed can be considered to be stable.

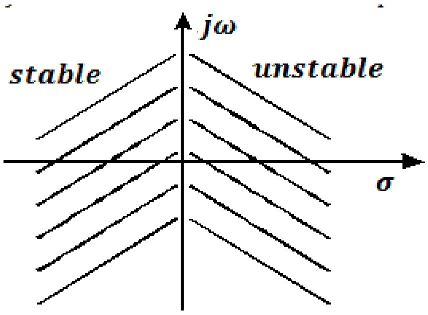
In other words, a system is said to be stable if

a) Bounded input gives bounded o/p

b) O/P should reduce to zero when input is removed.

The stability of the system can be determined with the knowledge location of poles of the system.

1) If all the poles of the system lie in the left half of s plane, then the system is stable.
2) If there are non-repeated poles on the jω axis, system is marginally stable.
3) If there are repeated poles of the system on jω axis, system is unstable.
4) If there is one or more than one pole in R.H. of imaginary axis, system is unstable.



Regions of Locations of Roots for satble & unstable Systems

3.1.1 ASYMPTOTICALLY STABLE SYSTEM

The stationary impulse response, h(t), is zero

i.e. $\lim_{t\to\infty} h(t) = 0$

For a system to be asymptotically or absolutely stable, each of the poles of the transfer function lies strictly in the left half plane (has strictly negative real part).

3.1.2 MARGINALLY STABLE SYSTEM

The stationary impulse response is different from zero, but limited

i.e. $0 < \lim_{t\to\infty} h(t) < \infty$

For a system to be marginally stable, one or more poles lies on the imaginary axis (have real part equal to zero), and all these poles are distinct. Besides, no poles lie in the right half plane. A marginally stable system has its output oscillates with constant frequency & amplitude.

3.1.3 UNSTABLE SYSTEM

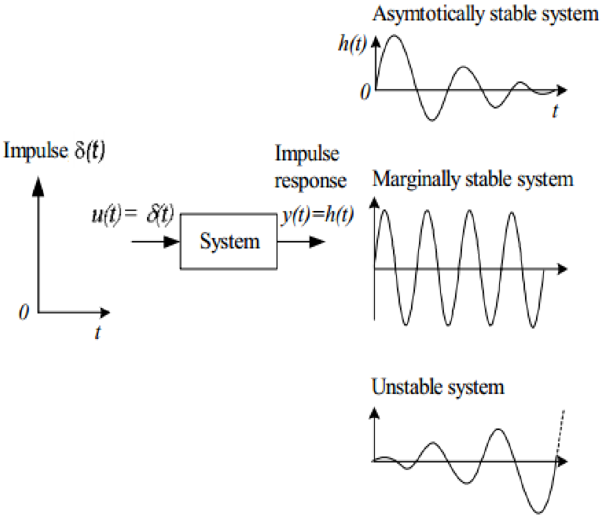
The stationary impulse response is unlimited

i.e. $\lim_{t\to\infty} h(t) = \infty$

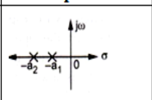
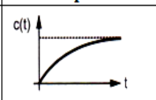
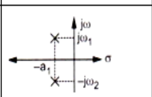
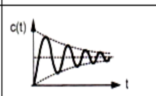
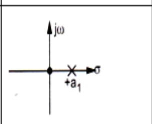
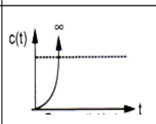
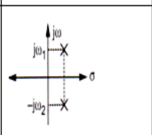
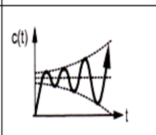
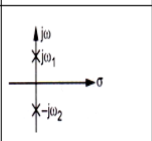
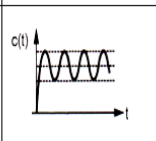
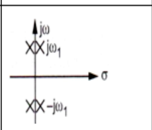
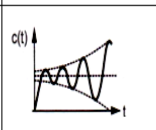
For a system to be unstable, at least one pole lies in the right half plane (has real part greater than zero). Or: There are multiple poles on the imaginary axis.

Note:

A stable system is either asymptotically stable or marginally stable.

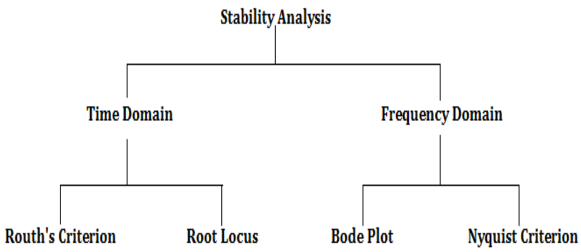


3.1.4 STEP RESPONSE FOR DIFFERENT POLE LOCATION

Sr. No.	Nature of closed loop poles	Location of poles in s-plane	Response to step input	Stability
1	Real, negative			Absolutely stable
2	Complex conjugate with –ve real part			Absolutely stable
3	Real , positive			Unstable
4	Complex conjugate with positive real part			Unstable
5	Non repeated pair on imaginary axis			Marginally or critically stable
6	Repeated pair on imaginary axis			Unstable

3.2 ROUTH'S STABILITY CRITERION

There are certain other methods to verify the stability of the control systems.



The Routh stability criterion provides a convenient method of determining control systems stability. It determines

1) The number of characteristic roots within the unstable right half of the s-plane, and the number of characteristic roots in the stable left half.
2) The number of roots on the imaginary axis. It does not locate the roots. The Routh-Hurwitz test is performed on the denominator of the transfer function, it is the characteristic equation. For instance, in a closed-loop transfer function with G(s) in the forward path, and H(s) in the feedback loop, we have:

$$T(s)=\frac{G(s)}{1+G(s)H(s)}$$

If we simplify this equation, we will have an equation with a numerator N (s), and a denominator D(s):

$$T(s)=\frac{N(s)}{D(s)}$$

The Routh-Hurwitz criteria will focus on the denominator polynomial D(s). Here are the three tests of the Routh-Hurwitz Criteria. For convenience, we will use n as the order of the polynomial (the value of the highest exponent of s in D(s)). The equation D(s) can be represented generally as follows:

$$D(s)=a_0s^n+a_1s^{n-1}+\ldots+a_{n-1}s^1+a_n$$

3.2.1 ALGORITHM FOR APPLYING ROUTH'S STABILITY CRITERION

The algorithm described below, like the stability criterion, requires the order of D(s) to be finite.

1) Remove the roots at origin to obtain the polynomial, and multiply by −1 if necessary, to obtain

$a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s^1 + a_n = 0$

Where, $a_0 \neq 0 \ \& a_n > 0$.

2) If the order of the resulting polynomial is at least two and any coefficient a_i is zero or negative, the polynomial has at least one root with nonnegative real part. To obtain the precise number of roots with nonnegative real part, proceed as follows. Arrange the coefficients of the polynomial, and values subsequently calculated from them as shown below:

s^n	a_0	a_2	a_4	a_6	$\ldots$
s^n	a_1	a_3	a_5	a_7	$\ldots$
s^n	b_1	b_2	b_3	b_4	$\ldots$
s^n	c_1	c_2	c_3	c_4	$\ldots$
s^n	d_1	d_2	d_3	d_4	$\ldots$
M	M	M	M	M	$\ldots$
s^n	e_1	e_2			
s^n	f_1				
s^n	g_1				

Where the coefficients b_i are generated until all subsequent coefficient are zero.

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

Similarly, cross multiply the coefficient of the two previous rows to obtain the c_i, d_i, etc.

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

Until the n^{th} row of the array has been completed. Missing coefficient are replaced by zeros. The resulting array is called the Routh array. The powers of s are not considered to be part of the array. We can think of them as labels. The column beginning with a_0 is considered to be the first column of the array.

The Routh array is seen to be triangular. It can be shown that multiplying a row by a positive number to simplify the calculation of the next row does not affect the outcome of the application of the Routh criterion.

3) Count the number of sign changes in the first column of the array. It can be shown that a necessary and sufficient condition for all roots of $a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s^1 + a_n = 0$ to be located in the left-half plane is that all the a_i are positive and all of the coefficients in the first column be positive.

Note:

Let us apply Routh's stability criterion to the following third-order polynomial:

$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$

Where, all the coefficients are positive numbers. The array of coefficients becomes

s^3	a_0	a_2
s^2	a_1	a_3
s^1	$\frac{a_1 a_2 - a_0 a_3}{a_1}$	
s^0	a_3	

The condition that all roots have negative real parts is given by

$$\boxed{a_1 a_2 > a_0 a_3}$$

Example:

Consider the following polynomial

$s^4 + 2s^2 + 3s^2 + 4s + 5 = 0$

Comment on stability.

Solution:
Let us follow the procedure just presented and construct the array of coefficients. (The first two rows can be obtained directly from the given polynomial. The remaining terms are obtained from these. If any coefficients are missing, they may be replaced by zeros in the array.)

s^4	1	3	5
s^3	2	4	0
s^2	1	5	
s^1	-6		
s^0	5		

The number of changes in sign of the coefficients in the first column is two (1 to -6&-6 to 5). This means that there are two roots with positive real parts. So the system is unstable.

Note:
The result is unchanged when the coefficients of any row are multiplied or divided by a positive number in order to simplify the computation.

3.2.2 SPECIAL CASE 1

If a first-column term in any row is zero, but the remaining terms are not zero or there is no remaining term, then the zero term is replaced by a very small positive number $\in$ and the rest of the array is evaluated.

Example:
Consider a characteristics equation $s^5+s^4+2s^3+2s^2+3s+5$
Determine the stability.
Solution:
The array of coefficients is

s^5	1	2	3
s^4	1	2	5
s^3	0		
s^2			
s^1			
s^0			

While solving for coefficient of 3rd row we get zero while the other element is -2 i.e. non zero so it is special case-1 of routh's array. For further calculations take this zero as small +ve number ∈.

s^5	1	2	3
s^4	1	2	5
s^3	$\in$	-2	
s^2	$\dfrac{2\in+2}{\in}$	5	
s^1	$\dfrac{-4\in-4-5\in^2}{2\in+2}$		
s^0	5		

Now, first element of 3rd row is $\in=0$ (will be considered as positive)
First element of 4th row is

$$\lim_{\in\to0}\frac{2\in+2}{\in}=+\infty$$

First element of 5th row is

$$\lim_{\in\to0}\frac{-4\in-4-5\in^2}{2\in+2}=-2$$

There are 2 sign changes
$+\infty$ to -2 & -2 to $+5$ Hence there are 2 poles on RHS of s-plane. Therefore the system is unstable.

3.2.3 SPECIAL CASE 2

When all the elements of any row are zero, it is special case-2. The Routh's array can be solved by following the procedure

- Form an auxiliary equation using the elements of the row just above the row with all zero elements.
- Take derivative of the auxiliary equation & the coefficients of the resultant equation will replace the row with all zeros.

Example:
$s^5+2s^4+24s^3+48s^2-25s-50=0$
Solution:
The array of coefficients is

s^5	1	24	-25	
s^4	2	48	-50 ⟵	Auxiliary polynomial P(s)

s^3 | 0 0

Here all the coefficients of s^3 row are zero. Now form the auxiliary equation using the coefficients of s^4 row. The auxiliary equation P(s) is

$P(s) = 2s^4 + 48s^2 - 50$

Now differentiate the equation we get

$$\frac{dp(s)}{ds} = 8s^3 + 96s$$

The terms in the s^3 row are replaced by the coefficients of the last equation, that is, 8 and 96. The array of coefficients then becomes

s^5	1	24	-25
s^4	2	48	-50
s^3	8	96	$\leftarrow$ coefficient of $\frac{dp(s)}{ds}$
s^2	24	-50	
s^1	112.7	0	
s^0	-50		

There is 1 sign hence the system is unstable.

Note:

Whenever there is all zero elements in any row, there will be

1) Either pair of real roots with opposite sign (1 +ve & 1 –ve)
2) Or complex conjugate roots
3) Or a pair of roots on imaginary axis

   This can be found out by solving the auxiliary equation.

   Now, solving the auxiliary equation of the above example $2s^4 + 48s^2 - 50 = 0$.

   We get $s^2 = 1$, $s^2 = -25$

   or $s = \pm 1$, $s = \pm j5$

   These two pairs of roots are a part of the roots of the original equation.

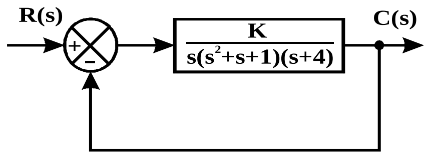
   Clearly, the original equation has one root with a positive real part (s = +1) also there are 2 roots on imaginary axis.

3.2.4 APPLICATION OF ROUTH'S STABILITY CRITERIA

1) To determine the range of gain K for stable system.

Example:

Consider a system with closed loop transfer function



$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2+s+1)(s+4)+K}$$

Therefore, the characteristic equation is

$$s(s^2+s+1)(s+4)+K=0$$

$$s^4+5s^3+5s^2+4s+K=0$$

Now, the Routh's array for this equation is

s^4	1	5	K
s^3	5	4	12
s^2	21/5	K	
s^1	$\left(\frac{84}{5}-5K\right)/\frac{21}{5}$		
s^0	K		

For the system to be stable all the elements of the 1st column should be +ve

i.e. $K > 0 \,\&\, (84/5 - 5K) > 0 \Rightarrow K < 84/25$

Therefore for stability, K should lie in the range

$$0 < K < \frac{84}{25}$$

If K becomes greater than 84/25, the system becomes unstable.

Note:

In the above example K cannot be greater than 84/25, this is the last value of K for which the system will be stable & this is called as marginal value of K i.e. for K = 84/25 the system will be marginally stable (on the verge of instability)

2) To determine the frequency of oscillations

In the above example the auxiliary polynomial for $K_{marginal} = 84/25$ is

$$(21/5)s^2 + 84/25 = 0$$

$$\Rightarrow s^2 = -4/5$$

$$\Rightarrow s = j\sqrt{4/5}$$

Put s = jω

$\therefore \quad \omega = \sqrt{4/5} \text{rad/sec}$

Example:

The characteristic equation of a feedback control system is

$s^3 + 3Ks^2 + (K + 2)s + 4 = 0$.

Determine the range of K for which system is stable

Solution:

Routh's array is

s^3	1	K + 2
s^2	3K	4
s^1	$\frac{3K^2+6K-4}{3K}$	
s^0	4	

For stability, 3K > 0 i.e. K > 0

And $3K^2 + 6K - 4 > 0$ i.e. K> -1 ± 1.53

i.e. K > 0.53

The range of K is thus ∞ > K > 0.53

Example:

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{s(s+3)(s^2+s+1)}$$

Determine the values of K that will cause sustained oscillations in the closed loop system. Also find the oscillation frequency.

Solution:

The characteristic equation is

$$1 + G(s) = 1 + \frac{K}{s(s+3)(s^2+s+1)} = 0$$

$s(s + 3)(s^2 + s + 1) + K = 0$

$s^4 + 4s^3 + 4s^2 + 3s + K = 0$

The Routh's array is

s^4	1	4	K
s^3	4	3	0
s^2	$\frac{13}{4}$	K	
s^1	$\frac{\left(\frac{39}{4} - 4K\right)}{\frac{13}{4}}$		
s^0	K		

The condition for system stability is

$$K > 0 \ \& \left(\frac{39}{4} - 4K\right) > 0$$

Therefore for stability, K should lie in the range

When K = $\frac{39}{16}$ here will be a zero at the first entry in the fourth row. This will indicate presence of symmetrical roots, which as shown below, will be pure imaginary.

$K_{marginal} = \frac{39}{16}$ will cause sustained oscillations.

The subsidiary equation of third row for $K = \frac{39}{16}$, is $\frac{13}{4}s^2 + \frac{39}{16} = 0$

s = ± j 0.75

Thus the frequency of sustained oscillations is 0.75 rad/sec

Example:

$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

The Routh's array is

s^6	1	4	5	2	$2s^4 + 4s^2 + 2 = 0$
s^5	3	6	3		$s^4 + 2s^2 + 1 = 0$
s^4	2	4	2		$4s^3 + 4s = 0$
s^3	0	0	0		$s^3 + s = 0$
	1	1			$2s^2 + 2 = 0$
					$s^2 + 1 = 0$
s^2	2	2			$2s = 0$
s^1	0	0			
	2				
s^0	1				

There are two rows which become zero and there is no sign change in the first column of the Routh's array.

Note:

We cannot comment on stability until the roots of auxiliary equation are not known.

Now the auxiliary equations is

$s^4 + 2s^2 + 1 = 0$

$\Rightarrow s = \pm j \ \& \ s = \pm j$

As there are repeated roots on imaginary axis, the system is unstable.

Example:

A feedback control system has an open loop transfer function of

$$G(s)H(s) = \frac{Ke^{-s}}{s(s^2+2s+1)}$$

Determine the maximum value of K for the close loop stability

Solution:

For low frequencies

$e^{-s} = (1-s)$

$$G(s)H(s) = \frac{K(1-s)}{s(s^2+2s+1)}$$

$$1+G(s)H(s) = \frac{K(1-s)}{s(s^2+2s+1)} = 0$$

$$\therefore \quad s(s^2+2s+1)+K(1-s)=0$$

$$s^3+2s^2+s+K-Ks=0$$

$$s^3+2s^2+s(1-K)+K=0$$

The Routh's array is

s^3	1	(1-K)
s^2	2	K
s^1	(2(1-K)-K)/2	
s^0	K	

For stability K > 0 and

2(1 – K) – K > 0 or 2 – 3K > 0

$$K < \frac{3}{2}$$

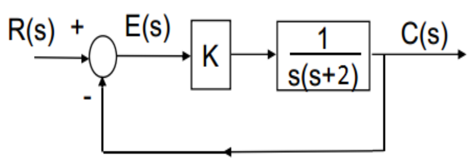
Hence the restriction on K is

$$0 < K < \frac{3}{2}$$

3.3 ROOT LOCUS

Root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly gain with in a feedback system. This is a technique used as a stability criterion in the field of control systems developed by Walter R. Evans which can determine stability of the system.

Consider the closed loop system:



The open loop poles of the system are:

$s = 0 \ \& \ S = -2$

The closed-loop transfer function is:

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+2s+K}$$

The characteristic equation is:

$$s^2+2s+K=0$$

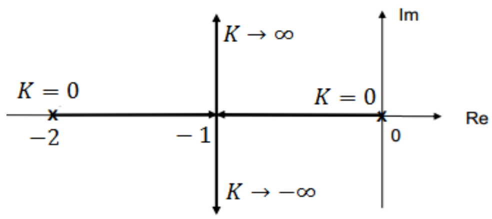
The roots of the characteristics equation (i.e. closed loop poles) are:

$$S = -\frac{2}{2} \pm \frac{\sqrt{2^2-4K}}{2} = -1 \pm \frac{\sqrt{4-4K}}{2}$$

From the above equations, it is clear that the roots depend on gain K of the system. If the gain is varied from 0 to ∞, the closed loop poles will also change. Following table shows the variation of closed loop poles with gain K:

K	s_1	s_2
0	0	-2
0.5	$\frac{-2+\sqrt{2}}{2}$	$\frac{-2-\sqrt{2}}{2}$
1	-1	-1
2	$-1+i$	$-1-i$
5	$-1+2i$	$-1-2i$
10	$-1+3i$	$-1-3i$
50	$-1+7i$	$-1-7i$
∞	$-1+\infty i$	$-1-\infty i$

When all the values of closed loop poles are plotted on a graph we get:



The locus of the roots of the closed loop system (closed loop poles) as a function of a gain of K, as it is varied from 0 to infinity results in **Root-Locus**.

3.3.1 ANGLE CONDITION

Consider a characteristics equation

$$1+G(s)H(s)=0$$

$$\Rightarrow \ G(s)H(s) = -1+j0$$

Now, $\angle G(s)H(s) = \pm(2q+1)180^\circ$

Note:

$-1+j0$ is a point on –ve real axis which can be traced at the angles $\pm180^o, \pm540^o, \pm900^o$ ……… with respect to the +ve real axis.
Angle condition is used to whether a point in s-plane lies on root locus or not.

Example:

Check whether the point $s=-0.75$ lies on the root locus of

$G(s)H(s)=\dfrac{K}{s(s+2)(s+4)}$ or not.

Solution:

Put $s=-0.75$ in the given transfer function

$$G(s)H(s)=\frac{K}{-0.75(-0.75+2)(-0.75+4)}$$

Now, $\angle G(s)H(s)=\dfrac{0^o}{180^o.0^o.0^o}=-180^o$

Therefore $s=-0.75$ lies on root locus.

3.3.2 MAGNITUDE CONDITION

Consider a characteristics equation

$1+G(s)H(s)=0$

$\Rightarrow \; G(s)H(s)=-1+j0$

Now, $|G(s)H(s)|=1$

Once it is confirmed that a point s lies on the root locus using angle condition, we can find the corresponding gain K of the system at that point.

Example:

Find the gain of the system with transfer Function $G(s)H(s)=\dfrac{K}{s(s+2)(s+4)}$ at a point $s=-0.75$.

Solution:

In the last example we confirmed that $s=-0.75$ lies on the root locus & now the corresponding gain can be found out using magnitude condition.

Put $s=-0.75$ in the given transfer function

$$G(s)H(s)=\frac{K}{-0.75(-0.75+2)(-0.75+4)}$$

Now,

$$|G(s)H(s)|_{s=-0.75}=\frac{K}{0.75\times1.25\times3.25}=1$$

$\therefore K=3.04$

3.3.3 RULES FOR CONSTRUCTING ROOT LOCUS

Rule 1: Symmetry

The root locus is always symmetrical about real axis.

Rule 2: Number of branches

The number of branches is equal to the number of poles of the open-loop transfer function.

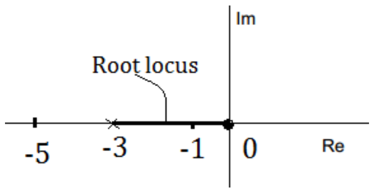
e.g. $G(s)=\dfrac{1}{s(s+2)(s+4)}$ the poles are at $0,-2,-4$ As the total number of open loop poles is 3, the number of branches of root locus is 3.

Rule 3: Real-axis root locus

If the total number of open loop poles and zeros to the right of any point on the real axis is odd, then this point lies on the root locus.

e.g. $G(s)=\dfrac{s}{(s+3)}$

poles $\Rightarrow s=-3$ & zeroes $\Rightarrow s=0$



a) To the point $s=-1$, on the right side there is 1 open loop zero & no pole i.e. total = 1 + 0 = 1 which is odd. Hence the point $s=-1$ lies on the root locus.
b) To the point $s=-5$, on the right side there is 1 open loop zero & 1 open loop pole i.e. total = 1+ 1 = 2 which is even. Hence the point $s=-5$ does not lie on the root locus.

Rule 4: Root locus end-points

The locus starting point (K=0) are at the open-loop **poles** and the locus ending points (K=∞) are at the open loop **zeros.** If there P number of open loop poles & Z number of open loop zeros, then m number of branches will end at zeros and $P - Z$ branches terminate at infinity.

e.g. $G(s) = \dfrac{s+3}{s(s+2)(s+4)}$

poles $\Rightarrow s = 0, -2, -4$; total $= 3$

zeroes $\Rightarrow s = -3$; total $= 1$

There will be 3 (no. of open loop poles) root locus branches, out of which 1 (no. of open loop zero) branch will terminate at a zero &$3 - 1 = 2$ number of branches will terminate at infinity.

Rule 5: Slope of asymptotes

The branches of root locus which terminates at infinity follows the path along the straight line called asymptotes. The number of a symptotes = P – Z.

The angles of asymptotes with +ve real axis are: $\varnothing = \dfrac{180(2q+1)}{(P-Z)}$

Where, q = 0, 1, 2, 3... (P-Z-1)

e.g. $G(s) = \dfrac{s+3}{s(s+2)(s+4)}$ poles

$\Rightarrow s = 0, -2, -4$

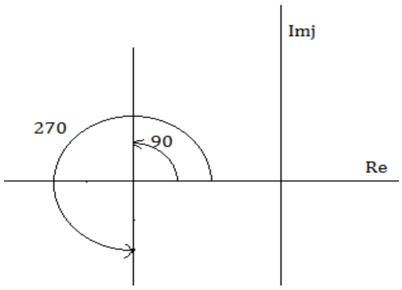
Total = 3 & zeroes $\Rightarrow s = -3$ total $= 1$

The number of asymptotes is $3 - 1 = 2$.

The angles of asymptotes with real axis are:

$$\varnothing_1 = \frac{180(2\times 0+1)}{(3-1)} = 90^o$$

$$\varnothing_2 = \frac{180(2\times 1+1)}{(3-1)} = 270^o$$



Rule 6: Intersection of asymptotes

The asymptotes for the root locus intersects at a point on the real axis called centroid & it is given by

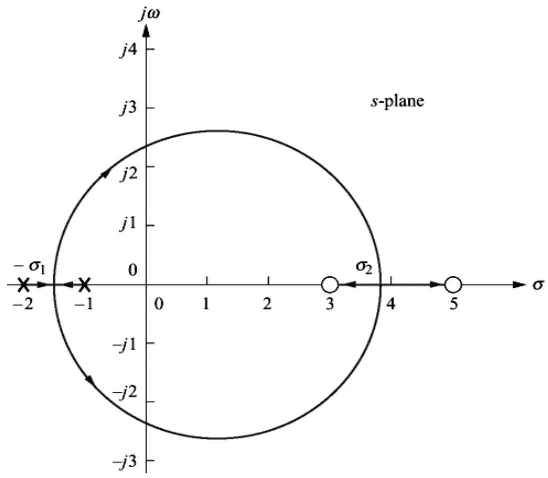
$$\sigma = \frac{\sum \text{real part of poles of } G(s)H(s) - \sum \text{real part of zeros of } G(s)H(s)}{(P-Z)}$$

e.g. $G(s) = \dfrac{s+3}{s(s+2)(s+4)}$

$$\sigma = \frac{(-4-2-0)-(-3)}{(3-1)} = \frac{-6+3}{2} = \frac{-3}{2}$$

Rule 7: Real Axis Breakaway and Break-in points:

A breakaway point is a point on root locus where multiple roots of the characteristics equation occur. Numerous root loci appear to break away from the real axis as the system poles move from the real axis to the complex plane. At other times the loci appear to return to the real axis as a pair of complex poles becomes real.



The figure shows a root locus leaving the real axis between -1 and −2 and returning to the real axis between +3 and +5. The point where the locus leaves the real axis, $-\sigma_1$, is called the breakaway point, and the point where the locus returns to the real axis, σ_2, is called break-in point.

Note:

At the breakaway or break-in point, the branches of the root locus form an angle of 180/n with the real axis, where n is the number of closed loop poles arriving or departing from the single breakaway or break-in point on the real axis. Thus for the poles shown in the figure the branches at the breakaway point form 90°angles with the real axis.

Procedure to find breakaway or break-in point:

1) Let $G(s)=\dfrac{K}{s(s+a)}$

   Write the characteristics equation for the above transfer function:

   $s^2+as+K=0$

2) Find the expression for K

   $K=-(s^2+as)$

3) Find $\dfrac{dK}{ds}$ & equate it to zero.

   $\dfrac{dK}{ds}=-\left(2s+a\right)=0$

4) The roots of the $\dfrac{dK}{ds}=0$ gives the breakaway or break-in point.

   Root is $s=-a/2$, hence breakaway point is $-a/2$.

5) For the breakaway or break-in point if G(s) satisfies angle condition, it is a valid break away point.

Note:

For a point on root locus between two adjacently placed poles, there exists a breakaway point if the number of poles & zeros to the right hand side is even.

Example

Determine the breakaway points for the root locus of $G(s)H(s)=\dfrac{K}{s\left(s+1\right)\left(s+4\right)}$

Solution:

The characteristics equation is

$s^3+5s^2+4s+K=0$

Now, $K=-s^3-5s^2-4s$

$\therefore \dfrac{dK}{ds}=-3s^2-10s-4=0$

Roots of this equation are

$s=-0.46$ & $s=-2.86$

Now, substituting these values in the expression of K

$K=+0.88$ for $s=-0.46$

$K=-6.06$ for $s=-2.86$

As K is +ve for $s=-0.46$, it is a valid breakaway point & $s=-2.86$ is not a valid breakaway point.

Rule 8: Intersection with imaginary axis

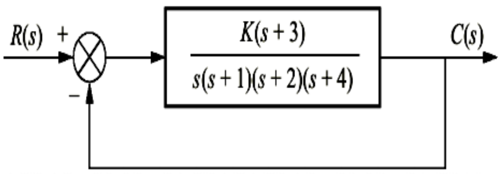
The jω axis crossing is a point on the root locus that separates the stable operation of the system from the unstable operation. The value of ω at the axis crossing yields the frequency of oscillation. The intersection of root-locus with imaginary axis can be obtained by solving the auxiliary equation from the Routh's array.

e.g. If the auxiliary equation is $s^2+4=0$, solving for s we get $s=\pm j2$.

Therefore the root locus intersects at $+j2$ & $-j2$.

Example:

For the system of the following figure, find the frequency and gain, K, for which the root locus crosses the imaginary axis. For what range of K is the system stable?



Solution:

The closed loop transfer function for the system is

$$T(s)=\frac{K(s+3)}{s^4+7s^3+14s^2+\left(8+K\right)s+3K}$$

The routh's array will be

s^4	1	14	3K
s^3	7	8+K	
s^2	90 -K	21K	
s^1	$\dfrac{-K^2-65K+720}{90-K}$		
s^0	21 K		

For system to be stable all the elements of 1st column should be positive.

i.e. $21K>0$

$\Rightarrow K > 0$ and $-K^2 - 65K + 720 > 0$

$\Rightarrow K < 9.65$

For marginal stability, $K_{mar} = 9.65$

Auxiliary equation is:

$(90 - K)s^2 + 21K = 0$

$\therefore 80.35s^2 + 202.7 = 0$

Solving equation we get, $s = \pm j1.59$

Therefore the root locus touch the imaginary axis at $\pm j1.59$ for $K_{mar} = 9.65$.

Rule 9: Angles of Departure and Arrival

In order to sketch the root locus more accurately, we want to calculate the root locus departure angle from the complex poles and the arrival angle to the complex zeros.

The angle of departure & arrival are given by $\varnothing_d = 180^\circ - \varnothing$ and $\varnothing_a = 180^\circ + \varnothing$

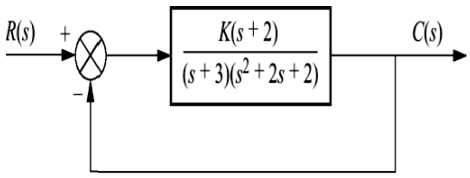
Where $\varnothing = \sum \varnothing_P - \sum \varnothing_Z$

$\sum \varnothing_P$ is the angle made by the other poles with the pole at which angle of departure is to be calculated or with the zero at which angle of arrival is to be calculated.

$\sum \varnothing_Z$ is the angle made by the other zeros with the pole at which angle of departure is to be calculated or with the zero at which angle of arrival is to be calculated.

Example:

Given the unity feedback system of following figure, find the angle of departure from the complex poles and sketch the root locus.

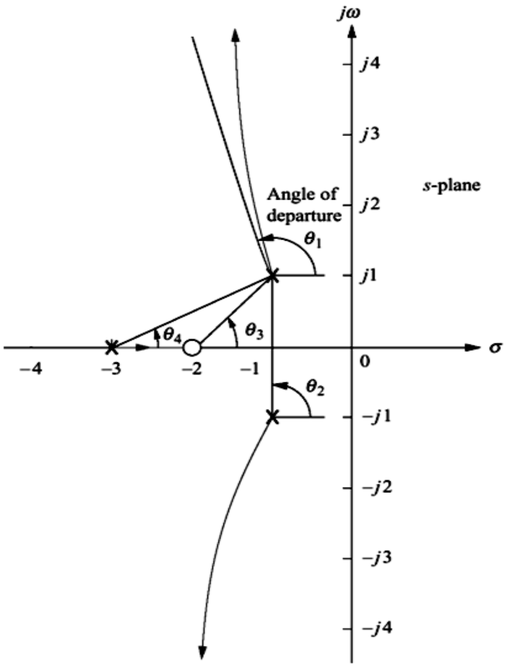


Solution:

Using the poles and zeros of

$G(s) = K(s+2)/[(s+3)(s^2+2s+2)]$

as plotted in the figure, we calculate the sum of angles drawn to a point ε close to the complex pole, $-1 + j1$, in the second quadrant.



Here

$\theta_2 = 90^\circ$,

$\theta_4 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^\circ$ &

$\theta_3 = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$

Now, $\varnothing = \sum \varnothing_P - \sum \varnothing_Z = \theta_2 + \theta_4 - (\theta_3)$

$= 90^\circ + 26.56^\circ - (45^\circ) = 71.56^\circ$

$\therefore \varnothing_d = 180^\circ - \varnothing = 180^\circ - 71.56^\circ$

$= 108.4°$

Example

Determine number asymptotes for

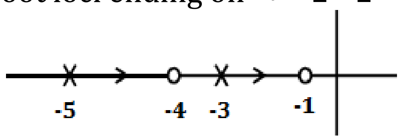
$G(s)H(s) = \frac{(s+1)(s+4)}{(s+3)(s+5)}$.

Solution

No. of open loop poles n = 2

No. of open loop zeros m = 2

No. of root loci ending on $\infty = 2 - 2 = 0$



No. of asymptotes = 0

Example

Given that $KG(s)=\dfrac{K}{s(s+2)(s+4)}$ Sketch the root locus of 1 + KG(s) = 0 and compute the value of K that will yield a "dominant" second order behavior with a damping ratio, $\xi = 0.7$.

Solution

We have, n = 3 and m = 0.

Open loop zero: none

Open loop poles: s = 0, −2, −4

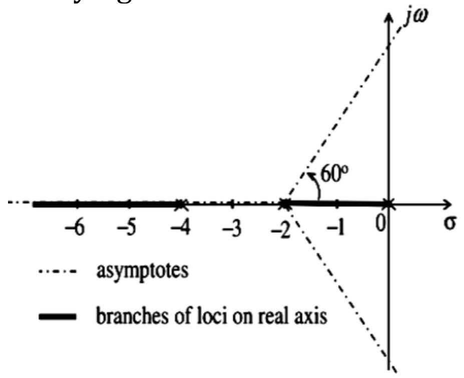
1) As= 3, the number of root locus branches=3.
2) n – m = 3 – 0 = 3, therefore 3 branches will terminate at infinity.
3) Number of asymptotes = 3.
4) Angle of asyptotes are

$$\phi_1 = \frac{(2\times 0+1)\times 180^0}{3} = 60^0$$

$$\phi_2 = \frac{(2\times 1+1)\times 180^0}{3} = 180^0$$

$$\phi_3 = \frac{(2\times 2+1)\times 180^0}{3} = 300^0$$

5) Centroid $\sigma = \dfrac{(-4-2-0)-0}{3} = -2$
6) Identifying the root locus branches



7) Crossing the imaginary axis

The characteristics equation is

$s^3 + 6s^2 + 8s + K = 0$

s^3	1	8	0
s^2	6	K	
s^1	$\dfrac{48-K}{6}$		
s^0	K		

When K = 48, the s^1 row will have all the elements zero. Auxiliary equation will be $6s^2 + 48 = 0$

$\therefore\ s = j2\sqrt{2}$

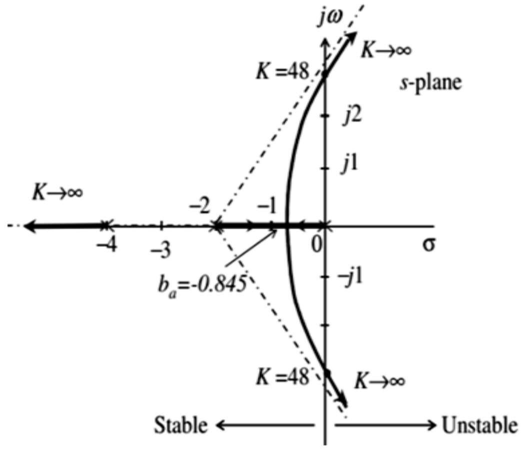
8) Breakaway point:

The breakaway point is the solution of $\dfrac{dK}{ds} = 0$

$K = -s(s+2)(s+4) = -(s^3 + 6s^2 + 8s)$

$\dfrac{dK}{ds} = -(3s^2 + 12s + 8)$

Solving we get $s = -0.845$



Example

$G(s)H(s) = \dfrac{K(s+1)(s+3)}{(s+4)(s+5)}$ find Break away point/Break in point.

Solution:

The characteristics equation is

1 + G(s)H(s) = 0

$$\therefore\quad 1 + \frac{K(s+1)(s+3)}{(s+4)(s+5)} = 0$$

$$K = \frac{s^2 + 9s + 20}{s^2 + 4s + 3}$$

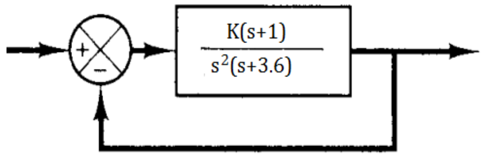
$$\frac{dk}{ds} = \frac{\{(s^2+4s+3)(2s+9) - (s^2+9s+20)(2s+4)\}}{(s^2+9s+20)^2} = 0$$

$(s_1 = -2.43, -4.3)$

Both the points satisfy angle condition hence they are valid break away point/break in point. $s = -4.3$ lies between 2 poles hence it is a breakaway point while s = −2.43 lies between 2 zeros hence it is break in point.

Example:

Sketch the root loci for the system shown in Figure



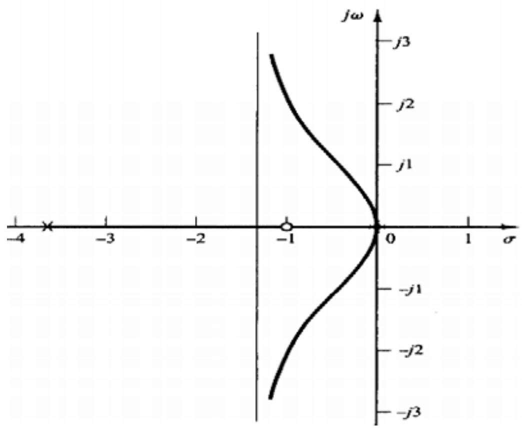
Solution:

1) No of open loop poles $= P = 3, s = 0, 0, -3.6$
2) No of open loop zeros $= Z = 1, s = -1$
3) No of root locus branches $= P = 2$
4) No of branches terminating at infinity $= P - Z = 3 - 1 = 2$
5) Number of asymptotes = 2
   $$\sigma = \frac{(0+0-3.6)-(-1)}{2} = -1.3$$
   Angle of asymptotes are +90 & -90
6) Intersection with $j\omega$ axis characteristics equation is $s^3 + 3.6s^2 + Ks + K = 0$
   Routh's array will be

s^3	1	K
s^2	3.6	K
s^1	$\frac{2.6K}{3.6}$	
s^0	K	

   For stability K > 0 & 2.6K/3.6 > 0 $\Rightarrow K_{mar} = 0$
   Auxiliary equation from s^2 is
   $3.6s^2 + K = 0$
   $3.6s^2 + 0 = 0 \Rightarrow s^2 = 0 \Rightarrow s = 0, 0$
   Therefore root locus will touch imaginary axis at origin.
7) Break away point
   The characteristics equation is
   $s^3 + 3.6s^2 + Ks + K = 0$
   $$K = -\frac{s^2(s+3.6)}{(s+1)}$$
   $$\frac{dK}{ds} = -\frac{(3s^2+7.2s)(s+1)-(s^3+3.6s^2)}{(s+1)^2} = 0$$
   $s^3 + 3.3s^2 + 3.6s = 0$
   Solving we get, $s = 0$
   $s = -1.65 + 0.9367j$
   $s = -1.65 - 0.9367j$

Point $s = 0$ corresponds to the actual breakaway point. But points $s = -1.65 + 0.9367j$ & $s = -1.65 - 0.9367j$ are neither breakaway nor break-in points, because they do not satisfy angle condition.



Example

For a unity feedback system the open Loop transfer function is given below

$$G(s) = \frac{K}{s(s^2 + 6s + 25)}$$ Draw the root locus for $0 \le K \le \infty$.

Solution:

1) No of poles = P = 3
   $s = 0, -3 + 4j, -3 - 4j$
2) No of zeros $= Z = 0$
3) No of root locus branches $= P = 3$
4) No of branches terminating at infinity $= P - Z = 3 - 0 = 3$
5) Number of asymptotes $= P - Z = 3 - 0 = 3$
   $$\text{Centroid } \sigma = \frac{(0-3-3)-0}{3} = -2$$
   Angle of asyptotes are
   $$\phi_1 = \frac{(2\times0+1)\times180^0}{3} = 60^0$$
   $$\phi_2 = \frac{(2\times1+1)\times180^0}{3} = 180^0$$
   $$\phi_3 = \frac{(2\times2+1)\times180^0}{3} = 300^0$$
6) Intersection with $j\omega$ axis
   Characteristics equation is
   $s^3 + 6s^2 + 25s + K = 0$
   Routh's array will be

s^3	1	25
s^2	6	K
s^1	$\frac{150-K}{6}$	
s^0	K	

For stability $K > 0$ & $150 - K > 0$

$\Rightarrow K < 150 \Rightarrow K_{mar} = 150$

Auxiliary equation from s^2 row will be

$6s^2 + 150 = 0$

solving we get $s = \pm j5$

therefore the root locus will touch imaginary axis at $\pm j5$.

7) Breakaway point

From the charcteristics equation

$K = -(s^3 + 6s^2 + 25s)$

$$\frac{dK}{ds} = -\left(3s^2 + 12s + 25\right) = 0$$

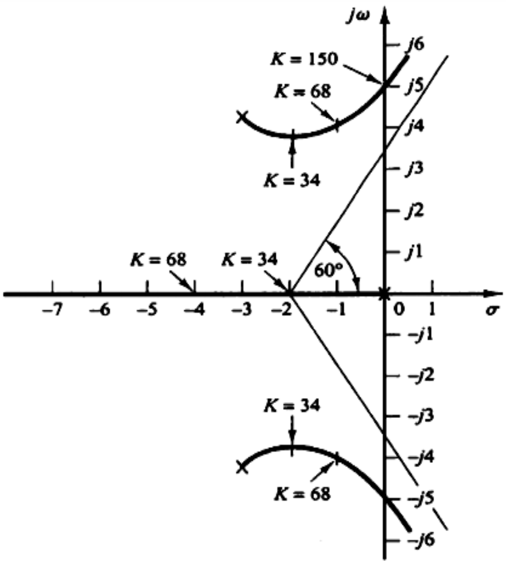
solving we get, $s = -2 + j2.0817, -2 - j2.0817$

Both the points does not satisfy the angle condition hence they are not valid breakaway points.

8) Angle of departure

The angle of departure from the complex pole in the upper half s plane is

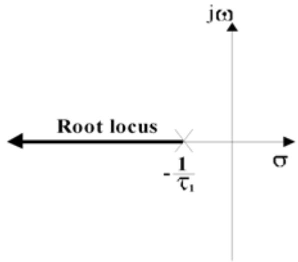
$\varnothing_d = 180^o - (126.87^o + 90^o) = -36.87^o$



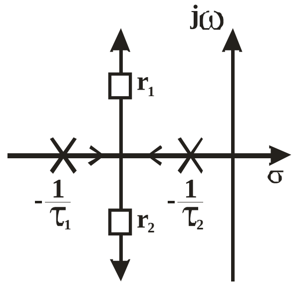
Note:

1) Addition of a pole pulls root locus towards right hand side & the stability of the system decreases.

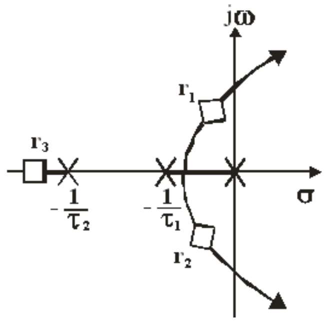
$$\frac{K}{s\tau_1 + 1}$$



$$\frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)}$$

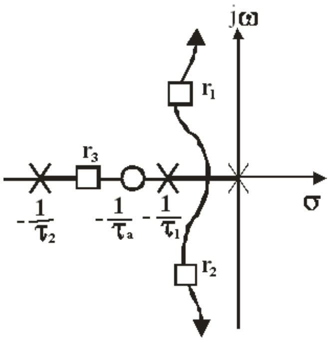


$$\frac{K}{s(s\tau_1 + 1)(s\tau_2 + 1)}$$



2) Addition of a zero pulls root locus to the left & stability of the system increases

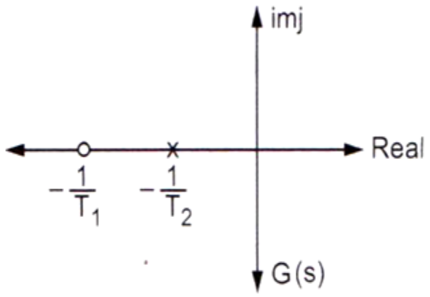
$$\frac{K(s\tau_a + 1)}{s(s\tau_1 + 1)(s\tau_2 + 1)}$$



3.4 MINIMUM PHASE SYSTEM

A system whose transfer function has all the poles and zeros in the left half of the s-plane is called as **minimum-phase system**.

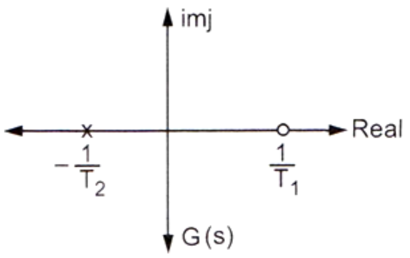
e.g. $G(s) = \dfrac{1+sT_1}{1+sT_2}$



3.5 NON MINIMUM PHASE SYSTEM

A system whose transfer function has one or more zeros in the right half s-plane is known as **non minimum phase system**.

e.g. $G(s) = \dfrac{1-sT_1}{1+sT_2}$



Note:

A common example of a minimum phase element is transportation lag which has the transfer function

$G(s) = e^{-sT}$

Put $s = j\omega$

$\therefore G(j\omega) = e^{-j\omega T}$

Now, $|G(j\omega)| = 1$

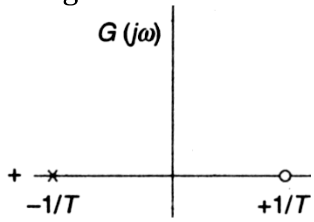
and $\angle G(j\omega) = -57.3\omega T$

3.6 ALL PASS SYSTEM

An all pass system has a transfer function having a pole-zero patterns which is anti symmetric about the imaginary axis, i.e. for every pole in the left half plane; there is a zero in the mirror image position. A common example of such a transfer function is

$$G(j\omega) = \frac{1-j\omega T}{1+j\omega T}$$

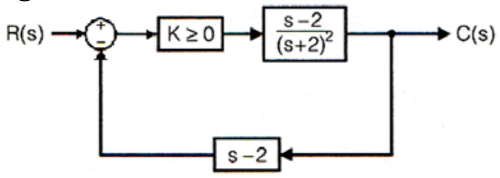
Pole-zero configurations is shown in figure:



It has a magnitude of unity at all frequencies and a phase angle ($-2\tan^{-1}\omega T$) which varies from 0^o to -180^o as ω is increased from 0 to ∞.

GATE QUESTIONS(EC)(Stability Analysis)

Q.1 The feedback control system in the figure is stable.



a) for all $K \geq 0$ b) only is $K \geq 0$
c) only if $0 \leq K < 1$ d) only if $0 \leq K \leq 1$

[GATE-2001]

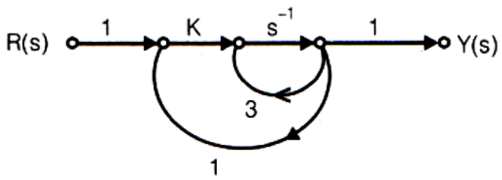
Q.2 The phase margin of a system with the open-loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+s)(2+s)}$$

a) 0° b) 63.4°
c) 90° d) ∞

[GATE-2002]

Q.3 The system shown in the figure remains stable when



a) $K < -1$ b) $-1 < K < 1$
c) $1 < K < 3$ d) $K < -3$

[GATE-2002]

Q.4 The characteristic polynomial of a system is $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$. The system is

a) stable b) marginally stable
c) unstable d) oscillatory

[GATE-2002]

Q.5 The gain margin for the system with open loop transfer function $G(s)H(s) = \frac{2(1+s)}{s^2}$ is

a) ∞ b) 0
c) 1 d) -∞

[GATE-2004]

Q.6 The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s^2+s+2)(s+3)}$$

The range of K for which the system is stable is

a) $\frac{21}{4} > K > 0$ b) $13 > K > 0$
c) $\frac{21}{4} < K < \infty$ d) $-6 < K < \infty$

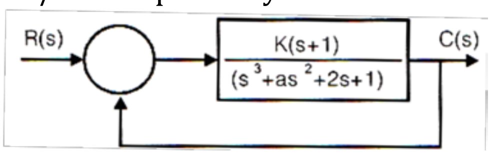
[GATE-2004]

Q.7 For the polynomial $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$, the number of roots which lie in the right half of the s-plane is

a) 4 b) 2
c) 3 d) 1

[GATE-2004]

Q.8 The positive values of "K" and "a" so that the system shown in the figure below oscillates at a frequency of 2 rad/ sec respectively are



a) 1, 0.75 b) 2, 0.75
c) 1, 1 d) 2, 2

[GATE-2006]

Common Data for Questions Q.9 & Q.10:
Consider a unity-gain feedback control system whose open-loop transfer function is $G(s) = \frac{as+1}{S^2}$

Q.9 The value of "a" so that the system has a phase–margin equal to $\pi/4$ is approximately equal to

a) 2.40 b) 1.40
c) 0.84 d) 0.74

[GATE -2006]

Q.10 With the value of "a" set for phase–margin of $\pi/4$, the value of unit–impulse response of the open-loop system at $t=1$ second is equal to

a) 3.40 b) 2.40
c) 1.84 d) 1.74

[GATE -2006]

Q.11 If the closed-loop transfer function of a control system given as $T(s)=\dfrac{s-5}{(s+2)(s+3)}$, then it is

a) an unstable system
b) an uncontrollable system
c) a minimum phase system
d) a non-minimum phase system

[GATE -2007]

Q.12 A certain system has transfer function $G(s)=\dfrac{s+8}{s^2+\alpha s-4}$, where α is a parameter. Consider the standard negative unity feedback configuration as shown below.

Which of the following statements is true?

a) The closed loop system is never stable for any value of α
b) For some positive values of α the closed loop system is stable, but not for all positive values
c) For all positive values of, the closed loop system is stable
d) The closed loop system is stable for all values of, both positive and negative

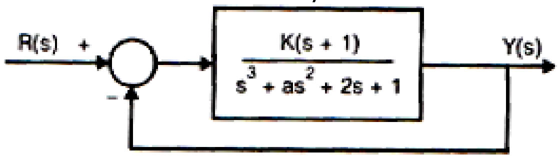
[GATE -2008]

Q.13 The number of open right half plane poles of $G(s)=\dfrac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$ is

a) 0 b) 1
c) 2 d) 3

[GATE -2008]

Q.14 The feedback system shown below oscillates at 2rad/s when



a) K=2 and a=0.75 b) K=3 and a=0.75
c) K=4 and a=0.5 d) K=2 and a=0.5

[GATE -2012]

Q.15 The forward path transfer function of a unity negative feedback system is given by $G(s)=\dfrac{K}{(s+2)(s-1)}$

The value of K which will place both the poles of the closed-loop system at the same location, is ______

[GATE-2014]

Q.16 Consider a transfer function $G_p(s)=\dfrac{ps^2+3ps-2}{s^2+(3+p)s+(2-p)}$ with pa positive real parameter. The maximum value of p until which G_p remain stable is ______.

[GATE-2014]

Q.17 Match the inferences X, Y, and Z, about a system, to the corresponding properties of the elements of first column in Routh's Table of the system characteristic equation.

X: The system is stable...
P: ... when all elements are positive
Y: The system is unstable...
Q: ... when any one element is zero
Z: The test breaks down...

R: ... when there is a change in sign of coefficients

a) $X \to P$, $Y \to Q$, $Z \to R$
b) $X \to Q$, $Y \to P$, $Z \to R$
c) $X \to R$, $Y \to Q$, $Z \to P$
d) $X \to P$, $Y \to R$, $Z \to Q$

[GATE-2016]

Q.18 The transfer function of a linear time invariant system is given by $H(s) = 2s^4 - 5s^3 - 5s - 2$.

The number of zeros in the right half of the s-plane is ________.

[GATE-2016]

Q.19 The first two rows in the Routh table for the characteristic equation of a certain closed-loop control system are given as

$$\begin{array}{c|cc} s^3 & 1 & (2K+3) \\ s^2 & 2K & 4 \end{array}$$

The range of K for which the system is stable is

a) —2.0 <K<0.5 b) 0<K<0.5
c) 0< K <∞ d) 0.5 < K <∞

[GATE-2016]

Q.20 Which one of the following options correctly describes the locations of the roots of the equation $s^4+s^2+1=0$ on the complex plane?

a) Four left half plane (LHP) roots.
b) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis.
c) Two RHP roots and two LHP roots.
d) All four roots are on the imaginary axis.

[GATE-2017-01]

Q.21 Consider $p(s) = s^3 + a_2s^2 + a_1s + a_0$ with all real coefficients. It is known that its derivative $p'(s)$ has no real roots. The number of real roots of $p(s)$ is

a) 0 b) 1 c) 2 d) 3

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(d)	(d)	(c)	(a)	(a)	(b)	(b)	(c)	(c)	(d)	(c)	(c)	(a)	2.25
16	**17**	**18**	**19**	**20**	**21**									
2	(d)	3	(d)	(c)	(b)									

EXPLANATIONS

Q.1 (c)

$$T.F. = \frac{G_1G_2}{1+G_1G_2H}$$

$$= \frac{\frac{K(s-2)}{(s+2)^2}}{1+\frac{K(s-2)(s-2)}{(s+2)^2}}$$

$$= \frac{K(s-2)}{(s+2)^2+K(s-2)^2}$$

∴ char. equation

$= s^2+4+4s+Ks^2-4Ks+4K$

$(1+K)s^2+K(4-4K)s+4K+4=0$

Routh table is

1) $1+K4K+4$

2) $4-4K$

$\Downarrow$ 0

$4(1-K)$

3) $4(K+1)$

For system to be stable

$1-K>0$

$1>K \,\&\, K\geq 0$ (Given in question)

From 3rd row

$K>-1 \therefore 0\leq K<1$

Q.2 (d)

ω_g where $|G(s)H(s)|=1$ is

$$\left|\frac{1-s}{(1+s)(2+s)}\right| = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}} = 1$$

$\sqrt{4+\omega^2}=1$

$\Rightarrow 4+\omega^2=1$

$\omega^2=-3$ (imaginary)

So no gain crossover frequency

$\therefore PM=\infty$

Q.3 (d)

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s}}{1-\left(\frac{3}{s}+\frac{K}{s}\right)} = \frac{K}{s-(3+K)}$$

For system to be stable,

$3+K<0$

$\Rightarrow K<-3$

Q.4 (c)

Routh table is

s^5	2(1)	4(2)	2(1)
s^4	1	2	1
s^3	0	0	0
s^2			
s^1			
s^0			

$$\therefore \frac{d}{ds}\left(s^4+2s^2+1\right)=0$$

$4s^3+4s=0$

$s=\pm j, s=\pm j$

$$\frac{d}{ds}\left(s^2+1\right)=0$$

$2s=0 \Rightarrow s=0$

Double roots on imaginary axis so system is unstable.

Q.5 (a)

$\angle G(s)H(s) = -180° + \tan^{-1}\omega$

For $\omega_\Phi = -180° + \tan^{-1}\omega = -180°$

$$\omega = 0 \; |G(s)H(s)| = \frac{2\sqrt{1+\omega^2}}{\omega^2} = \infty$$

$$G.M = \frac{1}{\infty} = 0$$

In db $G.M = \infty$

Q.6 **(a)**

$$G(s)=\frac{K}{s(s^2+s+2)(s+3)}$$

$$H(s)=1$$

$$1+G(s)H(s)$$

$$=1+\frac{K}{s(s^3+3s^2+s^2+3s+2s+6)}$$

$$=\frac{s^4+4s^3+5s^2+6s+K}{s(s^3+4s^2+5s+6)}$$

$$1+G(s)H(s)=0$$

s^4	1	5 K
s^3	4	6
s^2	$\frac{7}{2}\left(\frac{14}{4}\right)$	K
s^1	$\frac{\frac{7}{2}\times 6-4K}{\frac{7}{2}}$	0
s^0	K	

For system to be stable, K>0

$$(21-4K)\frac{2}{7}>0$$

$$\frac{21}{4}>K\Rightarrow K<\frac{21}{4}$$

$$\therefore \frac{21}{4}>K>0$$

Q.7 **(b)**

$$P(s)=s^5+s^4+2s^3+2s^2+3s+15$$

s^5	1	2	3
s^4	1	2	15
s^3	$0(\in)$	-12	0
s^2	$\frac{2\in+12}{\in}$	15	0
s^0	$\frac{\frac{-12(2\in+12)}{\in}-15\in}{\left(\frac{2\in+12}{\in}\right)}$		

$$\frac{2\epsilon+12}{\epsilon}=t$$

Let ϵ be a small positive no.

$$\frac{-12t-15\epsilon}{t}$$

$$s\Rightarrow -12-\frac{15\epsilon}{t}$$

s^0 15

∴Two sign change from

$s^2$to s and s to s^0

∴2 roots on RHS of s plane.

Q.8 **(b)**

$$1+G(s)H(s)$$

$$=1+\frac{K(s+1)}{s^3+as^2+2s+1}=0$$

$$\Rightarrow\frac{s^3+as^2+2s+1+Ks+K}{s^3+as^2+2s+1}=0$$

$$s^3+as^2+(2+K)s+K+1=0$$

s^3	1	$2+K$
s^2	a	$K+1$
s	$\frac{a(2+K)-(K+1)}{a}$	

For oscillation,

$$\frac{a(2+K)-(K+1)}{a}=0$$

$$a=\frac{K+1}{K+2}$$

$$as^2+K+1=0$$

$$s=j\omega, s^2=-\omega^2=-4$$

$$\Rightarrow -4a+K+1=0$$

$$a=\frac{K+1}{4}\Rightarrow\frac{K+1}{4}=\frac{K+1}{K+2}\Rightarrow K=2$$

$$a=0.75$$

Q.9 **(c)**

$$PM=\frac{\pi}{4}$$

$$\Rightarrow 180+\tan^{-1}a\omega-180°=\frac{\pi}{4}$$

$$\tan\frac{\pi}{4}=a\omega\Rightarrow a\omega=1$$

Now for gain crossover frequency

$$|G(s)|=1 \qquad \Rightarrow\frac{\sqrt{1+a^2\omega^2}}{\omega^2}=1$$

$$\sqrt{1+1}=\omega^2 \qquad (\text{as } a\omega=1)$$

$$\omega^2=\sqrt{2}$$

$\omega = (2)^{1/4}$

$a = \frac{1}{2^{1/4}} = 0.84$

Q.10 (c)

$G(s) = \frac{0.84s+1}{s^2}$

$H(s) = 1, R(s) = 1$

$\therefore C(s) = G(s).R(s)$

$C(s) = \frac{0.84s+1}{s^2}$

$c(t) = L^{-1}\left[\frac{1+0.84s}{s^2}\right]$

$= L^{-1}\left[\frac{1}{S^2} + \frac{0.84}{S}\right]$

$c(t) = [t + 0.84]U(t)$

At $t = 1, c(t) = 1 + 0.84 = 1.84$

Q.11 (d)

As there is a right half zero, the system is a non-minimum phase system.

Q.12 (c)

Closed loop gain is

$\frac{G(s)}{1+G(s)} = \frac{s+8}{s^2 + \alpha s - 4 + s + 8}$

Characteristic equation

$q(s) = s^2 + (\alpha+1)s + 4$

Closed loop system is stable only for $\alpha > -1$.Therefore, for all positive of, the closed loop system is stable.

Q.13 (c)

Characteristic equation

$q(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

Putting $s = \frac{1}{z}$

$q^{'(z)} = 3z^5 + 5z^4 + 6z^3 + 3z^2 + 2z + 1$

Routh array is

z^5	3	6	2
z^4	5	3	1
z^3	21/5	7/5	
z^2	$\frac{\frac{63}{5}-\frac{35}{5}}{21} = \frac{4}{3}$	1	
z	$\frac{\frac{28}{15}-\frac{21}{5}}{4/3} = \frac{-35/15}{4/3} = \frac{-7}{4}$		
z	1		

Since sign changes twice in Routh–array therefore, there are two poles on right half plane.

Q.14 (a)

Characteristic equation is

$1 + G(s)H(s) = 0$

$1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$s^3 + as^2 + (2+k)s + (1+k)$

$= 0$

Routh array for this is

s^3	1	$(2+k)$
s^2	a	$(1+k)$
s^1	$\frac{a(2+k)-(1+k)}{a}$	
s^0	$(1+k)$	

For oscillation $\frac{a(2+k)-(1+k)}{a} = 0$

$\Rightarrow a = \left(\frac{1+k}{2+k}\right)$

Now

$as^2 + (1+k) = 0$

$-a\omega^2 + (1 + k) = 0$

Given $\omega = 2\text{rad/sec}$

$-4a + (1+k) = 0$

$-4\frac{(1+k)}{(2+k)} + (1+k) = 0$

$-4(1+k) + (2+k)(1+k) = 0$

$(1+k)[(2+k) - 4] = 0$

$k = -1, 2$

But $k = -1$is not possible as system will not oscillate for this as $a = 0$ so $k = 2$

$$a = \frac{1+k}{2+k} = \frac{3}{4} = 0.75$$

Q.15 (2.25)

Given $G(s) = \frac{K}{(s+2)(s-1)}$

H(s)=1

Characteristic equation:

$1 + G(s)H(s) = 0$

$$1 + \frac{K}{(s+2)(s-1)} = 0$$

The poles are $S_{1,2} = -1 \pm \sqrt{\frac{9}{4} - 4K}$

If $\frac{9}{4} - K = 0$, then both poles of the closed loop system at the same location.

So, $K = \frac{9}{4} \Rightarrow 2.25$

$$\phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

Q.16 (2)

Given $G_p(s) = \frac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$

By R - H criteria

The characteristic equation is $s^2 + (3+p)s + (2-p) = 0$

i.e. $s^2 + (3+p)s + (2-p) = 0$

By forming R-H array,

s^2	1	(2-p)
s^1	$(3+\phi)$	0
s^0	(2-p)	

For stability, first column elements must be positive and non-zero

i.e. (1)(3 + p) > 0 ⟹ p > - 3

and (2)(2-p) > 0 ⟹ p < 2

i.e. – 3< p< 2

The maximum value of p unit which G_p remains stable is 2

Q.17 (d)

Q.18 (3)

We can proceed here by taking this polynomial as characteristic equation and conclusion can be draw by using RH criterion. As we are interested to know how many roots are lying on right half of s plane.

s^4	2	0	-2
s^3	-5	+5	0
s^2	2	-2	since row of zero occurs the auxillary equation is $A.e: 2s^2 - 2$; $\frac{d}{ds}(At) = 4$
s^1	4	0	
s^0	-2		

→ The number of roots i.e. the number of zeros in this case in right half of plane is number of sign changes

→ Number of sign changes = 3

Q.19 (d)

S^3	1	2k+3
S^2	2k	4

From the table we can find characteristic equation

$S^3 + 2ks^2 + (2k+3)s + 4 = 0$

For stability $(2k)(2k+3) > 4$

$4k^2 + 6k - 4 > 0$ 4

$(k - \frac{1}{2})(k+2) > 0$

So the conditions are $k > \frac{1}{2}$ and $k > -2$ combining $k > -2$

Q.20 (c)

Q.21 (b)

Given

$p(s) = s^3 + a_2 s^2 + a_1 s + a_0$

$p'(s) = 3s^2 + 2a_2 s + a_1$

Given $p'(s)$ has no real roots.

We know that, if $p(s)$ has 'n ' real roots, then $p'(s)$ will have at least 'n –1' real roots.

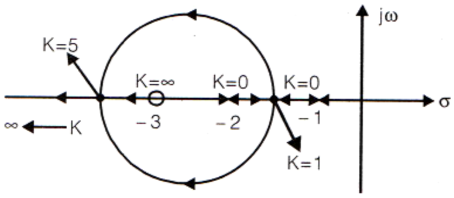
Hence, from the given condition,

$n - 1 = 0$

$n = 1$

GATE QUESTIONS(EC)(Root Locus)

Q.1 The root-locus diagram for a closed-loop feedback system is shown in the figure. The system is over damped.



a) only if $0 \le K \le 1$
b) only if $1 < K < 5$
c) only if $K > 5$
d) if $0 \le K < 1$ or $K > 5$

[GATE -2001]

Q.2 Which of the following points is NOT on the root locus of a system with the open–loop transfer function $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$?

a) $s = -j\sqrt{3}$ b) $s = -1.5$
c) $s = -3$ d) $s = -\infty$

[GATE -2002]

Q.3 The root locus of the system $G(s)H(s) = \dfrac{K}{s(s+2)(s+3)}$ has the break-away point located at

a)(-0.5,0) b)(-2.5478,0)
c)(-4,0) d)(-0.784,0)

[GATE -2003]

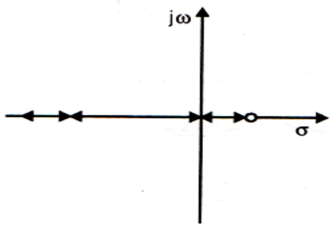
Q.4 Given $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$, the point of intersection of the asymptotes of the root loci with the real axis is

a)-4 b)1.33
c)-1.33 d)4

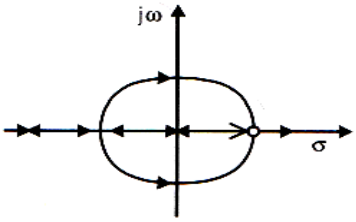
[GATE -2004]

Q.5 A unity feedback system is given as $G(s) = \dfrac{K(1-s)}{s(s+3)}$. Indicate the correct root locus diagram

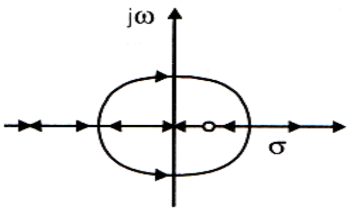
a)



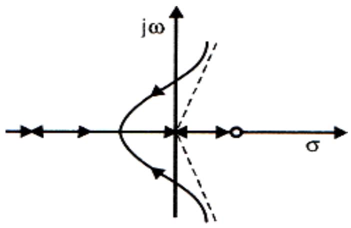
b)



c)



d)



[GATE -2005]

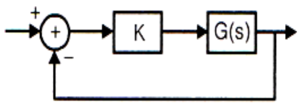
Q.6 A unity feedback control system has an open-loop transfer function $G(s) = \dfrac{K}{s(s^2+7s+12)}$. The gain K for which $s = -1 + j1$ will lie on the root locus of this system is

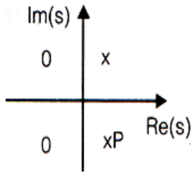
a)4 b)5.5
c)6.5 d)10

[GATE-2007]

Q.7 The feedback configuration and the pole–zero locations of

$G(s)=\frac{s^2-2s+2}{s^2+2s+2}$ are shown below. The root locus for Negative values of K, i.e. for $-\infty < K < 0$, has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to

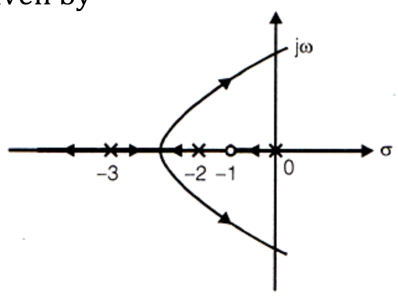




a) $\pm\sqrt{2}$and0° b) $\pm\sqrt{2}$and45°
c) $\pm\sqrt{3}$and0° d) $\pm\sqrt{3}$and45°

[GATE-2009]

Q.8 The root locus plot for a system is given below. The open loop transfer function corresponding to this plot is given by



a) $G(s)H(s)=K\frac{s(s+1)}{(s+2)(s+3)}$

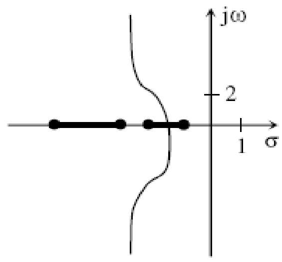
b) $G(s)H(s)=K\frac{(s+1)}{s(s+2)(s+3)^2}$

c) $G(s)H(s)=K\frac{1}{s(s-1)(s+2)(s+3)}$

d) $G(s)H(s)=K\frac{(s+1)}{s(s+2)(s+3)}$

[GATE-2011]

Q.9 In the root locus plot shown in the figure, the pole/zero marks and the arrows have been removed. Which one of the following transfer functions has this root locus?



a) $\frac{s+1}{(s+2)(s+4)(s+7)}$

b) $\frac{s+4}{(s+1)(s+2)(s+7)}$

c) $\frac{s+7}{(s+1)(s+2)(s+4)}$

d) $\frac{(s+1)(s+2)}{(s+7)(s+4)}$

[GATE-2014]

Q.10 The open-loop transfer function of a unity-feedback control system is

$$G(s)=\frac{K}{s^2+5s+5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is

[GATE-2016]

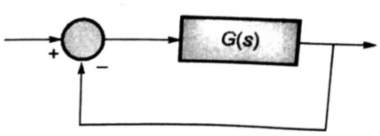
Q.11 The forward-path transfer function and the feedback-path transfer function of a single loop negative feedback control system are given as $G(s)=\frac{K(s+2)}{s^2+2s+2}$ and H(s)=1, respectively, If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is ___.

[GATE-2016]

Q.12 A linear time invariant system with the transfer function

$$G(s)=\frac{K(s^2+2s+2)}{(s^2-3s+2)}$$

Is connected in unity feedback configuration as shown in the figure.



For the closed loop system shown, the root locus for 0<K<∞ intersects the imaginary axis for K=1.5. The closed loop system is stable for

a) K>1.5
b) 1<K<1.5
c) 0<K<1.5
d) No positive value of K

ANSWER KEY:

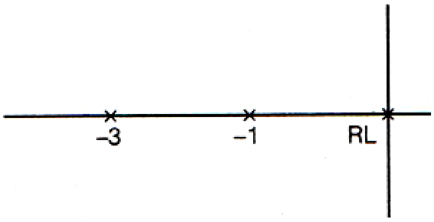
1	2	3	4	5	6	7	8	9	10	11
(d)	(b)	(d)	(c)	(c)	(d)	(b)	(b)	(b)	1.25	-3.414

EXPLANATIONS

Q.1 **(d)**

For over damping roots of characteristic equation should lie on negative axis and be unequal.

Q.2 **(b)**



RL lies where no. of poles and zeros to the right of the pole is odd.

$\therefore s = -1.5$ doesn't lie on RL

Q.3 **(d)**

$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \dfrac{K}{s(s+2)(s+3)} = 0$

$\Rightarrow 1 + \dfrac{K}{s(s^2+5s+6)} = 0$

$\Rightarrow K = -1.(s^3 + 5s^2 + 6s)$

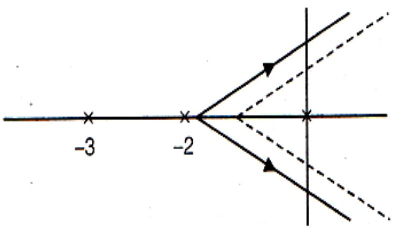
$\Rightarrow \dfrac{dK}{ds} = -(3s^2 + 10s + 6s)$

$\dfrac{dK}{ds} = 0$

$\Rightarrow 3s^2 + 10s + 6 = 0$

$s = \dfrac{-10 \pm \sqrt{100-72}}{6}$

$= \dfrac{-10 \pm 5.3}{6} = -0.784, -2.55$



$\text{Centroid} = \dfrac{-2-3}{-3} = \dfrac{-5}{3} = -1.66$

Angle of asymptotes

$= (2K+1)\dfrac{\pi}{P-Z}$

$= (2K+1)\dfrac{\pi}{3}, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

$\therefore$ Break away point is (-0.784,0)as -2.55 does not lie on RL.

Q.4 **(c)**

$\text{Centroid} = \dfrac{\Sigma P - \sum Z}{P - Z}$

$= \dfrac{-1-3}{3} = -1.33$

Q.5 **(c)**

$1 + G(s)H(s) = 0$

$K = \dfrac{s^2 + 3s}{1-s}$

For breakaway & break in point

$\dfrac{dK}{ds} = (1-s)(2s+3) + s^2 + 3s = 0$

$= -s^2 + 2s + 3 = 0$

$\Rightarrow s^2 - 2s - 3 = 0$

$\Rightarrow (s-3)(s+1) = 0$

$s = 3, -1$

-1is the breakaway point and 3 is the break in point.

Q.6 **(d)**

$\text{T.F.} = \dfrac{G(s)}{1+G(s)}$

As $H(s) = 1$

For the point $s = -1 + j1$ to lie on root locus

$1 + G(s) = 0$

$\Rightarrow 1 + \dfrac{K}{s(s^2+7s+12)} = 0$

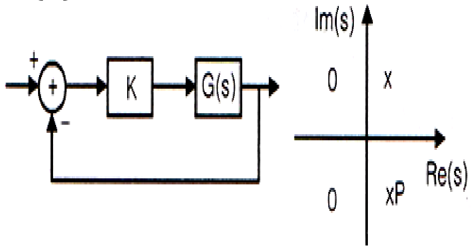
$(s^2+7s+12)+K=0$

Putting

$s=-1+j(-1+j)(1-2j-1-7+7j+12)+K=0$

$\Rightarrow K=+10$

Q.7 **(b)**



$1+G(s)H(s)=0$

$$1+\frac{K(s^2-2s+2)}{s^2+2s+2}=0$$

$$K=-\frac{s^2+2s+2}{s^2-2s+2}$$

Put $\frac{\partial K}{\partial s}=0$ we have

$(s^2-2s+2)(s+1)$

$-(s^2+2s+2)(s-1)=0$

$2s^2-4s^2+4=0$

$2s^2=+4$

$s=\pm\sqrt{2}$

Angle of departure is

$\Phi_D=180°+\Phi$

Where $\Phi=\sum\Phi_z-\sum\Phi_p$

$\Phi=135°$

$\Phi_D=180-135°=45°$

So (b) options is correct.

Q.8 **(b)**

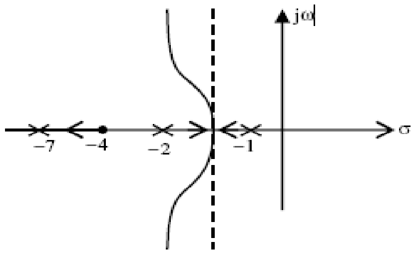
From plot we can observe that one pole terminates at one zero at position -1 and three poles terminates to ∞. It means there is four poles and 1 zero .Pole at -3 goes on both sides. It means there are two poles at -3.

Q.9 **(b)**

For transfer function

$$\frac{(s+4)}{(s+1)(s+2)(s+3)}$$

From pole zero plot



Q.10 **(1.25)**

In this first we need to find the break point by finding the root of $\frac{dk}{ds}=0$ and then by using magnitude condition value of k can be obtained.

$$G(s)=\frac{K}{s^2+5s+5}$$

$K=-(s^2+5s+5)$

$\frac{dk}{ds}=0$

$\Rightarrow 2s+5=0\Rightarrow s=-2.5$

$\rightarrow$ Applying magnitude condition $|G(s)|=1$

$$\left|\frac{K}{s^2+5s+5}\right|_{s=-2.5}=1$$

$$\Rightarrow\left[\frac{k}{(-2.5)^2+[5x(-2.5)]+5}\right]=1$$

$$\Rightarrow\left[\frac{k}{6.25-12.5+5}\right]=1$$

$$\Rightarrow\left|\frac{k}{-1.25}\right|=1\Rightarrow k=1.25$$

Q.11 **(-3.414)**

To find break point, from characteristic equation we need to arrange k as function of s, then the root of $\frac{dk}{ds}=0$ gives break point.

$s^2+2s+2+k+2k=0$

$\Rightarrow k(s+2)=-s^2+2s+2$

$$\Rightarrow k=-\left[\frac{s^2+2s+2}{s+2}\right]$$

$$\Rightarrow \frac{dk}{ds} = -\left[\frac{(s+2)\frac{d}{ds}(s^2+2s+2) - (s^2+2s+2)\frac{d}{ds}(s+2)}{(s+2)^2}\right]$$

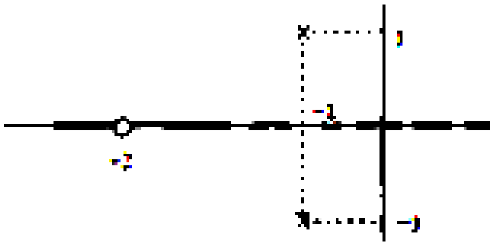
$$\Rightarrow \frac{dk}{ds} = -\left[\frac{(s+2)(2s+2) - (s^2+2s+2)}{(s+2)^2}\right]$$

$$\Rightarrow \frac{dk}{ds} = 0$$

$$\Rightarrow 2s^2 + 2s + 4s + 4 - s^2 - 2s - 2 = 0$$

$$\Rightarrow s^2 + 4s + 2 = 0$$

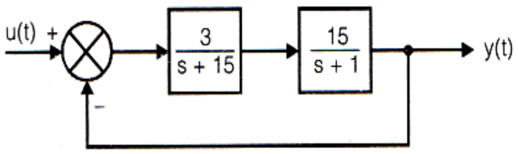
$$\Rightarrow s = -0.58 \text{ and } -3.414$$



- To find the valid break point we need to find that lies on root locus
- -3.414 lies on root locus
- So break point -3.414.

GATE QUESTIONS(EE)(Stability Analysis)

Q.1 The roots of the closed loop characteristic equation of the system are



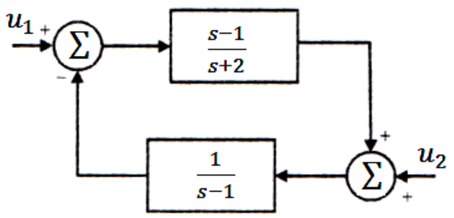
a) -1 and -15 b) 6 and 10
c) -4 and -15 d) -6 and -10
[GATE-2003]

Q.2 The loop gain GH of a closed loop system is given by the following expression $\frac{K}{s(s+2)(s+4)}$. The value of K for which the system just becomes unstable is
a) K=6 b) K=8
c) K=48 d) K=96
[GATE-2003]

Q.3 For the equation, $s^3-4s^2+s+6=0$ the number of roots in the left half of s-plane will be
a) zero b) one
c) two d) three
[GATE-2004]

Q.4 A unity feedback system, having an open loop gain $G(s)H(s)=\frac{k(1-s)}{(1+s)}$, becomes stable when
a) $|k|>1$ b) $k>1$
c) $|k|<1$ d) $k<-1$
[GATE-2005]

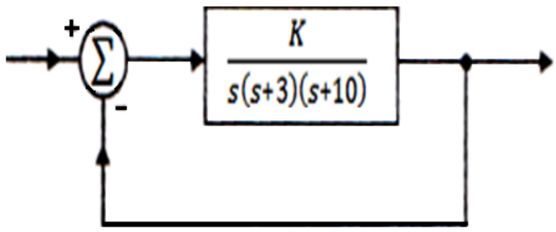
Q.5 The system shown in the figure is



a) Stable
b) Unstable
c) Conditionally stable
d) stable for input u_1, but unstable for input u_2
[GATE-2007]

Q.6 If the loop gain k of a negative feedback system having a loop transfer function $k(s+3)/(s+8)^2$ is to be adjusted to induce a sustained oscillation then
a) The frequency of this oscillation must be $4/\sqrt{3}$ rad/s
b) The frequency of this oscillation must be 4 rad/s
c) The frequency of this oscillation must be 4 or $4/\sqrt{3}$ rad/s
d) Such a k does not exist
[GATE-2007]

Q.7 Figures shows a feedback system where $k>0$



The range of k for which is stable will be given by
a) $0<k<30$ b) $0<k<39$
c) $0<k<390$ d) $k>390$
[GATE-2008]

Q.8 The first two rows of Routh's tabulation of a third order equation are as follows.

S^3	2	2
S^2	4	4

This means there are

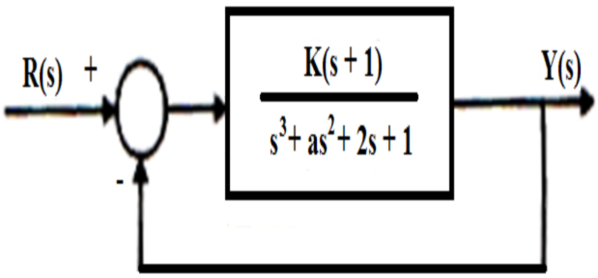
a) two roots at $s = \pm j$ and one root in right half s-plane
b) two roots at $s = \pm j2$ and one root in left half s-plane
c) two roots at $s = \pm j2$ and one root in right half s-plane
d) two roots at $s = \pm j$ and one root in left half s-plane

[GATE-2009]

Q.9 An open loop system represented by the transfer function $G(s)=\frac{(s-1)}{(s+2)(s+3)}$ is
a) stable and of the minimum phase type
b) stable and of the non-minimum phase type
c) unstable and of the minimum phase type
d) unstable and of the non-minimum phase

[GATE-2011]

Q.10 The feedback system shown below oscillates at 2rad/s when



a)K=2 and a=0.75 b)K=3 and a=0.75
c)K=4 and a=0.5 d)K=2 and a=0.5

[GATE -2012]

Q.11 A single-input single-output feedback system has forward transfer function G(s) and feedback transfer function H(s) It is given that I G(s).H(s)I<1. Which of the following is true about the stability of the system?
a) The system is always stable
b) The system is stable if all zeros of G(s).H(s) are in left half of the s-plane
c) The system is stable if all poles of G(s).H(s) are in left half of the s-plane
d) It is not possible to say whether or not the system is stable from the information given

[GATE-2014]

Q.12 Given the following polynomial equation $s^3+5.5s^2+8.5s+3=0$, the number of roots of the polynomial, which have real parts strictly less than -1, is

[GATE-2016]

Q.13 The open loop transfer function of a unity feedback control system is given by $G(s)=\frac{K(s+1)}{s(1+T_s)(1+2s)}$, $K>0, T>0$. The closed loop system will be stable if
a) $0<T<\frac{4(K+1)}{K-1}$ b) $0<K<\frac{4(T+2)}{T-2}$
c) $0<K<\frac{(T+2)}{T-1}$ d) $0<T<\frac{8(K+1)}{K-1}$

[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(c)	(b)	(c)	(d)	(b)	(c)	(d)	(b)	(a)	(a)	2	(c)

EXPLANATIONS

Q.1 (d)

$1+GH=0$ (Characteristic equation)

$$1+\frac{45}{(s+15)(s+1)}=0$$

$s^2+16s+60=0$

$\therefore (s+10)(s+6)=0$

$\therefore$ roots are (-6, -10).

Q.2 (c)

Characteristic equation

$=1+GH=0$

$$\Rightarrow 1+\frac{K}{s(s+2)(s+4)}=0$$

Characteristic eq.

$\Rightarrow \quad s^3+6s^2+8s+k=0$

The Routh-array is formed as follows

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	
s^0	K	

According to Routh-Hurwitz criterion, for a stable system there should be no change of sign in the first column of Routh array.

So, $\frac{48-K}{6}>0$ and $K>0$

$0<K<48$

Hence for K=48, the system just becomes unstable.

Q.3 (b)

$s^2-4s^2+s+6=0$

The Routh-array is formed as follows

s^3	1	1
s^2	−4	6
s^1	$\frac{-4\times 1-6\times 1}{-4}=2.5$	
s^0	6	

No. of sign changes in first column of Routh-array=2.

According to Routh-Hurwitz criterion, the number of changes of sign in the first column gives the number of positive real part roots of the polynomial.

So, no. of roots in RHS of s-plane=2.

Total no. of roots=3

Hence, no. of roots in LHS of s-plane =3-2=1

Q.4 (c)

Characteristic equation

$1+G(s)H(s)=0$

$$\Rightarrow \quad 1+\frac{k(1-s)}{1+s}=0$$

$$\Rightarrow \quad s+1+k(1-s)=0$$

$$\Rightarrow \quad s(1-k)+1+k=0$$

$$s=\frac{k+1}{k-1}$$

For a stable system pole lies in left hand side of s=plane, it means and must be negative for stable system.

$s<0 \quad \frac{k+1}{k-1}<0$

Case-I

$k+1<0$ and $k-1>0$

$k<-1$ and $k>1$

Which is not possible.

Case-II

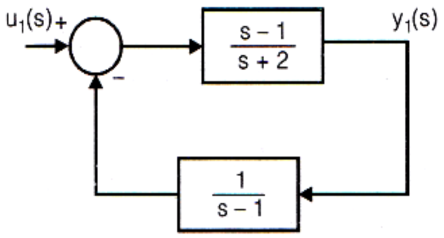
$k+1>0$ and $k-1<0$

$k>-1 \text{and} k<1$

$-1<k<1$

$|k| < 1$

Q.5 **(d)**



Pole is in LHS of s-plane, hence stable.

$$(T/F)_1 = \frac{\frac{(s-1)}{(s+2)}}{1 + \frac{(s-1)}{(s+2)} \times \frac{1}{(s-1)}} = \frac{(s-1)}{(s+3)}$$

$$(T/F)_2 = \frac{\frac{1}{(s-1)}}{1 + \frac{1}{(s-1)} \times \frac{(s-1)}{(s+2)}}$$

$$= \frac{s+2}{(s-1)(s+3)}$$

Hence unstable as it has pole at right hand side of S-plane.

Q.6 **(b)**

Loop transfer function

$$= G(s)H(s) = k\frac{(s+3)}{(s+8)^2}$$

$\therefore$ characteristic equation

$= 1 + G(s)H(s) = 0$

$\Rightarrow \quad 1 + k\frac{(s+3)}{(s+8)^2} = 0$

$\Rightarrow \quad s^2 + 16s + 64 + (s+3) = 0$

$\Rightarrow \quad s^2 + (16+k)s + 64 + 3k = 0$

Routh- Array

s^2	1	$64+3k$
s^1	$16+k$	
s^0	$64+3k$	

For sustained oscillation

16+k=0

$\Rightarrow k = -16s^2 + 64 + 3k$

$= s^2 + 64 + 3x(-16) = 0$

$s^2 + 16 = 0$

$\Rightarrow s = \pm j4$

So, frequency of oscillation 4 rad/sec.

Q.7 **(c)**

$G(s) = \frac{k}{s(s+3)(s+10)}$ and H(s)=1

Characteristic equation

$\Rightarrow 1 + G(s)H(s) = 0$

$1 + \frac{k}{s(s+3)(s+10)} = 0$

$\Rightarrow s(s+3)(s+10) + k = 0$

$s^3 + 13s^2 + 30s + k = 0$

Routh-Array

s^3	1	30
s^2	13	k
s^1	$\frac{13 \times 30 - k}{13}$	
s^0	k	

According to Routh-Hurwitz criterion. For a stable system, signs of first column do no change

$k > 0$ and $\frac{13 \times 30 - k}{13} > 0$

Therefore system to be stable

$0 < k < 390$

Q.8 **(d)**

Routh-Array

s^3	2	2
s^2	4	4
s^1	$\frac{2 \times 4 - 4 \times 2}{4} = 0$	

The third row vanishes. An auxiliary equation is formed using elements of 2nd row.

Auxiliary equation

$A(s) = 4s^2 + 4 = 0$

$\Rightarrow s = \pm j$.

The derivative of this auxiliary equation is taken wrt s and the coefficients of the differentiated equation are taken as the elements of 3rd row.

$$\frac{dA(s)}{ds} = 8s$$

Routh-Array

s^3	2	2
s^2	4	4
s^1	8	
s^0	4	

There is no root in RHS of s-plane.
Two roots of $s = \pm j$, so one root is in LHS of s-plane.

Q.9 (b)

$$G(s) = \frac{s-1}{(s+2)(s+3)}$$

one zero at s=1
two poles at s=-2 & -3
Since zero lies in RHS of s-plane.
It is non-minimum phase type system.
Since both poles lie in LHS of s-plane, system is stable.

Q.10 (a)

Characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + (2+k)s + (1+k) = 0$$

Routh array for this is

s^3	1	$(2+k)$
s^2	a	$(1+k)$
s^1	$\frac{a(2+k)-(1+k)}{a}$	
s^0	$(1+k)$	

For oscillation $\frac{a(2+k)-(1+k)}{a} = 0$

$$\Rightarrow a = \left(\frac{1+k}{2+k}\right)$$

Now

$$as^2 + (1+k) = 0$$

$$-a\omega^2 + (1+k) = 0$$

Given $\omega = 2\text{rad/sec}$

$$-4a + (1+k) = 0$$

$$-4\frac{(1+k)}{(2+k)} + (1+k) = 0$$

$$-4(1+k) + (2+k)(1+k) = 0$$

$$(1+k)\left[(2+k) - 4\right] = 0$$

$$k = -1, 2$$

But $k = -1$ is not possible as system will not oscillate for this as $a = 0$ so $k = 2$

$$a = \frac{1+k}{2+k} = \frac{3}{4} = 0.75$$

Q.11 (a)

Q.12 (2)

The polynomial is

$S^3 + 5.5S^2 + 8.5S + 3 = 0$, since we are interested to see the roots wrt S. -1 so in the above equation replace S by z-1 then the equation is

$$(Z-1)^3 + 5.5(Z-1)^2 + 8.5(Z-1) + 3 = 0$$

$$\Rightarrow Z^3 - 3Z^2 + 3Z - 1 + 5.5$$

$$(Z^2 + 1 - 2Z) + 8.5Z - 8.5 + 3 = 0$$

$$\Rightarrow Z^3 + 2.5Z^2 + 0.52 - 1 = 0$$

$$\Rightarrow Z^3 + Z^2(-3+5.5)$$

$$+Z(3+8.5-11) + (-1+5.5-8.5+3) = 0$$

Using RH table

Z^3	1	0.5
Z^2	2.5	-1
Z^1	0.9	
Z^0	-1	

The single sign change in 1st column indicate that out of 3 roots 1 root lie on the right half of S=-1 plane if memory remaining 2 lies on left half of S =-1 plane.

Q.13 (c)

To comment closed 100b system stability we need the characteristic equation. Here it is given that it is a unity feedback system.
Unity feedback system
So the characteristic equation is

$$S(1+TS)(1+2s) + K(S+1) = 0$$

$$\Rightarrow (S+TS^2)(1+2S) + KS + K = 0$$

$$\Rightarrow S + 2S^2 + TS^2 + 2T\ S^3 + KS + K = 0$$

$$\Rightarrow S^3(2T) + S^2(2+T) + S(1+k) + k = 0$$

$$\Rightarrow S^3\left(\frac{2+T}{2T}\right)S^2 + \left(\frac{k+1}{2T}\right)S + \frac{k}{2T} = 0$$

$\rightarrow$ for stability using criterion R(t)

$$\left(\frac{2+T}{2T}\right)\left(\frac{K+1}{2T}\right) > \frac{K}{2T}$$

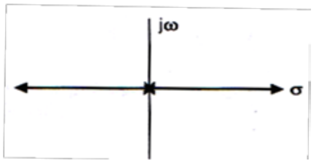
$$\Rightarrow (T+2)(K+1) > K \Rightarrow \frac{K+1}{K} > \frac{1}{T+2}$$

$$\Rightarrow \frac{1}{K} > \frac{1}{T+2} - 1 \Rightarrow \frac{1}{K} > -\left(\frac{T+1}{T+2}\right) \Rightarrow K < -\left[\frac{T+2}{T+1}\right]$$

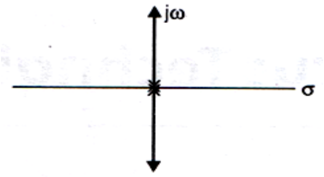
GATE QUESTIONS(EE)(Root Locus)

Q.1 A unity feedback system has an open loop transfer function, $G(s)=\frac{K}{s^2}$. Its root locus plot will be

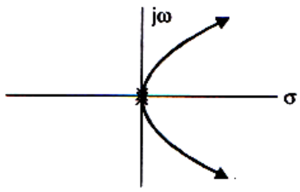
a)



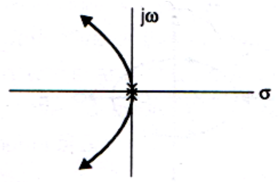
b)



c)

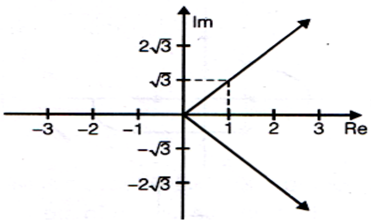


d)



[GATE-2002]

Q.2 Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is



a) $\frac{K}{s^3}$

b) $\frac{K}{s^2(s+1)}$

c) $\frac{K}{s(s^2+1)}$

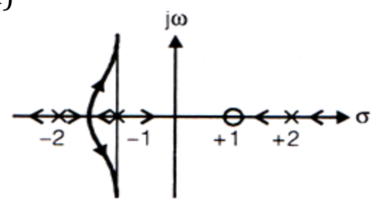
d) $\frac{K}{s(s^2-1)}$

[GATE-2005]

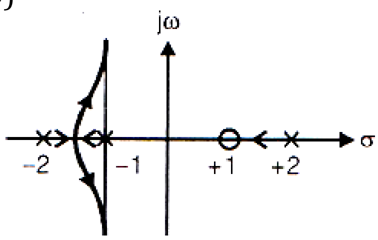
Q.3 A closed-loop system has the characteristic function $(S^2-4)(s+1)+K(s-1)=0$.

Its root locus plot against K is

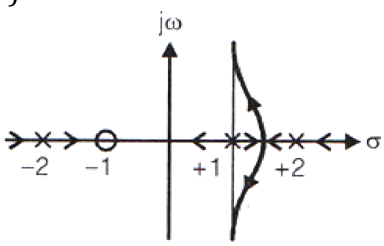
a)



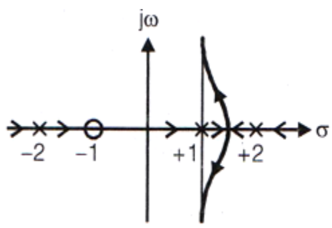
b)



c)



d)



[GATE-2006]

Q.4 The characteristic equation of a closed-loop system is $s(s+1)(s+3)+k\ (s+2)=0,\ k>0$.

Which of the following statements is true?

a) Its roots are always real
b) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$
c) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$
d) It may have complex roots in the right half plane

[GATE-2010]

Q.5 The open loop transfer function G(s) of a unity feedback control system is given as, $G(s)=\frac{k\left(s+\frac{2}{3}\right)}{s^2(s+2)}$

From the root locus, it can be inferred that when k tends to positive infinity

a) three roots with nearly equal real parts exist on the left half of the s-plane
b) one real root is found on the right half of the s-plane
c) the root loci cross the jω axis for a finite value of k:k≠0
d) three real roots are found on the right half of the s-plane

[GATE-2011]

Q.6 The open loop poles of a third order unity feedback system are at 0,-1, -2. Let the frequency corresponding to the point where the root locus of the system transits to unstable region be K. Now suppose we introduce a zero in the open loop transfer function at -3, while keeping all the earlier open loop poles intact. Which one of the following is TRUE about the point where the root locus of the modified system transits to unstable region?

a) It corresponds to a frequency greater than K
b) It corresponds to a frequency less than K
c) It corresponds to a frequency K
d) Root locus of modified system never transits to unstable region

[GATE-2015]

Q.7 An open loop transfer function G(s) of system is $G(s)=\frac{k}{s(s+1)(s+2)}$.
For a unity feedback system, the breakaway point of the root loci on the real axis occurs at,

a)-0.42
b) -1.58
c) -0.42 and -1.58
d) none of the above.

[GATE-2015]

Q.8 The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function G(s)$\frac{Ks}{(s-1)(s-4)}$ is

a) 1 b) 2
c) 5 d) 9

[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6	7	8
(b)	(a)	(b)	(c)	(a)	(d)	(a)	(a)

EXPLANATIONS

Q.1 **(b)**

$G(s)H(s) = \frac{K}{s^2}$

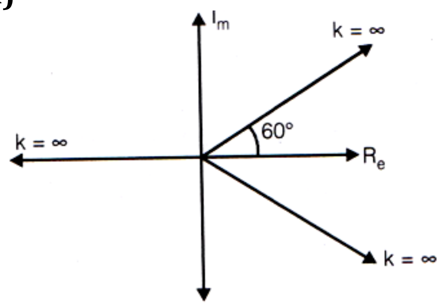
$\therefore\ P - Z = 2$

Centroid $= \frac{0}{2} = 0$

Angle of asymptotes $= 90^\circ, 270^\circ$

$\therefore$ Option (b) is correct.

Q.2 **(a)**



These are three asymptotes with angle $60^\circ, 180^\circ$ and 300°

Angle of asymptotes

$= \frac{(2k+1)\times 180^\circ}{P - Z}$

Where k=0, 1, 2 upto (P-Z) -1 as angles are $60^\circ, 180^\circ$ and 300° it means

$P - Z = 3$

Intersection of asymptotes on real axis

$X = \frac{\Sigma poles - \Sigma zero}{P - Z}$

Since, system does not have zeros

$X = \frac{\Sigma poles}{P}$

As asymptotes intersect at origin, it means all the three poles are at origin.

Hence, option (a) is correct.

Q.3 **(b)**

Characteristic function

$\Rightarrow (S^2 - 4)(s+1) + K(s-1) = 0$

$\Rightarrow 1 + \frac{K(s-1)}{(S^2-4)(s+1)} \equiv 1 + G(s)H(s)$

Open loop transfer function

$= G(s)H(s) = \frac{K(s-1)}{(S^2-4)(s+1)}$

Zero of OLTF s=1; z=1

Poles of OLTF s=-1, -2, +2, P=3

The root locus starts from open-loop poles and terminates either on open-loop zero or infinity.

Root locus exist on a section of real axis it the sum of the open-loop poles and zeros to the right of the section is odd.

Number of branches terminating on infinity.= P-Z=3-1=2

Angles of asymptotes

$= \frac{(2k+1)\times 180^\circ}{P - Z} = \frac{(2k+1)\times 180^\circ}{2}$

$= 90^\circ \text{and} 270^\circ$

Intersection of asymptotes on real axis (centroid)

$= \frac{\Sigma poles - \Sigma zero}{P - Z} = \frac{(-1-2+2)-(1)}{2} = -1$

Option (d) is correct on the basic of above analysis.

Q.4 **(c)**

Characteristic equation

$s(s+1)(s+3) + k(s+2) = 0$

$1 + \frac{k(s+2)}{s(s+1)(s+3)} = 0$

Comparing with 1+G(s)H(s)=0

G(s)H(s) = Open-loop transfer function (OLTF)

$= \frac{k(s+2)}{s(s+1)(s+2)}$

no. of zero=Z=1 zero at -2

no. of poles=P=3 poles at 0,-1 &-3

No. of branches terminating at infinity

=P-Z=3-1=2

Angle of asymptotes

$$=\frac{(2k+1)\times 180^o}{P-Z}$$

$$=\frac{(2k+1)\times 180^o}{2}$$

$$=(2k+1)\times 90^o$$

$$=90^o \text{and} 270^o$$

Centroid $=\frac{\Sigma poles-\Sigma zero}{P-Z}$

$$=\frac{0-1-3-(-2)}{2}=-1$$

Breakaway point lies in the range -1 < Re[s] < 0 and two branches terminates at infinity along the asymptotes Re(s) = -1.

Q.5 (a)

$$G(s)=\frac{k\left(s+\frac{2}{3}\right)}{s^2(s+2)} \text{ and } H(s)=1$$

Characteristic equation

$$1+G(s)H(s)=0 \Rightarrow 1+\frac{k\left(s+\frac{2}{3}\right)}{s^2(s+2)}=0$$

$$\Rightarrow s^3+2s^2+k\left(s+\frac{2}{3}\right)=0$$

$$\Rightarrow s^3+2s^2+ks+\frac{2k}{3}=0$$

Routh Array

As k > 0, there is no sign change in the 1st column of routh array. So the system is stable and all the three roots lie on LHS of s-plane.

For $k>0(k\neq 0)$, none of the row of routh array becomes zero. So root loci does not cross the $j\omega$ axis.

no. of zero=Z=1

no. of poles=P=3

No. of branches terminating at infinity

=P-Z=3-1=2

Angle of asymptotes

$$=\frac{(2k+1)\times 180^o}{P-Z}$$

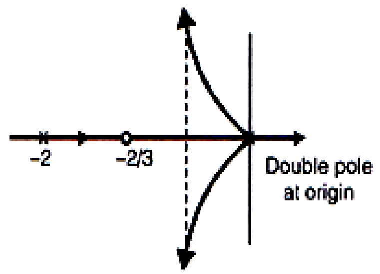
$$=\frac{(2k+1)\times 180^o}{2}$$

$$=(2k+1)\times 90^o$$

$$=90^o \text{and} 270^o$$

Centroid $=\frac{\Sigma poles-\Sigma zero}{P-Z}$

$$=\frac{0+0-2-\left(-\frac{2}{3}\right)}{2}=-\frac{2}{3}$$



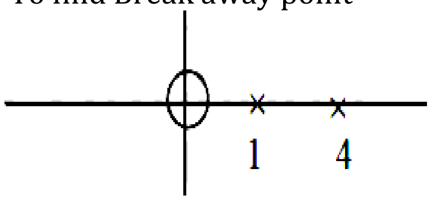
Since, all the three branches terminates at $Re(s)=-\frac{2}{3}$.

So all the three roots have nearly equal real part.

Q.8 (a)

$$G(s)\frac{Ks}{(s-1)(s-4)}$$

To find Break away point



We need to find the root of $\frac{dk}{ds}=0$

where

$$K=-\frac{(s-1)(s-4)}{s}=\left(\frac{s^2-5s+4}{s}\right)$$

$$\frac{dk}{ds}=\frac{s\frac{d}{ds}(s^2-5s+4)-(s^2-5s+4)\frac{d}{ds}(s)}{s^2}$$

$$\Rightarrow S(2S-5)(s^2-5s+4)=0$$

$$\Rightarrow 2S^2-5S-S^2+5S-4=0$$

$\Rightarrow S^2 - 4 = 0 \Rightarrow S = \pm 2$

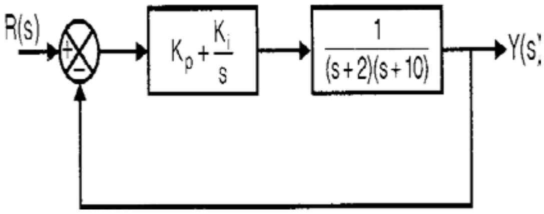
From the pole zero plot it is clean that Break away point must be as it is in between 2 poles

Now to find gain at this point use magnitude condition

$$\Rightarrow \left|\frac{KS}{(s-1)(s-4)}\right|_{s=2} = 1 \Rightarrow \left|\frac{KS}{(1)(-2)}\right| = 1 \Rightarrow K = 1$$

GATE QUESTIONS(IN)(Stability Analysis)

Q.1 The range of the controller gains (K_p, K_i) that makes the closed loop control system (shown in the following figure) stable is given as



a) $K_i < 0$ and $K_p < \frac{K_i}{12} - 20$

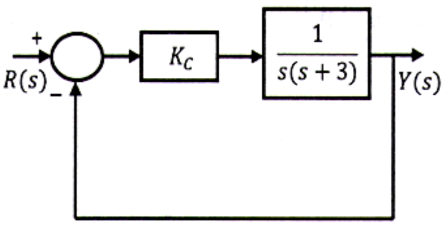
b) $K_i < 0$ and $K_p > \frac{K_i}{12} - 20$

c) $K_i < 0$ and $K_p > 0$

d) $K_i < 0$ and $K_p > \frac{K_i}{12} - 20$

[GATE-2006]

Q.2 A closed loop control system is shown below. The range of the controller gain K_c which will make the real parts of all the closed loop poles more negative than -1 is



a) K_c>-4 b) K_c>0

c) K_c>2 d) K_c<2

[GATE-2008]

Q.3 The open loop transfer function of a unity gain feedback system is given by : $G(s)\frac{k(s+3)}{(s+1)(s+2)}$. The range of positive values of k for which the closed loop system will remain stable is:

a) $1 < k < 3$ b) $0 < k < 10$

c) $5 < k < \infty$ d) $0 < k < \infty$

[GATE-2010]

Q.4 The first two rows of Routh's table of a third-order characteristic equation are

s^3	3	3
s^2	3	4

It can be inferred that the system has

a) one real pole in the right- half of s-plane

b) a pair of complex conjugate poles in the right-half of s-plane

c) a pair of real poles symmetrically placed around s=0

d) a pair of complex conjugate poles on the imaginary axis of the s-plane

[GATE-2011]

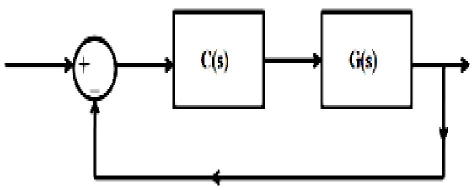
Q.5 The value of a_0 which will ensure that the polynomial s^3+2s+a_0 has roots on the left half of the s-plane is

a) 11 b) 9

c) 7 d) 5

[GATE-2016]

Q.6 For the feedback system given below, the transfer function $G(s) = \frac{1}{(s+1)^2}$. The system CANNOT be stabilized with



a) $C(s) = 1 + \frac{3}{s}$ b) $C(s) = 3 + \frac{7}{s}$

c) $C(s) = 3 + \frac{9}{s}$ d) $C(s) = \frac{1}{s}$

[GATE-2016]

Q.7 Consider a standard negative feedback configuration with $G(s) = \frac{1}{(s+1)(s+2)}$ and $H(s) = \frac{s+\alpha}{s}$. For the closed loop system to have a poles on the imaginary axis, the value of α should be equal to (up to one decimal place) ______________.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7
(d)	(c)	(d)	(d)	(d)	(c)	9

EXPLANATIONS

Q.1 (d)

Characteristic equation is

$s(s+2)(s+10)+(k_pS+k_i)=0$

$s^3+12s^2+20s+k_pS+k_i=0$

$$\begin{array}{c|cc} s^3 & 1 & 20+k_p \\ s^2 & 12 & k_i \\ s^1 & \frac{(240+12k_p-k_i)}{12} & \\ 1 & k_i & \end{array}$$

$\therefore K_i>0 \text{ and } 240+12k_p-k_i>0$

$\Rightarrow K_p>\frac{K_i}{12}-20$

Q.2 (c)

$s(s+3)+K_c=0$

$Gs^2+3s+K_c=0$

$S_{1,2}=\frac{-3\pm\sqrt{9-4K_c}}{2}=-1.5\pm\sqrt{\frac{9}{4}-K_c}$

$-1.5+\sqrt{\frac{9}{4}-K_c}<-1 \text{ or } \sqrt{\frac{9}{4}-K_c}<0.5$

$\frac{9}{4}-K_c<0.25, K_c>2$ OR

Put S=Z-1 and apply RH criterion for the polynomial in z

Q.3 (d)

$1+G(s)H(s)=0$

$\Rightarrow(s+1)(s+2)+k(s+3)=0$

System is stable, for all positive K. (from Routh Hurwitz criterion)

(or) $0<K<\infty$

Q.4 (d)

$$\begin{array}{c|cc} s^3 & 3 & 3 \\ s^2 & 4 & 4 \\ s & 0 & (\in>0) \\ 1 & & 4 \end{array}$$

$\therefore$ C.E is $3S^3+4S^2+3S+4$

$=0\Rightarrow(3S+4)(S^2+1)=0$

$S=-\frac{4}{3},\pm J$

Q.5 (d)

$$\begin{array}{c|cc} S^3 & 1 & 2 \\ S^2 & 3 & a_0 \\ S1 & \frac{6-a_0}{3} & \\ S^0 & a_0 & \end{array}$$

For Stability $\frac{6-a_0}{3}>0\Rightarrow a_0<6$

Q.6 (c)

The characteristic equation of system is 1+G(s) = 0

$\Rightarrow 1+\frac{C(S)}{S^2+2S+1}=0$

$\Rightarrow S^2+2S+1+C(S)=0$

if we take C(S)= $3+\frac{9}{5}$ then

S^3	1	4
S^2	2	9
S^1	$-1/2$	
S^0	9	

$$S^2+2S+1+3+\frac{9}{S}=0$$

$$\Rightarrow S^3+2S^2+4S+9=0$$

So system is unstable, remaining options gives stable.

Q.7 **9**

$$G(s)=\frac{1}{(s+1)(s+2)}$$

$$H(s)=\frac{s+\alpha}{s}$$

$$C.E=1+G(s)H(s)=0;$$

$$s(s+1)(s+2)+(s+\alpha)=0$$

$$s^3+3s^2+2s+s+\alpha=0$$

$$s^3+3s^2+3s+\alpha=0$$

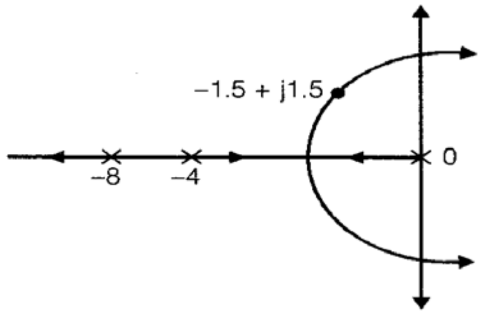
If system is marginal stable

$$3\times 3=\alpha$$

$$\alpha=9$$

GATE QUESTIONS(IN)(Root Locus)

Q.1 The roots locus of a plant is given in the following figure. The rot locus crosses imaginary at $\omega = 4\sqrt{2} rad/s$ with gain K= 384. It is observed that the point $s = -1.5 + j\,1.5$ lies in the root locus. The gain K at $s = -1.5 + j1.5$ is computed as



a) 11.3 b) 21.2
c) 41.25 d) 61.2

[GATE-2006]

Statement for linked Answer Questions Q.2 & Q.3:
A transfer function with unity DC gain has three poles at -1, -2 and -3 and no finite zeros. A plant with this transfer function is connected with this transfer function is connected with a proportional controller of gain K in the forward path, in a unity feedback configuration.

Q.2 The transfer function is

a) $\dfrac{S}{(s-1)(s-2)(s-3)}$

b) $\dfrac{6}{(s+1)(s+2)(s+3)}$

c) $\dfrac{S}{(s+1)(s+2)(s+3)}$

d) $\dfrac{6}{(s-1)(s-2)(s-3)}$

[GATE-2007]

Q.3 If the root locus plot of the closed loop system passes through the points $\pm j\sqrt{11}$, the maximum value of K for stability of the unity feedback closed loop system is

a) $\sqrt{11}$ b) 6
c) 10 d) $6\sqrt{11}$

[GATE-2007]

Q.4 The open loop transfer function of a unity feedback system is

$$G(s) = \frac{k(s+2)}{(s+1+j1)(s+1-j1)}$$

The root locus plot of the system has
a) Two breakaway points located at s = -0.59 and s = -3.41
b) One breakaway point located at s = -0.59
c) One breakaway point located at s = -3.41
d) One breakaway point located at s = -1.41

[GATE-2008]

Q.5 Consider the second- order system with the characteristic equation s(s+3)+K(s+5)=0 Based on the properties of the root loci, it can be shown that the complex portion of the root loci of the given system for 0<k <∞ is described by a circle, and the two breakaway points on the real axis are .

a) $-5 \pm \dfrac{\sqrt{5}}{2}$ b) $-5 \pm \sqrt{5}$
c) $-5 \pm \sqrt{10}$ d) $-5 \pm 2\sqrt{5}$

[GATE-2011]

Q.6 The open loop transfer function of a unity gain negative feedback control system is given by

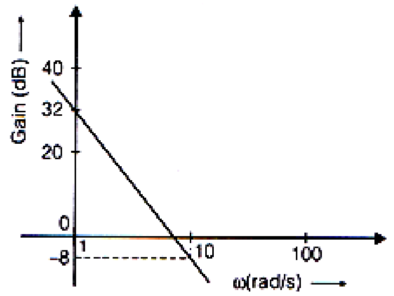
$G(s)=\frac{s^2+4s+8}{s(s+2)(s+8)}$.The angle θ, at which the root locus approaches the zeros of the system satisfies

a) $|\theta|=\pi-\tan^{-1}\left(\frac{1}{4}\right)$ b) $|\theta|=\frac{3\pi}{4}-\tan^{-1}\left(\frac{1}{3}\right)$

c) $|\theta|=\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ d) $|\theta|=\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3}\right)$

[GATE-2012]

Q.7 The Bode plot of transfer function G(s) is shown in the figure below.



The gain (20log |G(s))| is dB and -8 at 1 rad/s and 10rad/s respectively. The phase is negative for all ω then G(s) is

a) $\frac{39.8}{s}$ b) $\frac{39.8}{s^2}$

c) $\frac{32}{s}$ d) $\frac{32}{s^2}$

[GATE-2013]

Q.8 A loop transfer function is given by:

$G(S)H(S)=\frac{K(S+2)}{S^2(S+10)}$

The point of intersection of the asymptotes of G(s)H(s)on the real axis in the s-plane is at ________.

[GATE-2014]

Q.9 The open loop transfer function of a system $G(s)=\frac{s^2+6s+10}{s^2+2s+2}$. The angle of arrival of its root loci are

a) $\pm\frac{\pi}{4}$ b) $\pm\frac{\pi}{3}$

c) $\pm\frac{\pi}{2}$ d) $\pm\frac{5\pi}{6}$

[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6	7	8	9
(c)	(b)	(c)	(c)	(c)	(d)	(b)	-4	(a)

EXPLANATIONS

Q.1 (c)

$$G(s)=\frac{K}{s(s+4)(s+8)}$$

$$1+G(s)=0, G(s)=-|G(s)|$$

$$=1, \frac{K}{|s||s+4||s+8|}=1$$

At $s=-1.5+j1.5=1.5=1.5(-1+j1)$

$|s|=1.5\sqrt{2}, |s+4|$

$=\sqrt{2.5^2+1.5^2}=\sqrt{8.5},$

$|s+8|=\sqrt{6.5^2+1.5^2}=\sqrt{44.5}$

$\therefore K-1.5\sqrt{2}\sqrt{8.5}\sqrt{44.5}=41.25$

Note that the data given at the intersecting point with imaginary axis is not necessary

Q.2 (b)

With unity DC gain, poles at s=1,-2 and -3and no finite zeros

$$\text{Plant T.F}=\frac{6}{(s+1)(s+2)(s+3)}$$

Q.3 (c)

At $s=\pm J\sqrt{11}, |G(s).H(s)|$

$=1 \text{where } G(s).H(s)$

$$=\frac{6k}{(s+1)(s+2(s+3)}$$

$$\therefore \frac{6k}{2.3.5.2}=1 \Rightarrow k=10$$

Q.4 (c)

$$(s)=\frac{k(s+2)}{(s^2+2s+2)} \Rightarrow k=\frac{-(s^2+2s+2)}{(s+2)}$$

$$\frac{dk}{ds}=0 \Rightarrow \frac{(s^2+2s+2).1-(s+2).2(s+1)}{(s+2)^2}$$

$=0 \Rightarrow s^2+4s+2=0$

$S=-2\pm\sqrt{2}=-0.59/-3.41$

RL exists at -3.41

Q.5 (c)

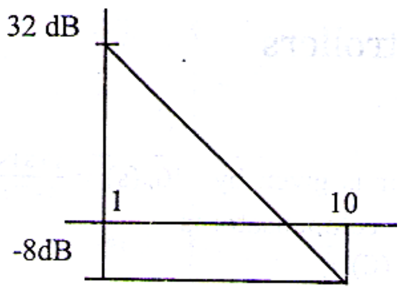
$$K=\frac{-s(s+3)}{(s+5)}$$

$$\frac{Dk}{ds}=0 \Rightarrow \frac{-((s+5).(2s+3)-(s^2+35).1)}{(S+5)^2}$$

$=0 \Rightarrow s^2+10s+15=0$

$$s=\frac{-10\pm\sqrt{100-60}}{2}=-5\pm\sqrt{10}$$

Q.7 (b)



$\omega=1$ to $\omega=10$

Is 1 dec are change & change is (G) is 40 dB

$\therefore$ S lope is 40dB / dec

$\therefore$ There are 2poles is orign

$$\text{So, } G(s)=\frac{K}{S^2}$$

$|G|_{at\omega=1}=32dB(\text{given})$

$$\Rightarrow 2\log\left|\frac{k}{\omega^2}\right|_{\omega=1}=32dB$$

$\Rightarrow 20\log k=32dB \Rightarrow k=39.8$

$$\therefore G=\frac{39.8}{S^2}$$

Q.8 (4)

The point of intersection of the asymptotes is nothing but a centroid.
Centroid.

$$\sigma = \frac{\sum \text{real part of poles} - \sum \text{real part of zeros}}{(P-Z)}$$

Given , $G(s)H(s) = \frac{K(s+2)}{s^2(s+10)}$

P=3; Z=1⇒ (P − Z) = 2

$$\sigma = \frac{[-10+0+0]-[-2]}{2} = \frac{-8}{2} = -4$$

Q.9 **(a)**

Angle of arrival is calculated on a complex zero and it is given by,

$\phi_a = 180 - \underline{|GH}$ (at a +ve imaginary zero)

$$G(s) = \frac{(s+3+i)(s+3-i)}{(s+1+i)(s+1-i)}$$

$$G(-3+i) = \frac{[-3+i+3+i][-3+i+3-i]}{[-3-i+1+i][-3+i+1-i]}$$

$$= \frac{[2i]}{[-2+2i][-2]}$$

$\underline{|G(-3+i)}$

$$= 90° - \left[180° - \tan^{-1}\frac{2}{2}\right] - [180°]$$

$$= 90° - 180° + 45° - 180° = 135°$$

$$\phi_a = 180° - 135° = 45° = \pi/4$$

Other angle will be same with opposite sign $\pm\pi/4$

4 FREQUENCY DOMAIN ANALYSIS

4.1 FREQUENCY RESPONSE

It is a measure of magnitude and phase of the output as a function of frequency, in comparison to the input. In simplest terms, if a sine wave is injected into a system at a given frequency, a linear system will respond at that same frequency with a certain magnitude and a certain phase angle relative to the input.

4.1.1 ADVANTAGE OF FREQUENCY RESPONSE

1) The design and parameter adjustment of the open-loop transfer function of a system for specified closed-loop performance is carried out somewhat more easily in frequency domain than in time domain.
2) Further the effects of noise disturbance and parameter variations are relatively easy to visualize and assess through frequency response. If necessary the transient response of a system can be obtained from its frequency response through the Fourier integral. An interesting and revealing comparison of frequency and time domain approaches is based on the relative stability studies of feedback systems.
3) The Routh criterion is a time domain approach which establishes with relative stability of a system, but its adoption to determine the relative stability is involved and requires repeated application of the criterion. The root locus method is a very powerful time domain approach as it reveals not only stability but also the actual time response of the system. On the other hand, the Nyquist criterion is a powerful frequency domain method of extracting the information regarding stability as well as relative stability of a system without the need to evaluate roots of the characteristic equation.

4.1.2 RELATION WITH TRANSFER FUNCTION

Consider the transfer of a system as:

$$T(s)=\frac{C(s)}{R(s)}$$

The frequency response function can be obtained simply by replacing s by $j\omega$

Then $T(j\omega)=\dfrac{C(j\omega)}{R(j\omega)}$

e.g. if $T(s)=\dfrac{1}{1+s}$ then $T(j\omega)=\dfrac{1}{1+j\omega}$

From this frequency response function we can calculate

1) Magnitude: The magnitude of the frequency response function is also called as the gain of system.

   e.g. if $T(j\omega)=\dfrac{1}{1+j\omega}$ then

   $$|T(j\omega)|=\frac{1}{\sqrt{1^2+\omega^2}}$$

2) Phase: The phase angle of the frequency response function is nothing but the phase shift provided by the system to the input.

   e.g. if $T(j\omega)=\dfrac{1}{1+j\omega}$ then

   $$\angle T(j\omega)=\frac{0}{\tan^{-1}\left(\frac{\omega}{1}\right)}=-\tan^{-1}(\omega)$$

4.2 CORRELATION BETWEEN TIME AND FREQUENCY RESPONSE

Consider a second order system with open loop transfer function

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Then $G(j\omega) = \dfrac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)}$

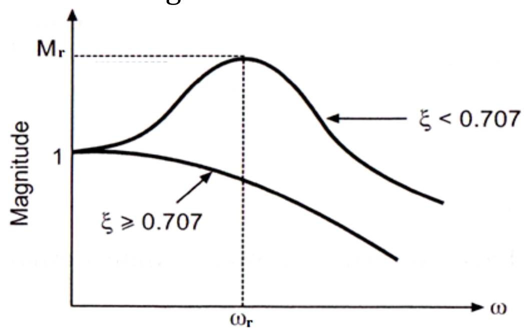
Where ξ is the damping factor ω_n is the undamped natural frequency of oscillations.

Now the closed loop transfer function with unity feedback will be

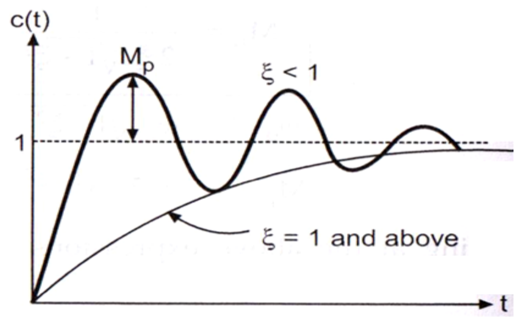
$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

4.2.1 RESONANT PEAK AND RESONANT FREQUENCY & BANDWIDTH

The frequency response magnitude & time response of a 2nd order system are as shown in the figure.



Frequency response magnutude of 2nd order system



Time response of 2nd order system

In the frequency response a 2nd order system shows a peak called resonant peak M_r and the corresponding frequency is called resonant frequency ω_r. For a second-order feedback control system, the peak resonant M_r, the resonant frequency ω_r, & the bandwidth are all uniquely related to the damping ratio ξ & the natural undamped frequency ω_n of the system.

The resonant peak is given by

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

The resonant frequency is

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

Bandwidth is

$$B.W. = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}$$

Note:

- From the equation $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$, M_r gets vanished when
  $\sqrt{1-2\xi^2} = 0$
  $2\xi^2 = 1$
  $\xi^2 = 1/2$
  $\therefore \ \xi = 0.707$
  i.e. at $\xi = 0.707, M_r = 1$
- For a second order system, the resonant peak M_r of its frequency response is indicative of its damping factor ξ for $0 < \xi \le 1/\sqrt{2}$.
- The resonant frequency ω_r of the frequency response is indicative of its natural frequency for a given ξ and hence indicative of its speed of response $\left(\text{as } t_s = \dfrac{4}{(\xi\omega_n)}\right)$.
- The frequency at which M has a value of $1/\sqrt{2}$ is of special significance and is called the cut-off frequency ω_c. The signal frequencies above cut-off are greatly attenuated in passing through a system.

Example

For a unity feedback system $G(s) = \dfrac{K}{s(1+s\tau)}$ determine the values of

K and τ, so that $M_r = 1.06$ and $\omega_n = 12 rad/sec$.

Solution

$$G(s) = \frac{K}{s(1+s\tau)}, H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(1+s\tau)}}{1+\frac{K}{s(1+s\tau)}} = \frac{K}{\tau s^2 + s + K} = \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}$$

Comparing denominator with

$s^2 + 2\xi\omega_n s + \omega_n^2$

$$\therefore \quad \omega_n^2 = \frac{K}{\tau}$$

i.e. $\omega_n = \sqrt{\frac{K}{\tau}}$ ... (1)

and $2\xi\omega_n = \frac{1}{\tau}$

i.e. $\xi = \frac{1}{2\sqrt{K\tau}}$ ... (2)

Now, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.06$

$\therefore \quad \xi\sqrt{1-\xi^2} = 0.4716$

i.e. $\xi^2(1-\xi^2) = 0.2225$

$\therefore \quad \xi^4 - \xi^2 + 0.2225 = 0$

i.e. $\xi^2 = 0.6658, 0.3341$

$\therefore \quad \xi = 0.8159, 0.578$ but ξ cannot be more that 0.707

$\therefore \quad \xi = 0.578$

Usingequation (1)

$$\omega_n = \sqrt{\frac{K}{\tau}}$$

i.e. $12 = \sqrt{\frac{K}{\tau}}$

i.e. K=144 τ

Using equations (2),

$$0.578 = \frac{1}{2\sqrt{144\tau \times \tau}}$$

i.e. $0.578 = \frac{1}{2 \times 12 \times \tau}$

$\therefore \quad \tau = 0.072, K = 10.3806$

4.3 BODE PLOTS

A Bode plot is a graph of the transfer function of a linear, time-invariant system versus frequency, plotted with a log-frequency axis, to show the system's frequency response. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response gain, and a Bode phase plot, expressing the frequency response phase shift i.e. a Bode plot consists of two graphs:

1) A plot of the magnitude in dB of a sinusoidal transfer functions against the frequency in logarithmic scale.
2) A plot of the phase angle against the frequency in logarithmic scale.

Note:
The magnitude & the phase angle can be calculated as shown in the example

e.g.

if $T(j\omega) = \frac{1}{1+j\omega}$ then the magnitude is

$|T(j\omega)| = \frac{1}{\sqrt{1^2+\omega^2}}$ & the phase angle is

$$\angle T(j\omega) = \frac{0}{\tan^{-1}\left(\frac{\omega}{1}\right)} = -\tan^{-1}(\omega)$$

4.3.1 ADVANTAGES OF USING LOGARITHMIC SCALE

- The main advantage of using the logarithmic plot is that multiplication of magnitude can be converted into addition.
- The logarithmic representation is useful in that it shows both the low-and high frequency characteristics of the transfer function in one diagram.
- Expanding the low frequency range by use of a logarithmic scale for the frequency is very advantageous since characteristics at low frequencies are most important in practical systems.

Note:
It is not possible to plot the curves right down to zero frequency because of the logarithmic frequency $(\log 0 = -\infty)$; this does not create a serious problem.

4.3.2 BASIC FACTORS OF G (jω) H (jω)

Consider the open loop transfer function of a system as

$$G(s)H(s) = \frac{K's^z(s+z_1)(s+z_2)}{s^p(s+p_1)(s+p_2)}$$

In the above equation either s^z or s^p will be there at a time. Converting the above equation into time constant form we get

$$G(s)H(s) = \frac{Ks^z(1+T_as)(1+T_bs)\dots}{s^p(1+T_1s)(1+T_2s)\dots}$$

Where $K = \frac{K'z_1 \times z_2 \dots}{p_1 \times p_2 \dots}$

T_1, T_2, T_a, T_b are the time constants of different poles & zeros.
Each term involved in the transfer function contributes some magnitude & phase to the system. Putting $s = j\omega$ in the above expression we get the factors in the transfer function.

1) Gain K
2) Poles or zeros at origin $(j\omega)^{\pm p}$
3) Simple poles & simple zeros $(1+j\omega T)^{\pm 1}$
4) Quadratic poles and zeros

$$\left(1+2\xi j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right)^{\pm 1}$$

4.3.2.1THE GAIN K

$G(s)H(s) = K$

$G(j\omega)H(j\omega) = K + j0$

$\therefore \ |G(j\omega)H(j\omega)| = K$ and

$\angle G(j\omega)H(j\omega) = 0$

Note:
As gain K is constant, its magnitude plot $20\log_{10} K$ will be constant.

4.3.2.2POLES OR ZEROS AT ORIGIN

$G(j\omega)H(j\omega) = (j\omega)^{\pm p}$

$\therefore \ |G(j\omega)H(j\omega)| = \omega^{\pm p}$

In dB $20\log_{10} \log_{10} \omega^{\pm p} = 20 \times \pm p \log_{10} \omega$

and $\angle G(j\omega)H(j\omega) = (90)^{\pm p}$

Note:
For poles and zeros at origin, the magnitude plot will be a straight line with slope of $\pm p \times 20\text{dB}/$ decade passing through $\omega = 1$.

e.g. If $G(j\omega)H(j\omega) = \frac{1}{j\omega}$ then

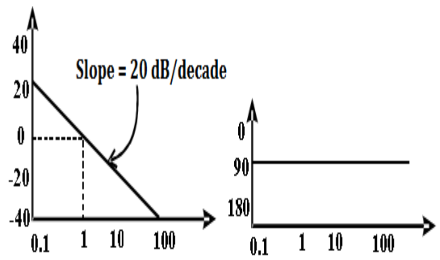
$|G(j\omega)H(j\omega)| = \frac{1}{\omega}$ in dB

$20\log_{10}\frac{1}{\omega} = -20\log_{10}\omega$

which is an equation of straight line $(y = mx + c)$ with a slope of $-20\text{dB}/\text{decade}$.

Also $\angle G(j\omega)H(j\omega) = -90^\circ$

Therefore the phase plot will be a straight line



Note:
The slope is sometime written as dB/octave $-20\text{db}/\text{decade} = -6\text{db}/\text{octave}$

4.3.2.3SIMPLE POLES AND ZEROS

$G(j\omega)H(j\omega) = (1+j\omega T)^{\pm 1}$

$\therefore \ |G(j\omega)H(j\omega)| = \left(\sqrt{1+(\omega T)^2}\right)^{\pm 1}$

In dB $20\log_{10}\left(\sqrt{1+(\omega T)^2}\right)^{\pm 1}$

$= 20 \times \pm 1 \log_{10}\sqrt{1+(\omega T)^2}$

and $\angle G(j\omega)H(j\omega) = \left[\tan^{-1}(\omega T)\right]^{\pm 1}$

Note:

- For simple poles and zeros the magnitude plot will be a straight line with slope $+20\ \text{dB/decade}$ and $-20\ \text{dB/decade}$ respectively.
- For low frequencies (ω<<1/T)
  $\pm 20\log_{10}\sqrt{1+(\omega T)^2} \approx \pm 20\log_{10} 1 = 0\text{dB}$
  Therefore for $\omega < \frac{1}{T}$ the magnitude plot will be a straight line coinciding with 0 dB line.
- For high frequencies (ω>>1/T)
  $\pm 20\log_{10}\sqrt{1+(\omega T)^2} \approx \pm 20\log_{10}\omega T\text{dB}$
  Therefore for $\omega > \frac{1}{T}$ the magnitude plot will be a straight line with slope of $\pm 20\ \text{dB/decade}$.
- The magnitude plot of simple pole & zero has some approximation & the error due to this approximation is for zero $\pm n \times 3\text{dB}$ (+ve & −ve for pole). Where n is the order of simple poles or zeros.

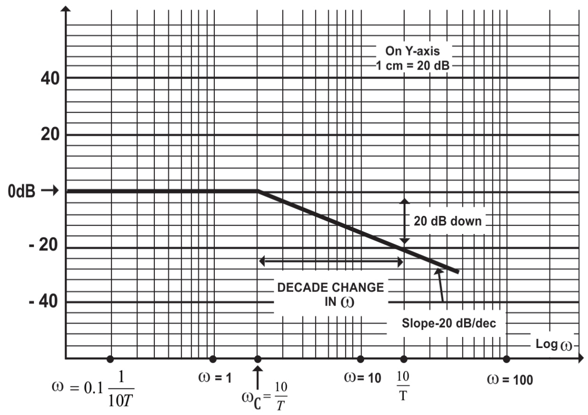
e.g. If $G(j\omega)H(j\omega) = \frac{1}{1+j\omega T}$ then

$$|G(j\omega)H(j\omega)| = \frac{1}{\sqrt{(1+(\omega T)^2}}$$

In dB $20\log_{10} \log_{10}\frac{1}{\sqrt{(1+(\omega T)^2}}$

$= -20\log_{10}\sqrt{(1+(\omega T))^2}\text{dB}$

which is a straight line with slope -20dB/decade & will start from $\omega_C = \frac{1}{T}$



Magnitude plot for simple pole

4.3.2.4 QUADRATIC POLES & ZEROS

$$G(j\omega)H(j\omega) = \left(1+2\xi j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right)^{\pm 1}$$

$$\therefore\ |G(j\omega)H(j\omega)| = \left(\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}\right)^{\pm 1}$$

$$\text{In dB} = 20\log_{10}\left(\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}\right)^{\pm 1}$$

$$= \pm 20\log_{10}\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}$$

and $\angle G(j\omega)H(j\omega)$

$$= \tan^{-1}\left(\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)^{\pm 1}$$

Note:

- The magnitude plot for quadratic poles & zeros is a straight line with slope $\pm 40\ \text{dB/decade}$.
- The corner frequency for Quadratic poles & zeros is $\omega_c = \omega_n$.

4.3.3 PROCEDURE FOR PLOTTING BODE PLOT

1) Write the transfer function in time constant form & put s = jω.
2) Then identify the corner frequencies associated with the basic factors.
3) Draw the asymptotic log-magnitude curves with proper slopes between the corner frequencies.
4) Shift the curve up or down by $20\log_{10} K$.
   $(\text{up for } K > 1 \text{ \& down for } K < 1)$
5) The phase angle curve of G (jω) H (jω) can be drawn by adding the phase angle curves of individual factors.

Example

Sketch the Bode plot for

$$G(s)H(s) = \frac{20}{s(1+0.1s)}$$

Solution

The transfer function is already in the time constant form. Identify the factors from the transfer function

1) Gain $K = 20$
   Its mgnitude $= 20\log_{10} 20 = +26\text{dB}$.
   Therefore the magnitude plot will be shifted up by *26 dB*.
2) 1 pole at origin $\frac{1}{s}$ Its magnitude plot is a straight line with slope $-20\text{dB}/\text{decade}$ passing through point $\omega = 1$
3) 1 simple pole $\frac{1}{1+0.1s}$ T=0.1 & the corner frequency $\omega_c = \frac{1}{T} = \frac{1}{0.1} = 10\text{rad}/\text{sec}$
   .The magnitude plot is a straight line with slope $-20\text{dB}/\text{decade}$ starts from $\omega_c = 10\text{rad}/\text{sec}$.

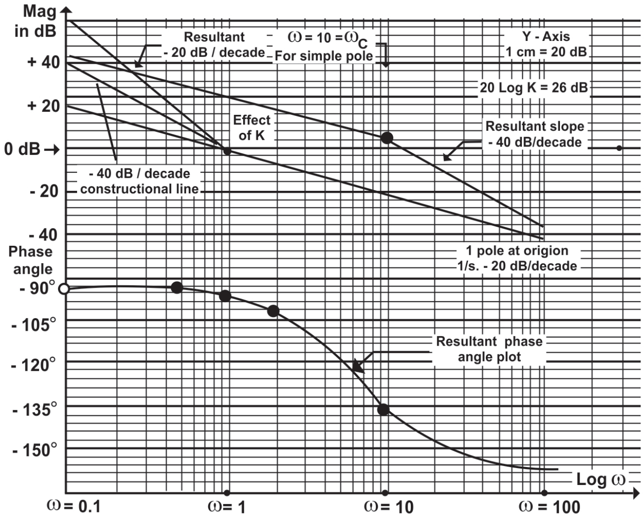
Note:

- As plot of pole at origin already has slope of −20 dB/decade, the slope after simple pole will be
  $$\left(-20\frac{\text{dB}}{\text{decade}}\right) + \left(-20\frac{\text{dB}}{\text{decade}}\right)$$
  $$= -40\text{dB}/\text{decade}$$
- After drawing the slopes for simple pole & pole at origin we will have to shift the graph up by *26dB*.

1) The total phase angle of the transfer function is as shown in the table

ω in rad/sec	∅ due to 1 pole at origin	∅ due to simple pole $= -\tan^{-1}(0.1\omega)$	$\varnothing_T$ Total
0.1	-90^o	-0.57^o	-90.57^o
0.5	-90^o	-2.86^o	-92.86^o
1	-90^o	-5.7^o	-95.7^o
2	-90^o	-11.3^o	-101.3^o
10	-90^o	-45^o	-135^o
50	-90^o	-78.79^o	-168^o

Plotting the magnitude & phase plot against frequency we get



4.3.4 GAIN CROSSOVER FREQUENCY

It is the frequency at which the gain of the system is unity or 0 dB i.e. at gain crossover frequency

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

In $20\log_{10} 1 = 0\text{dB}$

4.3.5 PHASE CROSSOVER FREQUENCY

It is the frequency at which the phase of the system is -180^o i.e. at phase crossover frequency

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = -180^{\circ}$$

4.3.6 GAIN MARGIN

As we seen earlier in root locus that, if gain K of the system is increased, after some value of gain the system becomes unstable. Gain margin of the system is defined as the allowable gain so that the system reaches on the verge of instability. Also the gain margin of system is defined as the reciprocal of the gain of system at phase crossover frequency.

i.e. $G.M. = \dfrac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$

In dB

$$G.M. = 20\log_{10}\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

$$G.M. = -20\log_{10}|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

Note: For stable systems the gain margin is +ve.

4.3.7 PHASE MARGIN

Phase margin of a system is an additional phase lag that can be introduced in the system at gain crossover frequency till it reaches on the verge of instability.

$$P.M. = 180^{\circ} + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}$$

Note:

For stable systems the phase margin should be positive.

4.3.8 CRITERION FOR STABILITY

Sr. No.	Stability	Frequency Condition	G.M. Condition	P.M. Condition
1	Stable	$\omega_{pc} > \omega_{gc}$	Positive	Positive
2	Unstable	$\omega_{pc} < \omega_{gc}$	Negative	Negative
3	Marginally Stable	$\omega_{pc} = \omega_{gc}$	Zero	Zero

Example:

For a unity feedback system $G(s) = \dfrac{242(s+5)}{s(s+1)(s^2+5s+121)}$ sketch Bode plot & determine ω_{gc}, ω_{pc} Gain margin, phase margin.

Solution:

First convert the transfer function into time constant form

$$G(s) = \frac{242\times 5\left(\frac{s}{5}+1\right)}{121\times s(s+1)\left(\frac{s^2}{121}+\frac{5s}{121}+1\right)}$$

$$= \frac{10\times\left(1+\frac{s}{5}\right)}{s(1+s)\left(1+\frac{5s}{121}+\frac{s^2}{121}\right)}$$

The factors in the transfer function in the increasin order of their corner frequencies are

1) Gain K = 10

   Its magnitude will be $20\log_{10}10 = +20\text{dB}$

   Therefore the magnitude plot will be shifted up by 20 dB.

2) 1 Pole at origin $\dfrac{1}{s}$

   It magnitude plot will be a straight line with slope −20 dB/decade passing through $\boldsymbol{\omega = 1}$. The phase angle contributed by this pole will be -90^{0}.

3) Simple pole $(1+s)$

   Its magnitude plot will be a straight line with slope −40 dB/decade starting from the corner frequenc $\omega_c = \dfrac{1}{T} = \dfrac{1}{1} = 1\text{rad/sec}$

Note:

In −40 dB/decade slope the slope of previous pole $\left(\text{i.e.}\dfrac{1}{s}\right)$ −20 dB/decade is also added.

The phase angle contributed by this pole will be $-\tan^{-1}\left(\dfrac{\omega}{1}\right)$.

1) Simple zero $\left(1+\frac{s}{5}\right)$

Its magnitude plot will be a straight line with slope −20 dB/decade starting from the corner frequenc $\omega_c = \frac{1}{T} = \frac{1}{\frac{1}{5}} = 5 \text{rad/sec}$.

Note:

For a simple zero the slope is always +20 dB/decade but the slope of previous 2 poles are added hence the resultant slope is

$$+20\frac{dB}{decade} + \left(-40\frac{dB}{decade}\right) = -20 dB/decade$$

The phase angle contribute by this zero will be $-\tan^{-1}\left(\frac{\omega}{5}\right)$.

2) Quadratic poles $\left(1+\frac{5s}{121}+\frac{s^2}{121}\right)$

The magnitude plot will be a straight line with slope −60 dB/decade starting from the corner frequency $\omega_c = \sqrt{121} = 11 \text{rad/sec}$.

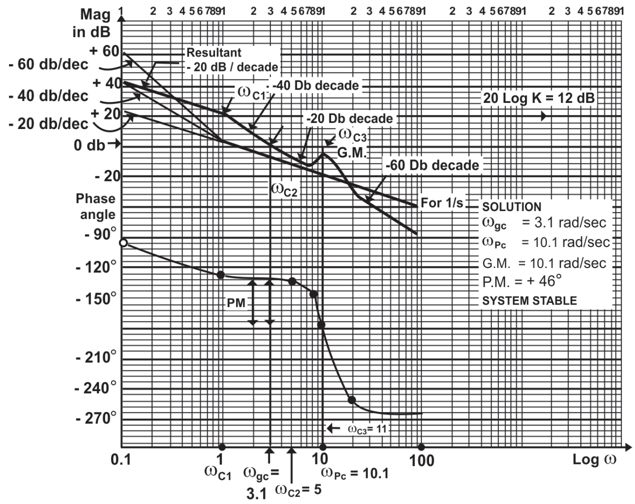
Note:

The slope for quadratic poles is −40 dB/decade but the slope of previous 3 factors will also be added hence the resultant slope is

$$-40\frac{dB}{decade} + \left(-20\frac{dB}{decade}\right) = -60 dB/decade$$

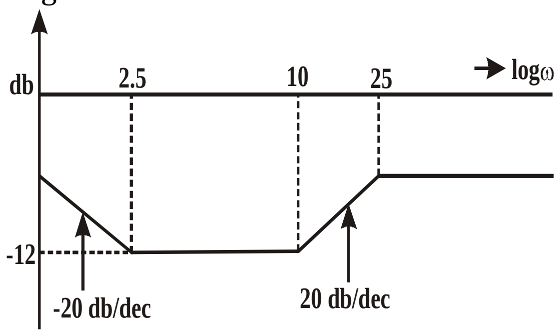
The phase angle contributed by these poles will be $-\tan^{-1}\left(\frac{\frac{5\omega}{121}}{1-\frac{\omega^2}{121}}\right)$.

ω	$\frac{1}{j\omega}$	$-\tan^{-1}(\omega)$	$-\tan^{-1}\left(\frac{\omega}{5}\right)$	$-\tan^{-1}\left(\frac{\frac{5\omega}{121}}{1-\frac{\omega^2}{121}}\right)$	$\varnothing_T$
0.1	−90°	+1.14°	−5.7°	−0.23°	−94.7°
1	−90°	+11.3°	−45°	−2.36°	−126°
5	−90°	+45°	−78.6°	−14.4°	−138°
8	−90°	+58°	−82.8°	−34.8°	−149.8°
10	−90°	+63.4°	−84.2°	−67.0°	−177.8°
20	−90°	+75.9°	−87.13°	+19.5° −180° = −160.4°	−261.63°
∞	−90°	+90°	−90°	−180°	−270°



Example:

Find the open-loop transfer function of a system whose approximate plot is shown in Fig.



Solution:

1) The starting slope of the graph is −20 dB/decade hence there is a pole of order1 at origin.
2) First corner frequency is at 2.5 rad/sec& at this frequency there is a change

of +20 dB/decade in slope hence there is a zero at 2.5 rad/sec.

3) Next corner frequency is at 10 rad/sec& at this frequency there is change +20 dB/decade in slope hence there is a zero at 10 rad/sec.

4) Next corner frequency is at 25 rad/sec& at this frequency there is change −20 dB/decade in slope hence there is a pole at 25 rad/sec.

5) Thus the transfer function is

$$G(j\omega)H(j\omega)=\frac{K\left(1+\frac{s}{2.5}\right)\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{25}\right)}$$

6) To find gain K write the equation for the straight line at the starting of the graph

$y=-20\log\omega+c$

At $\omega=2.5, y=-12dB$

$\therefore\ -12=-20\log 2.5+c$

$c=-4.04dB$

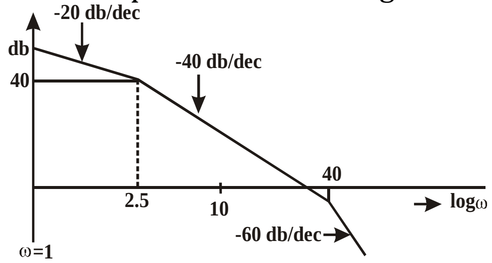
Now c is the shift in the graph

$c=20\log_{10}K$

$\therefore\ K=0.63$

Example

Determine the transfer function whose approximate plot is shown in Fig.



Solution:

1) The starting slope is −20 dB/decade hence there is a pole of order 1 at origin.

2) First corner frequency is at 2.5 rad/sec & at this frequency there is a change of −20 dB/decade in slope hence there is a simple pole at2.5 rad/sec.

3) Next corner frequency is at 40rad/sec & there is a slope change of −20 dB/ decade at this frequency hence there is another simple pole at 40rad/sec.

4) Thus the transfer function for the given Bode plot is

$$G(j\omega)H(j\omega)=\frac{K}{s\left(1+\frac{s}{2.5}\right)\left(1+\frac{s}{40}\right)}$$

5) To find gain K write the equation for the straight line at the starting of the graph

$y=-20\log\omega+c$

At $\omega=2.5, y=+40dB$

$\therefore\ +40=-20\log 2.5+c$

$c=+47.95dB$

Now c is the shift in the graph

$c=20\log_{10}K$

$\therefore\ K=250$

5 POLAR & NYQUIST PLOTS

5.1 POLAR PLOTS

In polar plots the magnitude of the frequency response is plotted against the phase angle for variations in frequency ω i.e. $|G(s)H(s)|$ is plotted against $\angle G(s)H(s)$.

To sketch the polar plot of G (jω) for the entire range of frequency ω, i.e., from 0 to infinity, there are four key points that usually need to be known:

1) The start of plot where ω = 0
2) The end of plot where ω = ∞
3) The point where polar plot cuts the –ve real axis gives the magnitude at phase crossover frequency
   I.e. $|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$.
4) The angle with respect to +ve real axis where the polar plot cuts the unit circle gives phase angle at gain crossover frequency i.e. $\angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$.

Example:

Draw the polar plot for $G(s)H(s) = \frac{5}{s}$.

Solution:

For the given transfer function, the magnitude & the phase angle are:

$$G(j\omega)H(j\omega) = \frac{5}{j\omega}$$

$$\therefore |G(j\omega)H(j\omega)| = \frac{5}{\omega}$$

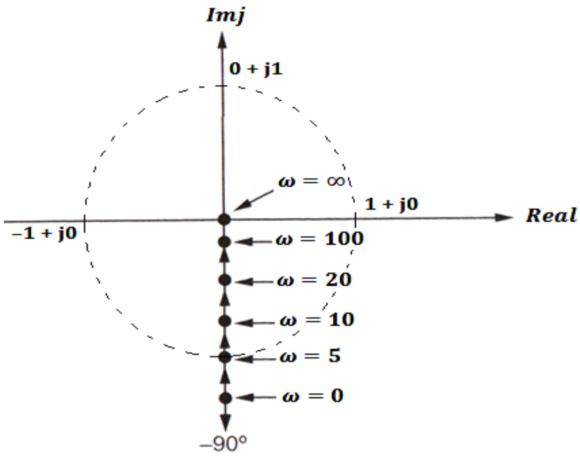
$$\text{and } \angle G(j\omega)H(j\omega) = \frac{0^o}{\tan^{-1}\left(\frac{\omega}{0}\right)}$$

$$= -\tan^{-1}\left(\frac{\omega}{0}\right) = -\tan^{-1}(\infty) = -90^o$$

Now find the magnitude & phase at different values of ω

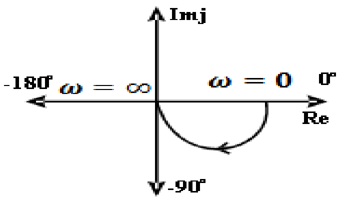
ω	$\lvert G(j\omega)H(j\omega)\rvert$	$\angle G(j\omega)H(j\omega)$
0	∞	-90^o
5	1	-90^o
10	0.5	-90^o
20	0.25	-90^o
100	0.05	-90^o
. . .	. . .	. . .
∞	0	-90^o

Plotting gain & phase on the graph we get

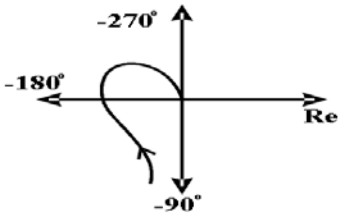


5.1.1 POLAR PLOTS OF SOME STANDARD FUNCTIONS

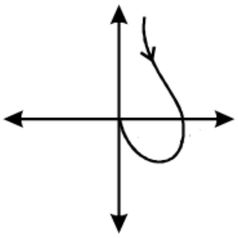
1) $G(s)H(s) = \frac{1}{(1+Ts)}$



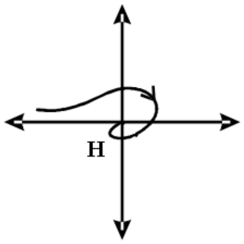
2) $G(s)H(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$



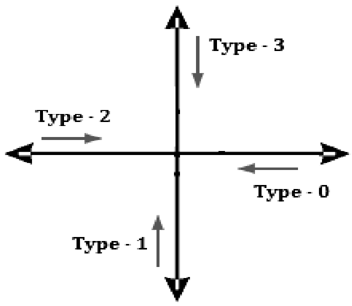
3) $G(s)H(s) = \dfrac{1}{s^3(1+T_1s)(1+T_2s)(1+T_3s)}$



4)
$G(s)H(s) = \dfrac{1}{s^2(1+T_1s)(1+T_2s)(1+T_3s)(1+T_4s)}$

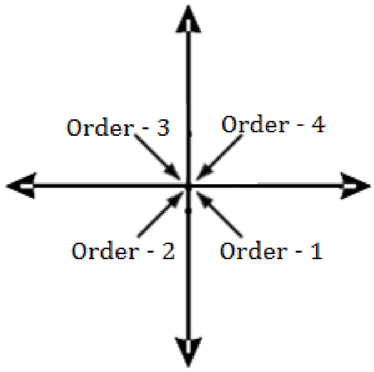


Note:
1) The start of the polar plots (at $\omega = 0$) depends on the type number of the system.



e.g. Type -0 system will always have its polar plot starting from +ve real axis.

2) The end of the polar plot (at $\omega = \infty$) depends on the order of the system.

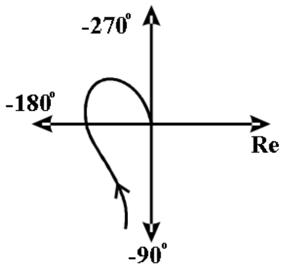


e.g. Order -1 system will always have its polar plot ending at origin in 4th quadrant.

5.1.2 POLAR PLOT FOR S_1

The curve S_1 is the whole +ve imaginary axis from $\omega = 0$ to $\omega = \infty$. The polar plot of S_1 will be the polar plot of given transfer function.

e.g. If $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$ the polar plot for S_1 will be the polar plot of given transfer function. As given transfer function is type -1 and order -3, its polar plot will be



5.1.3 POLAR PLOT FOR S_2

The curve S_2 covers the whole RHS s-plane from $+90^\circ$ to -90° & its polar will be plotted by putting $s = \lim_{R\to\infty} Re^{j\theta}$ where θ varies from $+90^\circ$ to -90°.

e.g. If $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$ put

$s = \lim_{R\to\infty} Re^{j\theta}$

As $R \to \infty$, s>>2 & s>>10 therefore the transfer function can be approximated as

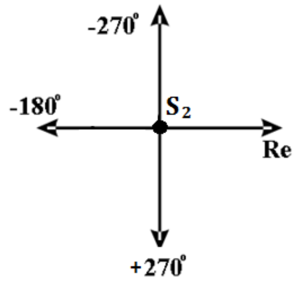
$G(s)H(s) = \frac{K}{s^3}$; Putting $s = \lim_{R\to\infty} Re^{j\theta}$

we get $G(s)H(s) = \frac{K}{\left(\lim_{R\to\infty} Re^{j\theta}\right)^3}$

$= \frac{K}{R^3 e^{j3\theta}} = 0e^{-j3\theta}$

As θ varies from $+90^\circ$ to -90°, $G(s)H(s)$ can be written as

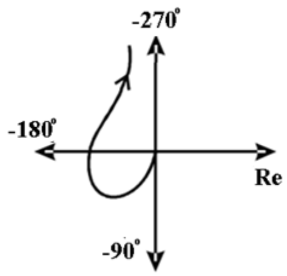
$G(s)H(s) = 0$ from -270° to $+270^\circ$.



5.1.4 POLAR PLOT FOR S_3

The curve S_3 is the whole –ve imaginary axis from $\omega = -\infty$ to $\omega = 0$. The polar plot for S_3 will be the inverse polar plot of given transfer function.

e.g. If $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$ the polar plot for S_3 will be the inverse polar plot of given transfer function. As given transfer function is type -1 and order -3, its inverse polar plot will be



5.1.5 POLAR PLOT FOR S_4

The curve S_4 is around origin from -90° to $+90^\circ$ & its polar plot will be plotted by putting $s = \lim_{R\to 0} Re^{j\theta}$ where θ varies from -90° to $+90^\circ$.

e.g. If $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$

put $s = \lim_{R\to 0} Re^{j\theta}$

As $R \to 0$, s<<2 & s<<10 therefore the transfer function can be approximated as

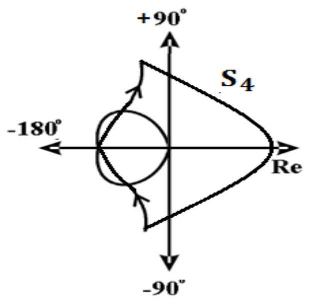
$G(s)H(s) = \frac{K}{20s}$ putting $s = \lim_{R\to 0} Re^{j\theta}$

we get

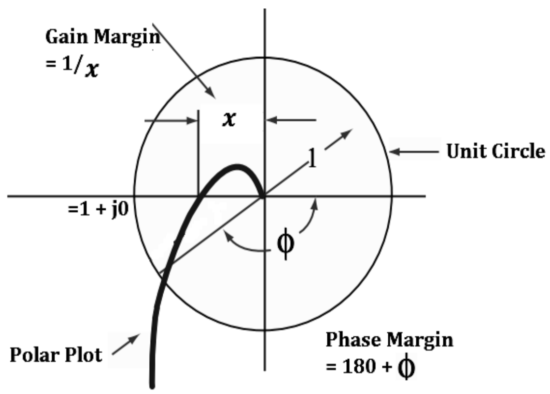
$G(s)H(s) = \frac{K}{20 \times \left(\lim_{R\to 0} Re^{j\theta}\right)} = \infty e^{-j\theta}$

as θ varies from -90° to $+90^\circ$, $G(s)H(s)$ can be written as

$G(s)H(s) = \infty$ from $+90^\circ$ to -90°.



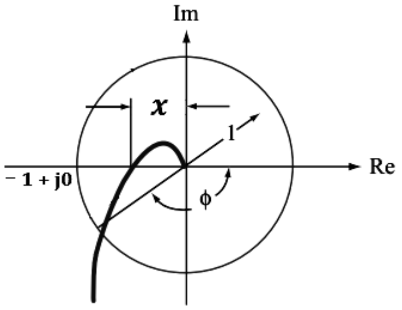
5.2 PROCEDURE TO FIND G.M. & P.M.



- The frequency at which polar plot intersects unity circle is the gain crossover frequency ω_{gc}.
- The frequency at which polar plot cuts –ve real axis is the phase crossover frequency ω_{pc}.

- The gain margin is reciprocal of the distance from origin to the point at which polar plot cuts the –ve real axis.
- Phase margin is the angle made by the line joining origin & the intersection point of polar plot with unit circle with the positive real axis $+180^o$.

5.2.1 STABLE SYSTEM



- In this figure the intersection of polar plot with –ve real axis is within unit circle i.e. $x < 0$

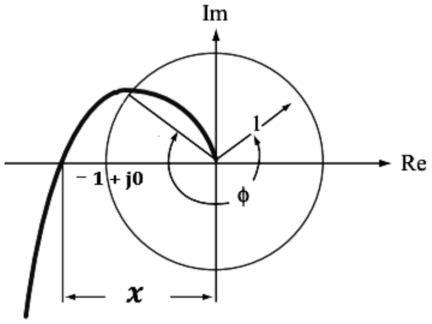
  $G.M. = 20\log_{10}\frac{1}{x}$ is positive

- The angle ϕ lies between $0^o \& -180^o$

  $P.M. = 180^o + \varnothing$ is positive

- As gain margin & phase margin both are positive therefore the system is stable.

5.2.2 UNSTABLE SYSTEM



- In this figure the intersection of polar plot with –ve real axis is within unit circle i.e. x>0

  $G.M. = 20\log_{10}\frac{1}{x}$ is negative

- The angle $\varnothing > -180^o$

  $P.M. = 180^o + \varnothing$ is negative

- As gain margin & phase margin both are negative therefore the system is unstable.

Note:
For a marginally stable system, the gain margin & phase margin both are 0.

Example
Draw the polar plot for the transfer function $G(s)H(s) = \frac{1}{s(1+Ts)}$

Solution
For the given transfer function, the magnitude & phase angle are

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

$$\therefore |G(j\omega)H(j\omega)|$$

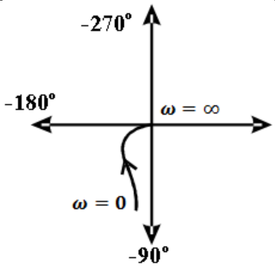
$$= \frac{1}{\omega\sqrt{1+(\omega T)^2}}$$

and $\angle G(j\omega)H(j\omega)$

$$= \frac{0^o}{+90^o + \tan^{-1}(\omega T)} = -90^o - \tan^{-1}(\omega T)$$

Find the gain & phase angle at $\omega = 0$ & $\omega = \infty$

ω	$\|G(j\omega)H(j\omega)\|$	$\angle G(j\omega)H(j\omega)$
0	∞	-90^o
∞	0	-180^o



Example
Draw the polar plot for

$$G(s)H(s) = \frac{1}{(1+T_1s)(1+T_2s)}.$$

Solution
For the given transfer function, the magnitude & phase angle are

$$G(j\omega)H(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$\therefore |G(j\omega)H(j\omega)|$$

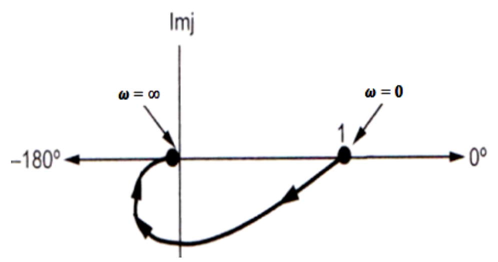
$$= \frac{1}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} \text{ and}$$

$$\angle G(j\omega)H(j\omega) = \frac{0^\circ}{+\tan^{-1}(\omega T_1)+\tan^{-1}(\omega T_2)}$$

$$= -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

Find the gain & phase angle at
$\omega = 0$ & $\omega = \infty$

ω	$\lvert G(j\omega)H(j\omega)\rvert$	$\angle G(j\omega)H(j\omega)$
0	1	0°
∞	0	-180°

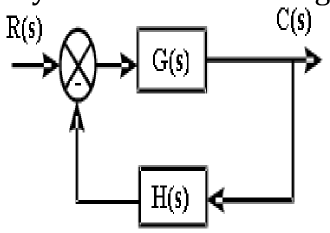


5.3 NYQUIST PLOT

Polar is the basis of Nyquist plot. For a closed loop system to be stable, the Nyquist plot of G(s)H(s) must encircle the (−1, j0) point as many times as the number of poles of G(s)H(s) that are in the right half of s-plane, and the encirclement, if any, must be made in the counter clockwise direction. Nyquist stability criterion is based on the complex analysis result known as Cauchy's principle of argument.

5.3.1 NYQUIST STABILITY CRITERION

Consider a system with block diagram



The open loop & closed loop transfer function of the system are

Open loop T.F.⇒ G(s)H(s)

Closed loop T.F. $\Rightarrow \frac{G(s)}{1+G(s)H(s)}$

For this system if

1) P is the number of open loop poles on the right hand side of s plane.

e.g. if $G(s)H(s) = \frac{1}{(1+s)(1-s)}$

The poles are at $s=-1$ & $s=1$ hence $P=1$. ($s=1$ is the open loop pole on RHS of s-plane)

2) Z is the number of closed loop poles on the right hand side of s plane.

e.g. if the closed loop transfer function is

$$\frac{G(s)}{1+G(s)H(s)} = \frac{1}{s^2+s-2}$$

The poles are at $s=-2$ & $s=1$ hence Z = 1. ($s=1$ is the closed loop pole on RHS of s-plane)

3) N is the number of encirclement of $-1+j0$ point by the Nyquist plot in anticlock wise direction.

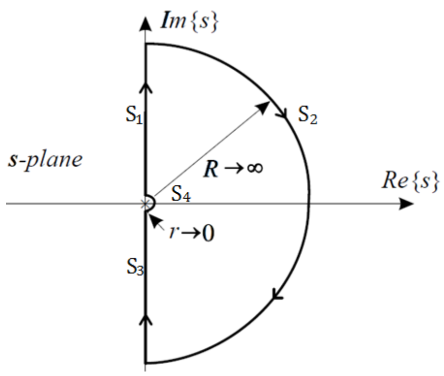
Then the relation between N, P & Z is $N = P - Z$. For a system to be stable the number of encirclement should be equal to the number of open loop poles on RHS of s-plane.

i.e. $N = P$

∴ $Z = 0$ (number of closed loop poles on RHS of s-plane is 0)

5.3.2 PROCEDURE TO DRAW NYQUIST PLOT

The Nyquist plot is a polar plot of the function $D(s) = 1+G(s)H(s)$ when travels around the contour given in figure

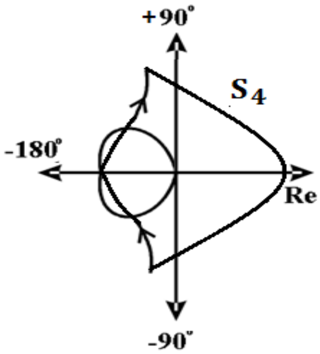


- If the poles of the system lie on the RHS of s-plane, the system will be unstable.
- The poles on the imaginary axis & at origin make the system marginally stable but not unstable they are not covered under the curve.
  The contour in this figure covers the whole unstable half plane of the complex plane s. The Nyquist plot for the above contour can be drawn by drawing the separate polar plot of the curves S_1, S_2, S_3 & S_4.

5.3.3 TO DETERMINE THE STABILITY OF THE SYSTEM

Consider a transfer function

$G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$, open loop poles on the RHS of s-plane i.e. P = 0. The polar plot for the above transfer function will be



Now to calculate intersection of polar plot with –ve real axis,

$$G(s)H(s) = \frac{K}{s(s+2)(s+10)} = \frac{K}{s^3+12s^2+20s}$$

$$G(j\omega)H(j\omega) = \frac{K}{-j\omega^3 - 12\omega^2 + j20\omega}$$

$$= \frac{K}{j(20\omega-\omega^3)-12\omega^2}$$

Equating imaginary part to zero we get,

$20\omega - \omega^3 = 0 \Rightarrow \omega^2 = 20 \Rightarrow \omega = \sqrt{20}$

This frequency is nothing but the phase crossover frequency ω_{pc}

Now substituting $\omega = \omega_{pc} = \sqrt{20}$ we get,

$$G(j\omega)H(j\omega) = \frac{K}{j(0) - 12\sqrt{20}^2}$$

$$= -\frac{K}{12\times 20} = -\frac{K}{240}$$

1) If K>240 then the intersection point will greater than $-1+j0$ i.e. $-1+j0$ lies inside the Nyquist plot, hence there are 2 encirclements of $-1+j0$ in the clockwise direction
   $\therefore N = -2$ and also from the transfer function
   $P = 0$ (i.e. No of open loop poles on RHS of s-plane)
   As we know
   $N = P - Z$
   i.e. $-2 = 0 - Z$
   Solving we get, $Z = 2$ it means that there are 2 closed loop poles on RHS of s-plane hence the system is **unstable** for **K>240**.
2) If K<240, then the intersection point will be smaller than $-1+j0$ i.e. $-1+j0$ lies outside the Nyquist plot, hence there is no encirclement of $-1+j0$.
   $\therefore N = 0$ and also from the transfer function
   P = 0 (i.e. No of open loop poles on RHS of s-plane)
   As we know
   $N = P - Z$
   i.e. $0 = 0 - Z$
   Solving we get, $Z = 0$ it means that there no closed loop poles on RHS of s-plane hence the system is **stable** for **K>240**.

Example

For a system with transfer function

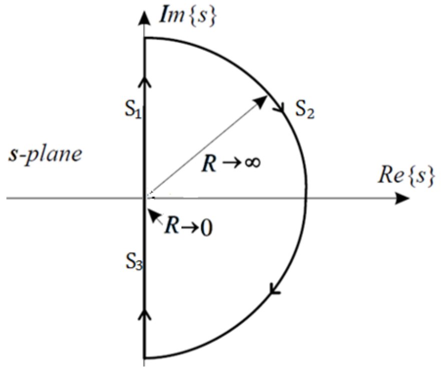
$(s)H(s) = \dfrac{40}{(s+4)(s^2+2s+2)}$, determine its stability & find gain margin.

Solution

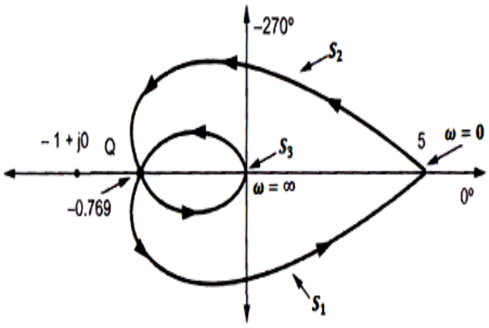
$$G(s)H(s) = \frac{40}{(s+4)(s^2+2s+2)}$$

$$= \frac{40}{(s+4)(s+(1-i))(s+(1+i))}$$

There are no open loop poles at origin or at imaginary axis; therefore we will draw polar plots only for $S_1, S_2, \& \ S_3$.



1) For S_1 draw the polar plot for given transfer function.
2) For S_2 draw the inverse polar plot for given transfer function.
3) For S_3 put $s = \lim_{R\to\infty} Re^{j\theta}$ where θ varies from $+90^o$ to -90^o



4) To find the intersection of plot with –ve real axis

$$G(s)H(s) = \frac{40}{(s+4)(s^2+2s+2)} = \frac{40}{s^3+6s^2++10s+8}$$

put $s = j\omega$

$$= \frac{40}{-j\omega^3 - 6\omega^2 + j10\omega + 8} = \frac{40}{j(10\omega - \omega^3) + (8 - 6\omega^2)}$$

now equate imaginary part to zero

$$10\omega - \omega^3 = 0 \Rightarrow \omega_{pc} = \sqrt{10}$$

Now, the intersection with –ve real axis is $|G(s)H(s)|_{\omega=\omega_{pc}} = \frac{40}{j0 + 8 - 6\times(\sqrt{10})^2}$

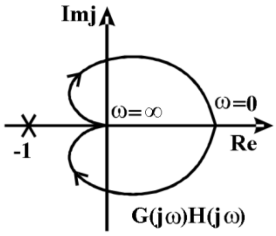
$= -0.769$

5) The point -1+j0 lies outside the Nyquist plot, hence N=0. Also from the transfer function the number of open loop poles on RHS of s-plane is 0 i.e. P=0. We know that N=P-Z
i.e. 0=0-Z $\therefore Z = 0$
Which means there are no closed loop poles on RHS of s-plane. Hence the system is stable.
6) The gain margin can calculated as

$$\text{G.M.} = 20\log_{10}\frac{1}{0.769} = +2.28\text{dB}$$

Example:
Consider a system with transfer function

$G(s)H(s) = \frac{K}{(T_1s+1)(T_2s+1)}$ & Nyquist plot

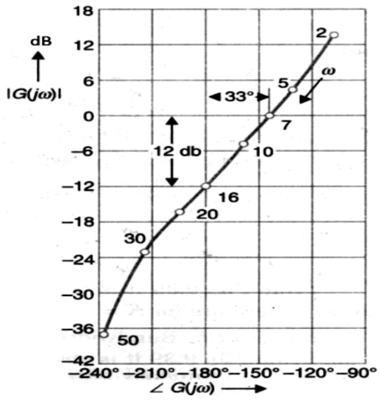


Examine the stability of the system
Solution:
1) From the transfer function, the number of open loop poles on the RHS of s-plane is 0 i.e.P = 0.
2) The point -1+j0 lies outside the plot hence N=0.
We know that,
$N = P - Z$
i.e. $0 = 0 - Z \Rightarrow Z = 0$ (no closed loop poles on RHS of s-plane). As Z=0 the system is stable

5.4 THE NICHOLS PLOT

The Nichols plot is similar to the Nyquist plot in that it is a locus as a function of ω, the difference being the chosen axes. On a Nichols plot these are the magnitude in dB on the vertical axis & the phase in degrees on the horizontal axis. The origin is chosen as $0\text{ dB and} -180^o$. For example, the Nichols plot for

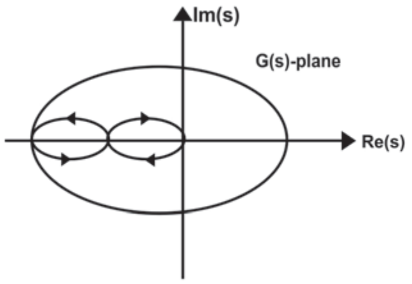
$G(s)H(s) = \frac{10}{s(1+0.1s)(1+0.05s)}$ is



The gain margin is the gain at the frequency at which phase angle is -180^{o} & the phase margin is the phase angle at the frequency at which gain is 0 dB.

GATE QUESTIONS(EC)

Q.1 The Nyquist plot for the open-loop transfer function G(s) of a unity negative feedback system is shown in the figure, if G(s) has no pole in the right-half of s-plane, the number of roots of the system characteristic equation in the right-half of s-plane is



a) 0 b) 1
c) 2 d) 3

[GATE-2001]

Q.2 The system with the open loop transfer function $G(s)H(s)=\frac{1}{s(s^2+s+1)}$ has a gain margin of

a) –6dB b) 0dB
c) 3.5dB d) 6dB

[GATE-2002]

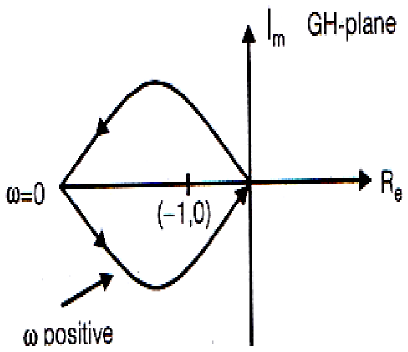
Q.3 The gain margin and the phase margin of a feedback system with $G(s)H(s)=\frac{s}{(s+100)^3}$ are

a) 0 dB, 0° b) ∞,∞
c) ∞, 0° d) 88.5 dB, ∞

[GATE-2003]

Q.4 In the figure, the Nyquist plot of the open-loop transfer function G(s) H(s) of a system is shown. If

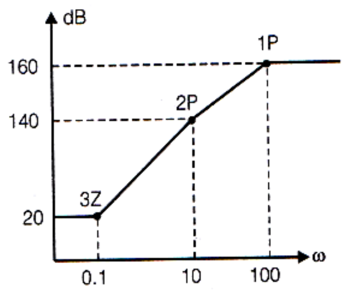
G(s)H(s) has one right-hand pole. The closed –loop system is



a) always stable
b) unstable with one closed-loop right hand pole
c) unstable with two closed-loop right hand poles
d) unstable with three closed-loop right hand poles

[GATE-2003]

Q.5 The approximate Bode magnitude plot of a minimum-phase system is shown in the figure. The transfer function of the system is



a) $10^8\frac{(s+0.1)^3}{(s+10)^2(s+100)}$

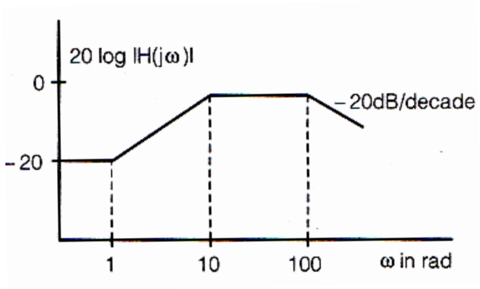
b) $10^7\frac{(s+0.1)^3}{(s+10)(s+100)}$

c) $10^8\frac{(s+0.1)^2}{(s+10)^2(s+100)}$

d) $10^9\frac{(s+0.1)^3}{(s+10)(s+100)^2}$

[GATE-2003]

Q.6 Consider the Bode magnitude plot shown in the figure. The transfer function H(s) is



a) $\dfrac{(s+10)}{(s+1)(s+100)}$ b) $\dfrac{10(s+1)}{(s+10)(s+100)}$

c) $\dfrac{10^2(s+1)}{(s+10)(s+100)}$ d) $\dfrac{10^3(s+100)}{(s+1)(s+10)}$

[GATE-2004]

Q.7 A system has poles at 0.01 Hz, 1 Hz and 80Hz; zeros at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

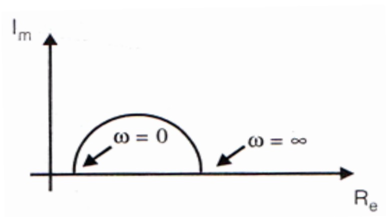
a) –90° b) 0°

c) 90° d) –180°

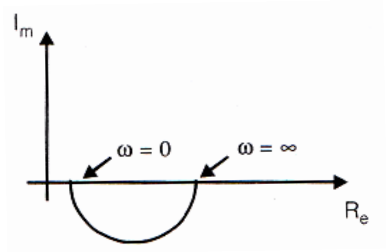
[GATE-2004]

Q.8 Which one of the following polar diagrams corresponds to a lag network?

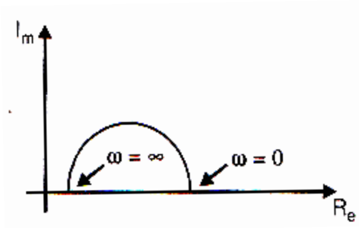
a)



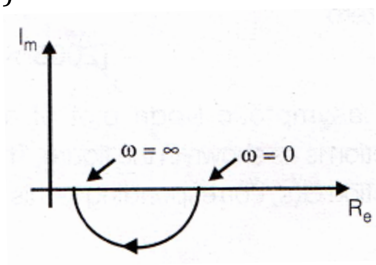
b)



c)

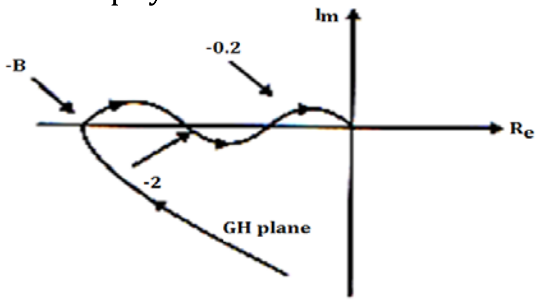


d)



[GATE-2005]

Q.9 The polar diagram of a conditionally stable system for open loop gain K=1 is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for



a) $K < 5$ and $\frac{1}{2} < K < \frac{1}{8}$

b) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

c) $K < \frac{1}{8}$ and $5 < K$

d) $K < \frac{1}{8}$ and $K < 5$

[GATE-2005]

Common Data for Questions Q.10 & Q.11:

The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{3e^{-2s}}{s(s+2)}$

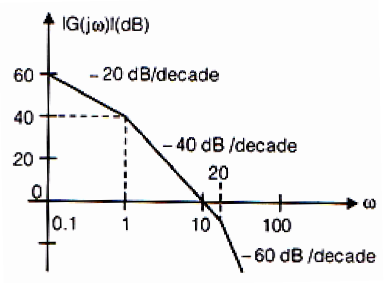
Q.10 The gain and phase crossover frequencies in rad/sec are, respectively
a) 0.632 and 1.26
b) 0.632 and 0.485
c) 0.485 and 0.632
d) 1.26 and 0.632
[GATE-2005]

Q.11 Based on the above results, the gain and phase margins of the system will be
a)-7.09 and87.5° b)7.09 and87.5
c)7.09 and-87.5° d)-7.09 and-87.5°
[GATE-2005]

Q.12 The open-loop transfer function of a unity-gain feedback control system is given by $G(s)=\frac{K}{(s+1)(s+2)}$.The gain margin of the system in dB is given by
a) 0 b) 1
c) 20 d) ∞
[GATE -2006]

Q.13 The Nyquist plot of $G(j\omega)H(j\omega)$ for a closed loop control system, passed through $(-1, j0)$ point in the GH plane. The gain margin of the system in dB is equal to
a)infinite b)greater that zero
c)less than zero d)zero
[GATE-2006]

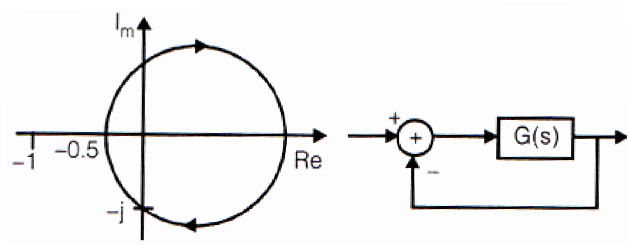
Q.14 The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function G(s) corresponding to this Bode plot is



a) $\frac{1}{(s+1)(s+20)}$ b) $\frac{1}{s(s+1)(s+20)}$
c) $\frac{100}{s(s+1)(s+20)}$ d) $\frac{100}{s(s+1)(1+0.05s)}$
[GATE-2007]

Q.15 The magnitude of frequency response of an under damped second order system is 5 at 0 rad/sec and peaks to $\frac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is
a) $\frac{500}{s^2+10s+100}$ b) $\frac{375}{s^2+5s+75}$
c) $\frac{720}{s^2+12s+144}$ d) $\frac{1125}{s^2+25s+225}$
[GATE-2008]

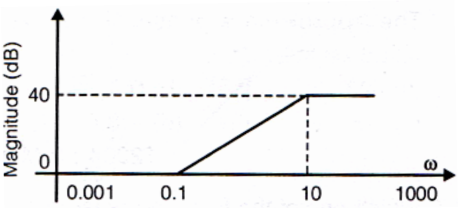
Common data for Questions Q.16 & Q.17:
The Nyquist plot of a stable transfer function G(s) is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown



Q.16 Which of the following statement is true?
a) G(s) is an all-pass filter
b) G(s) has a zero in the right –half plane
c) G(s) is the impedance of a passive network
d) G(s) is marginally stable
[GATE -2009]

Q.17 The gain and phase margins of G(s) for closed loop stability are
a) 6dB and 180° b) 3dB and 180°
c) 6dB and 90° d) 3dB and 90°
[GATE-2009]

Q.18 For the asymptotic Bode magnitude plot shown below, the system transfer function can be



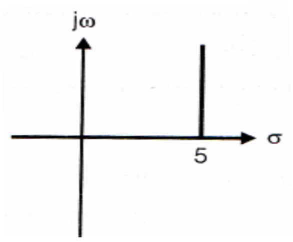
a) $\frac{10s+1}{0.1s+1}$ b) $\frac{100s+1}{0.1s+1}$

c) $\frac{100s}{10s+1}$ d) $\frac{0.1s+1}{10s+1}$

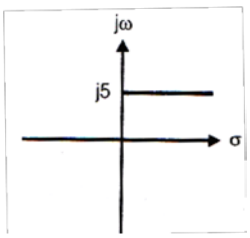
[GATE-2010]

Q.19 For the transfer function, $G(j\omega)=5+j\omega$, the corresponding Nyquist plot for positive frequency has the form

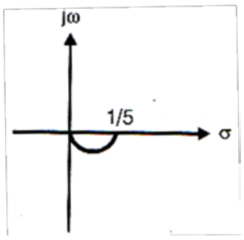
a)



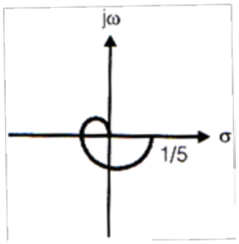
b)



c)



d)



[GATE -2011]

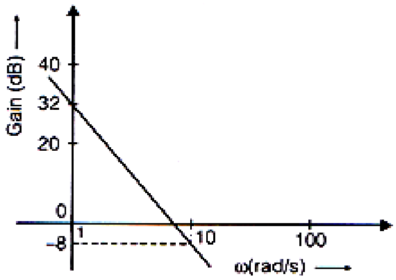
Q.20 The gain margin of the system under closed loop unity negative feedback is $G(s)H(s)=\frac{100}{S(S+10)^2}$

a)0 dB b)20 dB

c)26 dB d)46 dB

[GATE -2011]

Q.21 The Bode plot of transfer function G(s) is shown in the figure below.



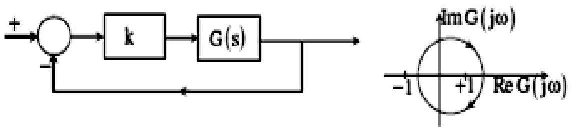
The gain (20log |G(s))| is dB and -8 at 1 rad/s and 10rad/s respectively. The phase is negative for all ω. Then G(s) is

a) $\frac{39.8}{s}$ b) $\frac{39.8}{s^2}$

c) $\frac{32}{s}$ d) $\frac{32}{s^2}$

[GATE-2013]

Q.22 Consider the feedback system shown in the figure. The Nyquist plot of G(s) is also shown. Which one of the following conclusions is correct?



a) G(s) is an all-pass filter

b) G(s) is a strictly proper transfer function

c) G(s) is a stable and minimum-phase transfer function

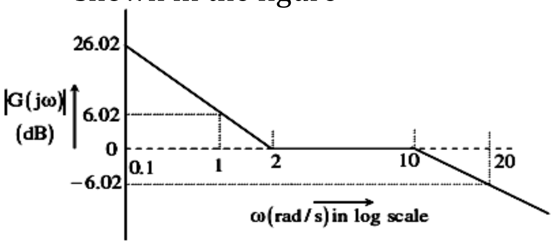
d) The closed-loop system is unstable for sufficiently large and positive k

[GATE-2014]

Q.23 The phase margin in degrees of $G(s)=\frac{10}{(s+0.1)(s+1)+(s+10)}$ calculated using the asymptotic Bode plot is _______.

[GATE-2014]

Q.24 The Bode asymptotic magnitude plot of a minimum phase system is shown in the figure



If the system is connected in a unity negative feedback configuration, the steady state error of the closed loop system, to a unit ramp input, is _____

[GATE-2014]

Q.25 In a Bode magnitude plot, which one of the following slopes would be exhibited at high frequencies by a 4th order all-pole system?

a) -80 dB/decade b) -40 dB/decade

c) +40 dB/decade d) +80 dB/decade

[GATE-2014]

Q.26 The polar plot of the transfer function $G(s)=\frac{10(s+1)}{s+10}$ for $0\le\omega<\infty$ will be in the

a) first quadrant b) second quadrant

c) third quadrant d) fourth quadrant

[GATE-2015]

Q.27 A closed-loop control system is stable if the Nyquist plot of the corresponding open-loop transfer function

a) encircles the s-plane point (-1+j0) in the counterclockwise direction as many times as the number of right-half s-plane poles.

b) encircles the s-plane point (0 - jl) in the clockwise direction as many times as the number of right-half s-plane poles.

c) encircles the s-plane point (-1+ j0) in the counterclockwise direction as many times as the number of left-half s-plane poles.

d) encircles the s-plane point (1+j0) in the counterclockwise direction as many times as the number of right-half s-plane zeros

[GATE-2016]

Q.28 The number and direction of encirclements around the point −1 + j0 in the complex plane by the Nyquist plot of $G(s)=\frac{1-s}{4+2s}$ is

a) zero

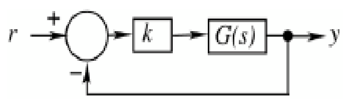
b) one, anti-clockwise

c) one, clockwise

d) two, clockwise.

[GATE-2016]

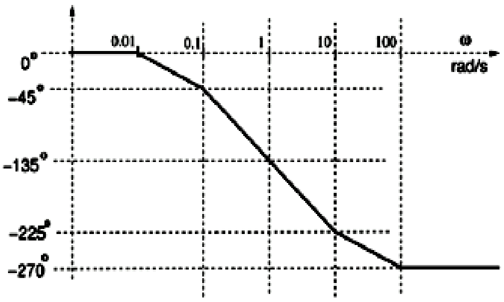
Q.29 In the feedback system shown below $G(s)=\frac{1}{(s+1)(s+2)(s+3)}$



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is _________.

[GATE-2016]

Q.30 The asymptotic Bode phase plot of $G(s)=\frac{1}{(s+0.1)(s+10)(s+p_1)}$, with k & p_1 both positive, is shown below.



The value of p_1 is ______.

[GATE-2016]

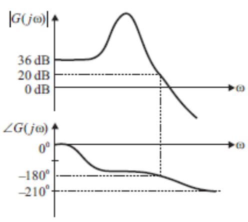
Q.31 The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE

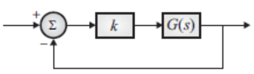
a) Both the criteria provide information relative to the stable gain range of the system.
b) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum phase systems.
c) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion.
d) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

[GATE-2018]

Q.32 The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s)=\frac{n_0}{s^3+d_2s^2+d_1s+d_0}$$





Consider the negative unity feedback configuration with gain k in the feed forward path. The closed loop is stable for $k<k_0$. The maximum value of k_0 is ________

[GATE-2018]

Q.33 For a unity feedback control system with the forward path transfer function

$$G(s)=\frac{K}{s(s+2)}$$

The peak resonant magnitude M_r of the closed-loop frequency response is 2. The corresponding value of the gain K (correct to two decimal places) is ______.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(b)	(a)	(a)	(c)	(a)	(d)	(b)	(d)	(d)	(d)	(d)	(d)	(a)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(b)	(c)	(a)	(a)	(c)	(b)	(d)	45	0.5	(a)	(a)	(a)	(a)	60	1
31	**32**	**33**												
(d)	0.1	14.92												

EXPLANATIONS

Q.1 **(a)**

$N = 0$ (1encircle mention CW direction
and other in CCW)
P=0 (no pole in right half)
So, $N = P - Z$
$Z = P - N = 0$
$\therefore$ No roots on RH of s-plane.

Q.2 **(b)**

$$G(s)H(s) = \frac{1}{s(s^2+s+1)}$$

ω_ϕ when $\angle$of $G(s)H(s) = -180°$

$$-180 = -90 - \tan^{-1}\frac{\omega}{1-\omega^2} - 9$$

$$= -\tan^{-1}\frac{\omega}{1-\omega^2}$$

$1 - \omega^2 = 0$

$\omega_\phi = 1$ rad/sec

Value of gain at $\omega_\phi = 1$

$$|G(s)H(s)| = \frac{1}{\sqrt{|(1-\omega^2)|^2 + |j\omega|^2}}$$

$\therefore$ G.M. $= -20\log 1 = 0$

Q.3 **(b)**

$$G(s)H(s) = \frac{s}{(s+100)^3}$$

G.M. and P.M. of the system cannot be determined.

Q.4 **(a)**

The encirclement of critical point (-1, 0) in A.C.W direction is once.
$\therefore N = 1, P = 1$ (given)
$Z = P - N = 0$
No zero in RH of s-plane. So system is stable.

Q.5 **(a)**

$\omega = 0.1$to$10, +120$dB change
$\therefore$ 3 zeros at 0.1
$\omega = 10$to$100, -40$dB
$[\text{i.e.} + 60\text{to} + 20]$ change
So two poles at ω=10
$\omega = 100, -20$dB change
$\therefore$ one pole at $\omega = 100$

$$\therefore T(s) = \frac{K(s+0.1)^3}{(s+10)^2(s+100)}$$

$$20\log\log(K/\omega)|_{\omega=10} = 140$$

$$\frac{K}{\omega}|_{\omega=10} = 10^7$$

$\therefore K = 10^8$

Q.6 **(c)**

$20\log K = -20$dB
$\Rightarrow K = 10^{-1} = 0.1$
Zero at $\omega = 1$ & poles at $\omega = 10, 100$

$$H(s) = \frac{K(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

$$= \frac{0.1(s+1)\times 10\times 100}{(s+10)(s+100)}$$

$$= \frac{100(s+1)}{(s+10)(s+100)}$$

Q.7 **(a)**

Pole at 0.01 and 1 Hz gives $-180°$ phase. Zero at 5 Hz gives $+90°$ phase
$\therefore$ at 20Hz $-90°$ phase shift is provided.

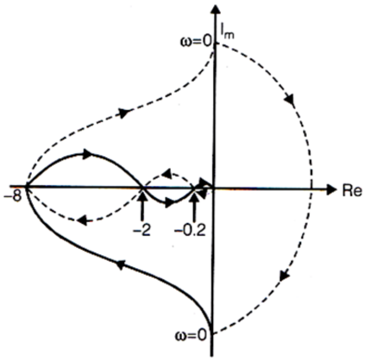
Q.8 **(d)**

Let $\frac{1}{s+1}$ be a lag network
At $\omega = 0$, Mag $= \infty$, $\angle = -\tan^{-1}0 = 0$

$\omega = \infty, \text{Mag} = 0, \angle = -\tan^{-1}\infty = -90°$

If in the direction of ω increasing phase shift is decreasing system is lag network.

Q.9 (b)



System is stable in region -0.2 to -2 & on the left side of -8 as no. of encirclement there is zero.

$0.2K < 1 \Rightarrow K < 5$

$2K > 1 \Rightarrow K > 0.5$

$\therefore 0.5 < K < 51 > 8K$

$K < \frac{1}{8}$ (negative sign only shows that it is on negative axis)

Q.10 (d)

1. Gain cross over frequency where gain is $|G(s)| = 1$

$$\Rightarrow \frac{3}{\omega(\omega^2+4)^{1/2}} = 1$$

$$\Rightarrow \frac{9}{\omega^2(\omega^2+4)} = 1$$

$\Rightarrow \omega_g = 1.26$

Phases cross over frequency

Where $\angle GH = 180°$

$$\Rightarrow \omega_\Phi = 0.632\left(\frac{\omega_\Phi}{2} = \cot 2\omega_\Phi\right)$$

Q.11 (d)

$$\text{G.M at } \omega_\Phi = \frac{3}{0.632(0.632^2+4)^{1/2}}$$

$a = 2.26$

$$\text{G.M} = 20\log\frac{1}{a}$$

$$\Rightarrow 20\log\frac{1}{2.26} = -7.09$$

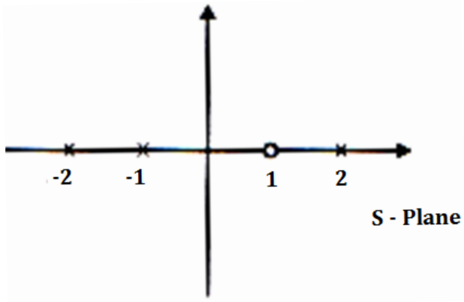
Since G.M is negative system is unstable.

$\therefore$ P.M.is also negative

$\text{P.M.} = -87.5°$

Q.12 (d)

For 2nd order system GM=∞



Q.13 (d)

$$\text{G.M} = 20\log\frac{1}{a}\text{dB}$$

a=1

$\therefore \text{G.M.} = 0$

Q.14 (d)

$$G(s) = \frac{K}{s(s+1)(s+20)}$$

$$= \frac{K \times 20}{s(1+s)\left(1+\frac{s}{20}\right)}$$

Bode plot is in (1+sT) form

$$20\log\frac{K}{\omega}|_{\omega=0.1} = 60\text{dB} = 1000$$

$\Rightarrow K = 5$

$$\therefore G(s) = \frac{100}{s(s+1)(1+0.5s)}$$

Q.17 (c)

$$\text{GainMargin} = 20\log\frac{1}{X}$$

$$= 20\log\frac{1}{0.5}$$

$= 20\log 2$

= 6dB

And phase margin = 90°

Q.18 (a)

System transfer function

$$G(s)H(s)=\frac{K\left(1+\frac{s}{0.1}\right)}{\left(1+\frac{s}{10}\right)}$$

Here, $20\log K=0$

$\Rightarrow K=1$

Therefore, $G(s)H(s)=\frac{10s+1}{0.1s+1}$

Q.19 (b)

$G(j\omega)=5+j\omega$

$|G(j\omega)|=\sqrt{25+\omega^2}$

At $\omega=0$

$|G(0)|=\sqrt{25+0}=5$

At $\omega=\infty$

$|G(\infty)|=\infty$

Q.20 (c)

$$G_{(S)}H_{(S)}=\frac{100}{s(s+10)^2}$$

$\Phi=-90°-2\tan^{-1}(\omega/10)$

For phase cross–over frequency, $\Phi=-180°$

$\therefore -180=-90°$

$-2\tan^{-1}(\omega/10)$

$\Rightarrow \omega=10\text{rad/sec}$

Put, $s=j\omega$

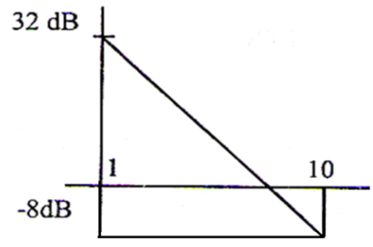
$$G_{(j\omega)}H_{(j\omega)}=\frac{100}{j\omega(j\omega+100)^2}$$

$$\left|G_{(j\omega)}H_{(j\omega)}\right|_{\omega=10}$$

$$=\frac{100}{\omega(\omega^2+100)}=\frac{100}{10(200)}=\frac{1}{20}$$

G.M.in dB $=20\log 20=26$dB

Q.21 (b)



$\omega=1$ to $\omega=10$

Is 1 dec are change & change is (G) is 40 dB

$\therefore$ S lope is 40dB / dec

$\therefore$ There are 2poles is orign

So, $G(s)=\frac{K}{S^2}$

$|G|_{at\omega=1}=32\text{dB(given)}$

$\Rightarrow 2\log\left|\frac{k}{\omega^2}\right|_{\omega=1}=32\text{dB}$

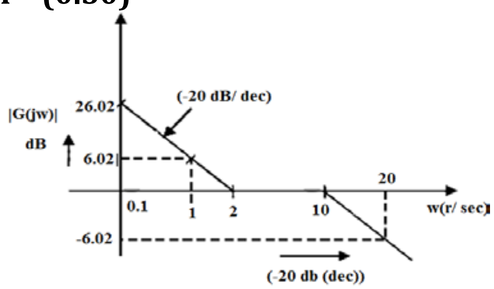
$\Rightarrow 20\log k=32\text{dB}\Rightarrow k=39.8$

$\therefore G=\frac{39.8}{S^2}$

Q.22 (d)

For larger values of K, it will encircle the critical point (-1+j0), which makes closed-loop system unstable.

Q.24 (0.50)



→ Due to initial slope, it is a type-1 system, and it has non zero velocity error coefficient (K_v)

→ The magnitude plot is giving 0dB at 2r/sec.

Which gives K_v

$\therefore \quad k_v=2$

The steady state error $e_{ss} = \frac{A}{k_v}$

given unit ramp input; A =1

$e_{ss} = \frac{1}{2}$

$e_{ss} = 0.50$

Q.25 (a)

→In a BODE diagram, in plotting the magnitude with respect to frequency, a pole introduce a line 4 slope-20dB / dc

→If 4th order all-pole system means gives a slope of (-20) * 4 dB / dec i.e. -80dB / dec

Q.26 (a)

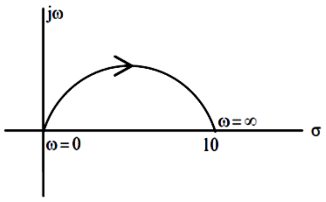
$G(s) = \frac{10(s+1)}{s+10}$

Put s= jω

$G(j\omega) = \frac{10(j\omega+1)}{(j\omega+10)}$

$\omega = 0, M = 1 < 0$

$\omega = \infty, M = 10 < 0$



So, zero is nearer to imaginary axis. Hence plot will move clockwise direction.

It is first quadrant.

Q.28 (a)

$G(j\omega) = \frac{1-j\omega}{4+2j\omega}$

$\omega = 0, |Gj\omega| = 0.25, \angle G(j\omega) = 0$

$\omega = \infty, 1G(j\omega) = 0.5, G(j\omega) = -180^o \ N = 0$

Q.29 (60)

The given condition implied marginal stability. One alternative way without going for gain margin, phase margin concepts is find k value for marginal stability using reflection.

C.E:- $s^3 + 11s^2 + 6s + 6k = 0$

s^3	1	6
s^2	11	6+k
s^1	$\frac{60-k}{11}$	
	6+k	

For marginal stability odd order row of S should be zero i.e.

$\frac{60-k}{11} = 0 \Rightarrow k = 60$

Q.30 (1)

Since it is the phase plot given we can't use the slope concept as these are non linear curves.

So we can take any phase angle of at a given frequency as reference and can obtain p_1

→ phase of transfer function

$\varnothing(\omega) = -\tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{P_1}\right)$

→ from the plot at $\omega = 0.1, \varnothing = -45^0$

$-45^0 = -\left[\tan^{-1}\frac{0.1}{0.1} + \tan^{-1}\frac{0.1}{10} + \tan^{-1}\frac{0.1}{P_1}\right]$

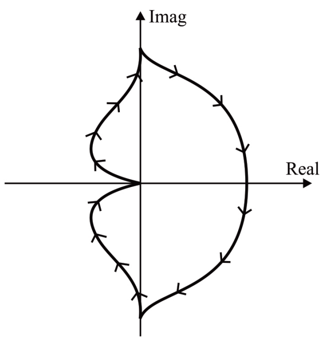
Solving for P, we get p_1 = 1.

Q.31 (d)

Let us consider O.L.T.F. of a unity feedback system as

$G(s) = \frac{K}{s(s+a)}$

Which represents a type-1 and order-2 system for which Nyquist plot will be as shown below:



Closed loop transfer function for the system is given as,

$$T(s)=\frac{G(s)}{1+G(s).1}$$

$$=\frac{K/s(s+a)}{1+[K/s(s+a)]}$$

$$T(s)=\frac{K}{s^2+as+K}$$

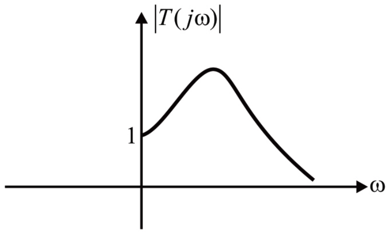
We can draw its Nyquist plot by simply substituting $s=j\omega$ and obtain polar co-ordinates of $T(j\omega)$ for different values of ω as,

$$T(j\omega)=\frac{K}{(j\omega)^2+ja\omega+K}$$

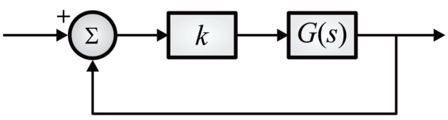
$$T(0)=\frac{K}{(0)+0+K}=1$$

$$T(\infty)=0$$

Hence statement given in option (D) is FALSE.



Q.32 0.1



Magnitude and phase angle plots for unstable system with

$$G(s)=\frac{n_0}{s^3+d_2s^2+d_1s+d_0}$$

Let $G'(s)=K.G(s)$

For the closed loop system to be stable

$GM>1$

$$\frac{1}{|G'(j\omega)|_{\omega=\omega_{pc}}}>1$$

$|G'(j\omega)|_{\omega=\omega_{pc}}<1$ (or 0 dB)

$20\log K+|G(j\omega)|_{\omega=\omega_{pc}}<0\text{dB}$

$20\log K+20\text{dB}<0\text{dB}$

$20\log K<-20\text{dB}$

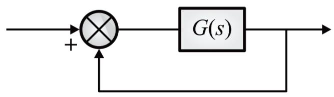
$\log K<-1$

Hence, Maximum value of K i.e. K_0, for which system is stable is given as

$K_{max}=K_0=(10)^{-1}=0.1$

Q.33 14.92

Given: For a unity feedback system $G(s)=\frac{K}{s(s+2)}$ and resonant peak $M_r=2$

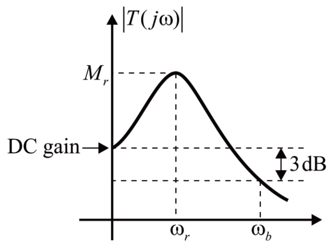


We can find its closed loop transfer function as,

$$\frac{C(s)}{R(s)}=T(s)=\frac{G(s)}{1+G(s)H(s)}$$

$$=\frac{K/s(s+2)}{1+\frac{K}{s(s+2)}.1}=\frac{K}{s^2+2s+K}$$

Here, one should note that the value of D.C. gain is 1, not '*K*'.



We have formula for resonant peak as,

$$M_r=\frac{\text{DCgain}}{2\xi\sqrt{1-\xi^2}}$$

Substituting the given value of M_r and obtained D.C. gain, we get

$$\frac{1}{2\xi\sqrt{1-\xi^2}}=2$$

$$4\xi\sqrt{1-\xi^2}=1$$

On squaring both the sides,

$16\xi^2\left(1-\xi^2\right)=1$

$16\xi^4-16\xi^2+1=0$

If we put $\xi^2=x$ then equation becomes

$16x^2-16x+1=0$

$x=0.067$

We have, $\xi^2=x=0.067$

$\xi=\sqrt{0.067}=0.258$

Here, characteristic equation is

$s^2+2s+K=0$

Comparing it with standard equation, we get

$\omega_n^2=K\Rightarrow\omega_n=\sqrt{K}$

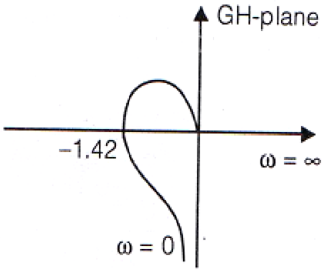
And $2\xi\omega_n=2$

$\xi=\dfrac{1}{\omega_n}=\dfrac{1}{\sqrt{K}}$

Hence, $K=\dfrac{1}{\xi^2}=14.92$

GATE QUESTIONS(EE)

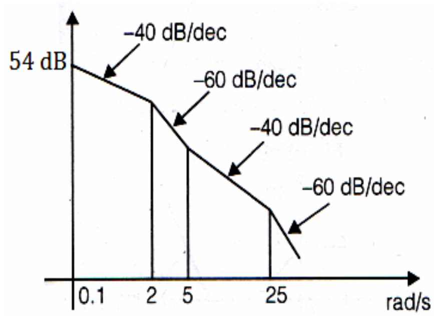
Q.1 The polar plot of a type-1, 3-pole, open loop system is shown in figure below. The closed loop system is.



a) Always stable.
b) Marginally stable.
c) Unstable with one pole on the right half s-plane.
d) Unstable with two poles on the right half s-plane.

[GATE-2001]

Q.2 The asymptotic approximation of the log-magnitude versus frequency plot of a minimum phase system with real poles and one zero is shown in figure. Its transfer function is

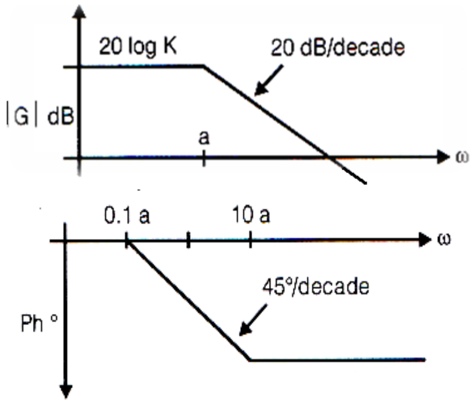


a) $\frac{20(s+5)}{s(s+2)(s+25)}$ b) $\frac{10(s+5)}{(s+2)^2(s+25)}$

c) $\frac{20(s+5)}{s^2(s+2)(s+25)}$ d) $\frac{50(s+5)}{s^2(s+2)(s+25)}$

[GATE-2001]

Q.3 The asymptotic Bode plot of the transfer function K/[1+(s/a)] is given in figure. The error in phase angle and dB gain at a frequency of ω = 0.5 a are respectively



a) 4.9°, 0.97dB b) 5.7°, 3dB
c) 4.9°, 3dB d) 5.7°, 0.97dB

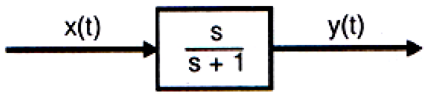
[GATE-2003]

Q.4 The Nyquist plot of loop transfer function G(s)H(s) of a closed loop control system passes through the point (-1, j0) in the G(s)H(s) plane. The phase margin of the system is

a) 0° b) 45°
c) 90° d) 180°

[GATE-2004]

Q.5 In the system shown in figure, the input $x(t) = \sin t$. In the steady-state, the response $y(t)$ will be



a) $\frac{1}{\sqrt{2}}\sin(t - 45^\circ)$ b) $\frac{1}{\sqrt{2}}\sin(t + 45^\circ)$

c) $\sin(t - 45^\circ)$ d) $\sin(t + 45^\circ)$

[GATE-2004]

Q.6 The open loop transfer function of a unity feedback control system is given as

$G(s)=\frac{as+1}{s^2}$. The value of 'a' to give a phase margin of 45° is equal to

a) 0.141 b) 0.441
c) 0.841 d) 1.141

[GATE-2004]

Q.7 A system with zero initial conditions has the closed loop transfer function.

$$T(s)=\frac{(s^2+4)}{(s+1)(s+4)}$$

The system output is zero at the frequency

a) 0.5 rad / sec b) 1 rad / sec
c) 2 rad / sec d) 4 rad / sec

[GATE-2005]

Q.8 The gain margin of a unity feedback control system with the open loop transfer function $G(s)=\frac{(s+1)}{s^2}$ is

a) 0 b) $\frac{1}{\sqrt{2}}$
c) $\sqrt{2}$ d) ∞

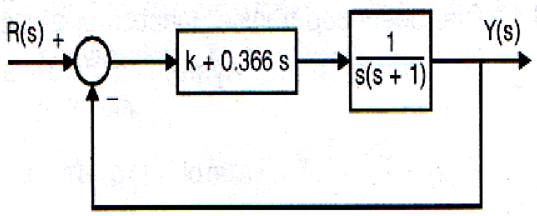
[GATE-2005]

Q.9 In the GH(s) plane, the Nyquist plot of the loop transfer function $G(s)H(s)=\frac{\pi e^{-0.25s}}{s}$ passes through the negative real axis at the point

a) (-0.25, j0) b) (-0.5, j0)
c) (-1, j0) d) (-0.2, j0)

[GATE-2005]

Q.10 If the compensated system shown in the figure has a phase margin of 60° at the crossover frequency of 1 rad/sec, then value of the gain k is

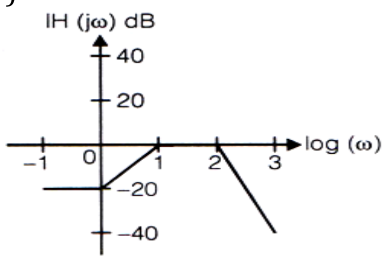


a) 0.366 b) 0.732
c) 1.366 d) 2.738

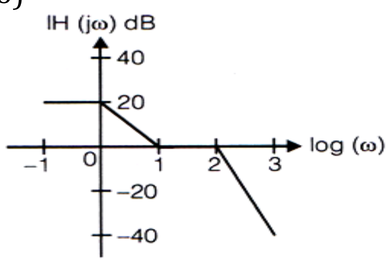
[GATE-2005]

Q.11 The Bode magnitude plot of $H(j\omega)=\frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$ is

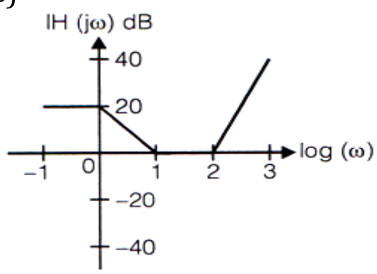
a)



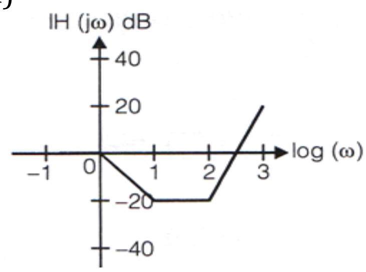
b)



c)

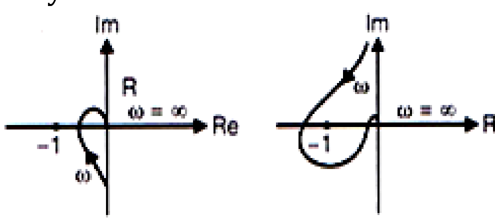


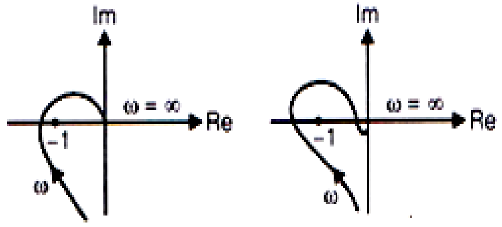
d)



[GATE-2006]

Q.12 Consider the following Nyquist plots of loop transfer functions over $\omega=0$ to $\omega=\infty$. Which of these plots represents a stable closed loop system?





a) (1) only b) all except (1)
c) all except (3) d)(1) and (2) only

[GATE-2006]

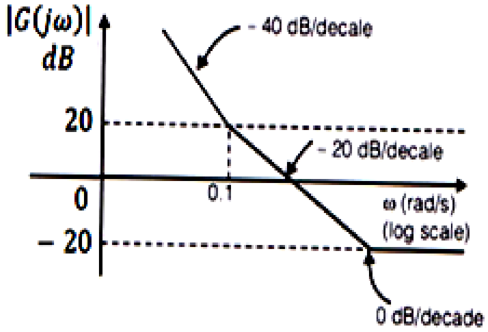
Q.13 If $x = \text{Re}\,G(j\omega)$, & $y = \text{Im}G(j\omega)$ then for $\omega \to 0^+$, the Nyquist plot for $G(s) = 1/s(s+1)(s+2)$

a) $x = 0$ b) $x = -3/4$

c) $x = y - 1/6$ d) $x = \dfrac{y}{\sqrt{3}}$

[GATE-2007]

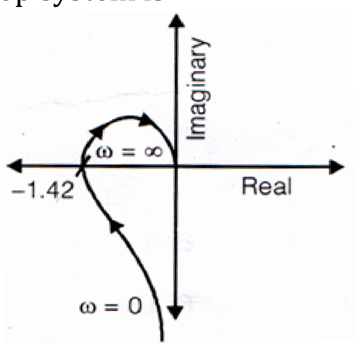
Q.14 The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure. This transfer function has



a) Three poles and one zero
b) Two poles and one zero
c) Two poles and two zeros
d) One pole and two zeros

[GATE-2008]

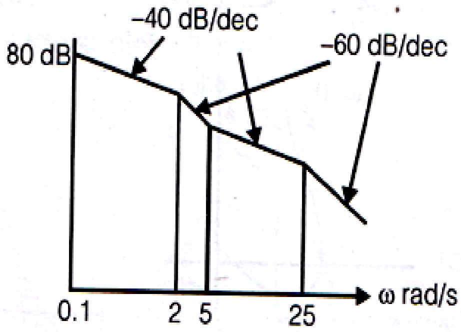
Q.15 The polar plot of an open loop stable system is shown below. The closed loop system is



a) always stable
b) marginally stable
c) unstable with one pole on the RHS s-plane
d) unstable with two poles on the RHSs-plane

[GATE-2009]

Q.16 The asymptotic approximation of the log-magnitude vs frequency plot of a system containing only real poles and zeros is shown. Its transfer function is



a) $\dfrac{10(s+5)}{s(s+2)(s+25)}$ b) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

c) $\dfrac{100(s+5)}{s(s+2)(s+25)}$ d) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

[GATE-2009]

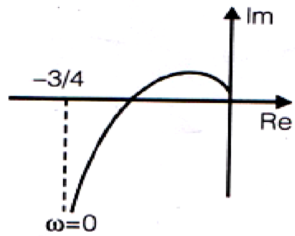
Q.17 The open loop transfer function of a unity feedback system is given by $G(s) = (e^{-0.1s})/s$. The gain margin of this system is

a) 11.95 dB b) 17.67dB
c) 21.33 dB d) 23.9dB

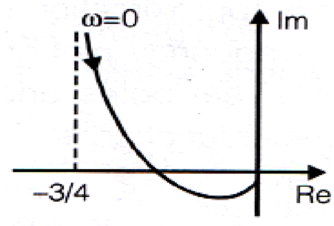
[GATE-2009]

Q.18 The frequency response G(s) =1/[s(s+1) (s+2)] plotted in the complex G(jω) plane (for 0 < ω<∞) is

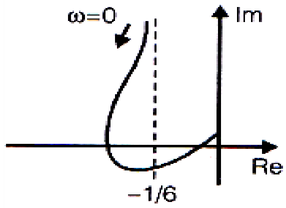
a)



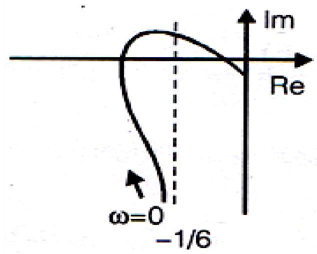
b)



c)



d)



[GATE-2010]

Q.19 The frequency response of a linear system is provided in the tabular form below

$\|G(j\omega)\|$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	-130°	-140°	-150°	-160°	-180°	-200°

The gain margin and phase margin of the system are

a)6dB and 30° b)6dB and -30°
c)-6dB and 30° d)-6dBand-30°

[GATE-2011]

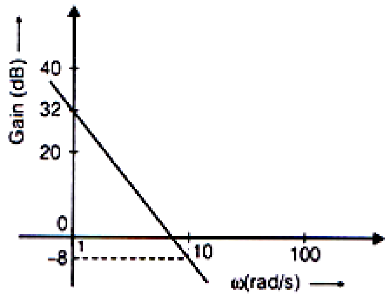
Q.20 A system with transfer function $G(s)=\dfrac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$ is excited sin (ωt).

The steady-state output of the system is zero at

a)ω =1 rad/s b)ω =2 rad/s
c)ω =3 rad/s d)ω =4 rad/s

[GATE-2012]

Q.21 The Bode plot of transfer function G(s) is shown in the figure below.



The gain (20log |G(s))| is dB and -8 at 1 rad/s and 10rad/s respectively. The phase is negative for all ωThen G(s) is

a) $\dfrac{39.8}{s}$ b) $\dfrac{39.8}{s^2}$
c) $\dfrac{32}{s}$ d) $\dfrac{32}{s^2}$

[GATE-2013]

Q.22 For the transfer function

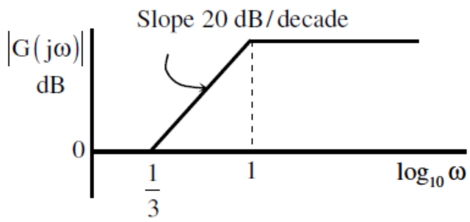
$$G(s)=\frac{5(S+4)}{s(s+0.25)(s^2+4s+25)}$$

The values of the constant gain term and the highest corner frequency of the Bode plot respectively are

a)3.2, 5.0 b)16.0, 4.0
c)3,2, 4.0 d)16.0, 5.0

[GATE-2014]

Q.23 The magnitude Bode plot of a network is shown in the figure

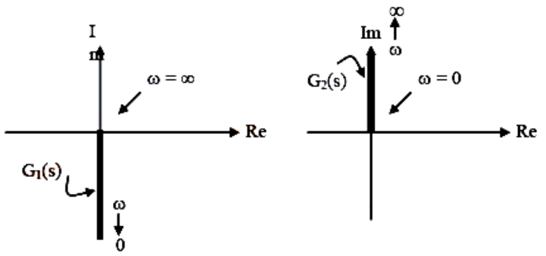


The maximum phase angle .m and the corresponding gain Gm respectively, are

a) -30° and 1.73dB
b) -30° and 4.77dB
c) +30° and 4.77dB
d) +30° and 1.73dB

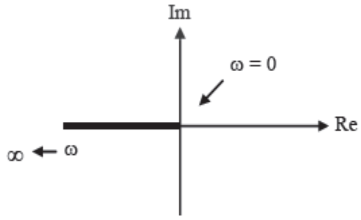
[GATE-2014]

Q.24 Nyquist plots of two functions $G_1(s)$ and $G_2(s)$ are shown in figure.

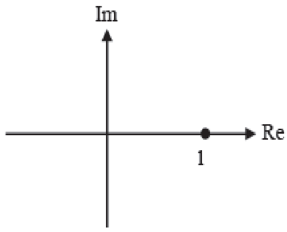


Nyquist plot of the product of $G_1(s)$ and $nG_2(s)$ is

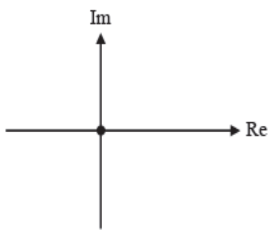
a)



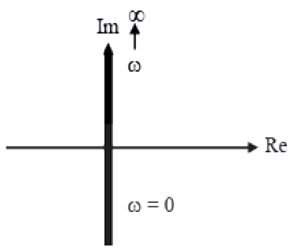
b)



c)



d)



[GATE-2015]

Q.25 The phase cross-over frequency of the transfer function $G(s) = \frac{100}{(s+1)^3}$ in rad/s is

a) $\sqrt{3}$

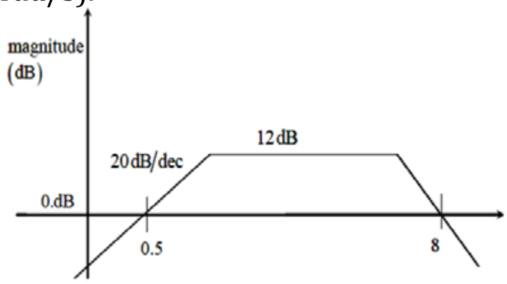
b) $\frac{1}{\sqrt{3}}$

c) 3

d) $3\sqrt{3}$

[GATE-2016]

Q.26 Consider the following asymptotic Bode magnitude plot (ω is in rad/s).



Which one of the following transfer function is best represented by the above Bode magnitude plot?

a) $\frac{2s}{(1+0.5s)(1+0.25s)^2}$

b) $\frac{4(1+0.5s)}{s(1+0.25s}$

c) $\frac{2s}{(1+2s)(1+4s)}$

d) $\frac{4s}{(1+2s)(1+4s)^2}$

[GATE-2016]

Q.27 Loop transfer function of a feedback system is $G(s)H(s) = \frac{s+3}{s^2(s-3)}$.Take the Nyquist contour in the clockwise direction. Then the Nyquist plot of G(s) H(s) encircles -1 + j 0

a) Once in clockwise direction

b) Twice in clockwise direction

c) Once in anticlockwise direction

d) Twice in anti clockwise direction

[GATE-2016]

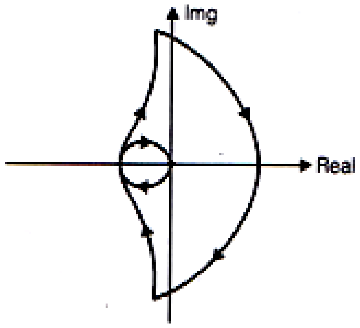
ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(d)	(d)	(a)	(b)	(c)	(c)	(a)	(b)	(c)	(a)	(a)	(b)	(c)	(d)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**			
(b)	(d)	(a)	(a)	(c)	(b)	(a)	(c)	(b)	(a)	(a)	(a)			

EXPLANATIONS

Q.1 (d)

Nyquist plot will be.



∴ number of encirclement of (-1, j0) =-2 and number of right sided pole in open loop system=0

$N = P - Z$

$\therefore -2 = 0 - 2$

$\Rightarrow Z = 2$

∴ Closed loop system is unstable with two poles on the right half of s-plane.

Q.2 (d)

Type : 2

Poles = 0, 0, 2, 25

Zero =5

$\text{Gain} = m\log\omega + 20\log k$

$54 = -40\log(0.1) + 20\log k$

$\Rightarrow k = 5$

$$\therefore T.F. = \frac{5(1+0.2S)}{s^2(1+0.5S)(1+0.04S)}$$

$$= \frac{50(S+5)}{s^2(S+2)(S+25)}$$

Q.3 (d)

$$\text{Transfer function} = G(s) = \frac{k}{1+\frac{s}{a}}$$

Put $s = j\omega$

$$G(j\omega) = \frac{k}{1+j\frac{\omega}{a}}$$

Magnitude of $G(j\omega)$

$$= |G(j\omega)| \frac{k}{\sqrt{1+\frac{\omega^2}{a^2}}}$$

Phase of $G(j\omega)\angle G(j\omega)$

$$= -\tan^{-1}\left(\frac{\omega}{a}\right)$$

at $\omega = 0.5a$

$$|G(j\omega)|_{\omega=0.5a} = \frac{k}{\sqrt{1+\frac{(0.5a)^2}{a^2}}} = \frac{k}{\sqrt{1.25}}$$

In dB

$= 20\log k - 20\log(1.25)^{1/2}$

$= 20\log k - 0.97dB$

From the plot

Magnitude $= 20\log k$

∴ error in dB gain =0.97dB

$$\angle G(j\omega)|_{\omega=0.5a} = -\tan^{-1}\left(\frac{0.5a}{a}\right)$$

$$\log_{10}\left(\frac{0.5a}{0.1a}\right) = \frac{\phi - 0}{-45^\circ}$$

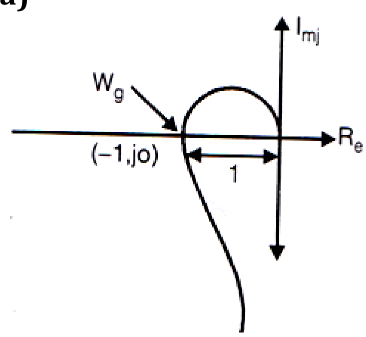
$$\angle G(j\omega)|_{\omega=0.5a} = \phi = -31.45^\circ$$

Error in phase-angle

$= -26.56^\circ - (-31.45^\circ)$

$= 5.89^\circ \approx 5.7^\circ$

Q.4 (a)



ω_g is gain cross over frequency at which the gain $|G(j\omega)H(j\omega)|$ becomes unity.

In this case, phase $\angle G(j\omega)H(j\omega) \text{ is} -180^\circ$ at $\omega = \omega_g$.

$\phi = -180^\circ$

So phase margin $= 180^\circ + \phi = 0^\circ$

Q.5 **(b)**

$x(t) = \sin \sin t = 1 < 0$

$\omega = 1 \text{rad/sec}$

Transfer function $= G(s) = \dfrac{s}{s+1}$

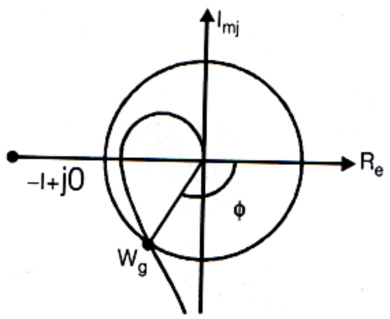
For sinusoidal input, $s = j\omega = j$

$$G(j\omega)\Big|_{\omega=1} = \frac{j\omega}{j\omega+1} = \frac{j}{j+1} = \frac{1}{\sqrt{2}}\angle 45^\circ$$

$$y(t) = \frac{1}{\sqrt{2}}\angle 45^\circ . 1\angle 0^\circ = \frac{1}{\sqrt{2}}\angle 45^\circ$$

$$y(t) = \frac{1}{\sqrt{2}}\sin\left(t + 45^\circ\right)$$

Q.6 **(c)**



$$G(s) = \frac{as+1}{s^2}$$

ω_g = gain-crossover frequency at which open gain is loop

Phase margin $= 180^\circ + \angle G(j\omega)$

$45^\circ = 180^\circ + \angle G(j\omega_g)$

$\angle G(j\omega_g) = -130^\circ$

$$\angle G(j\omega_g) = \frac{ja\omega_g + 1}{(j\omega_g)^2}$$

$= -2 \times 90^\circ + \tan^{-1} a\omega_g$

$\angle G(j\omega_g) = -180^\circ + \tan^{-1} a\omega_g = -135^\circ$

$\tan^{-1} a\omega_g = -45^\circ$

$a\omega_g = 1$

$$\omega_g = \frac{1}{a}$$

Gain $= \left|G(j\omega_g)\right| = 1$

$$\Rightarrow \quad \frac{\sqrt{1 + a^2\omega_g^2}}{\omega_g^2} = 1$$

$$\Rightarrow \quad \frac{\sqrt{1 + a^2 \times \frac{1}{a^2}}}{1/a^2} = 1$$

$$\frac{1}{a^2} = \sqrt{2}$$

$$a = \frac{1}{2^{1/4}} = 0.841$$

Q.7 **(c)**

$$T(s) = \frac{s^2 + 4}{(s+1)(s+4)}$$

For frequency response, put $s = j\omega$

$$T(j\omega) = \frac{(j\omega)^2 + 4}{(j\omega+1)(j\omega+4)}$$

$$= \frac{4 - \omega^2}{(1 + j\omega)(4 + j\omega)}$$

Magnitude of

$$T(j\omega) = \left|T(j\omega)\right| \frac{4 - \omega^2}{\sqrt{(1+\omega^2)(16+\omega)^2}}$$

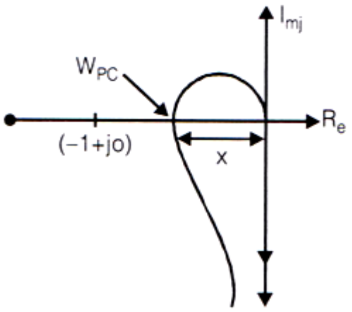
$$\left|T(j\omega)\right| \frac{4 - \omega^2}{\sqrt{(1+\omega^2)(16+\omega)^2}} = 0$$

$\omega = 2 \text{rad/sec}$

The system output is zero at 2 rad/sec.

Q.8 **(a)**

ω_{pc} = Phase cross over frequency at which phase angle of OLTF is -180° or at which OLTF cuts real axis



$T(s) = OLTF = G(s)H(s) = G(s).1 = G(s)$

$T(s) = \frac{s+1}{s^2}$

$T(j\omega_{pc}) = \frac{j\omega_{pc}+1}{(j\omega_{pc})^2}$

$\angle T(j\omega_{pc}) = -180^\circ + \tan^{-1}\omega_{pc}$

OLTF cuts real axis at ω_{pc}, so at ω_{pc} phase of OLTF is -180°

$\angle T(j\omega_{pc}) = -180^\circ + \tan^{-1}\omega_{pc}$

$-180^\circ = -180^\circ + \tan^{-1}\omega_{pc}$

$\omega_{pc} = 0$

$M = |T(j\omega_{pc})| = \frac{\sqrt{1+\omega_{pc}^2}}{\omega_{pc}^2} = \infty$

Gain margin $= \frac{1}{M} = \frac{1}{\infty} = 0$

Q.9 (b)

$G(s)H(s) = \frac{\pi e^{-0.25s}}{s}$

Putting $s = j\omega$, for frequency response

$= G(j\omega)H(j\omega) = \frac{\pi e^{-j0.25\omega}}{j\omega}$

Phase angle $\angle G(j\omega)H(j\omega)$

$= -90 - 0.25\omega$ ω_{pc} at nyquist plot cuts negative real axis and this frequency

$= \angle G(j\omega_{pc})H(j\omega_{pc})$ is -180°

$\angle G(j\omega_{pc})H(j\omega_{pc}) = -90^\circ - \frac{\omega}{4}$

$-180^\circ = -90^\circ - \frac{\omega}{4} \omega = 360^\circ \text{ or } 2\pi$

$|G(j\omega_{pc})H(j\omega_{pc})| = \frac{\pi e^{-j0.25\times 2\pi}}{j2\pi}$

$= -\frac{j}{2}\left[\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right] = -0.5 + j0$

Q.10 (c)

$G(s) = \frac{k+0.366s}{s(s+1)}$ and $H(s) = 1$

$OLTF = G(s)H(s) = \frac{k+0.366s}{s(s+1)}$

Phase margin $= 60^\circ$ at ω_{gc} (gain cross over frequency)

$\Rightarrow 180^\circ + \angle G(j\omega_{pc})H(j\omega_{pc}) = 60^\circ$

$\angle G(j\omega_{gc})H(j\omega_{gc}) = -120^\circ - 90^\circ$

$-\tan^{-1}\omega_{gc} + \tan^{-1}\left(\frac{0.366\omega_{gc}}{k}\right)$

$= -120^\circ$

$\omega_{gc} = 1\text{rad/sec} - 90^\circ - \tan^{-1}(1)$

$+\tan^{-1}\left(\frac{0.366\times 1}{k}\right) = -120^\circ$

$= -90^\circ - 45^\circ + \tan^{-1}\left(\frac{0.366}{k}\right)$

$= -120^\circ$

$\Rightarrow \quad \tan^{-1}\left(\frac{0.366}{k}\right) = 15^\circ$

$\Rightarrow \quad k\left(\frac{0.366}{\tan 15^\circ}\right) = 1.366$

Q.11 (a)

$H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$

$= \frac{(1+j\omega)}{10\times\left(1+\frac{j\omega}{10}\right)\times\left(1+\frac{j\omega}{100}\right)^2}$

$H(j\omega) = \frac{k(1+j\omega)}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)^2}$

Where $k = \frac{1}{10} = 0.1$

Corner frequencies

$\omega_1 = 1\text{rad}/\text{sec}$

$\omega_2 = 10\text{rad}/\text{sec}$

and $\omega_3 = 100\text{rad}/\text{sec}$

For frequency less than ω_1i.e.$\omega < \omega_1$

Gain of the system is constant as there is no pole at origin.

Gain $= 20\log k = 20\log 0.1 = 20\text{dB}$

At

$\omega = \omega_1 = 1\text{rad}/\text{sec or} \log\omega_1 = \log = 0$

There is Zero, so system gain increases with slope +20 dB/decade and system gain becomes 0 dB at $\omega = 10\text{rad}/\text{sec or}$

$\log\omega\big|_{\omega=10} = \log_{10} = 1$

At $\omega_2 = 10$ or $\log\ \omega\big|\omega_{2=10} = \log 10 = 1$

There is pole, so slope is -20 dB/ decade.

Overall slope $\omega_2 < \omega < \omega_3$

=20 dB/decade-20 dB/decade

=0 dB/decade

So, gain remains constant between $\omega_2 < \omega < \omega_3$

or $1 < \log\omega < 2$

At $\omega = \omega_3 = 100\text{rad}/\text{sec or}$

$\log\omega - \omega_3 = 100 = \log 100 = 2$

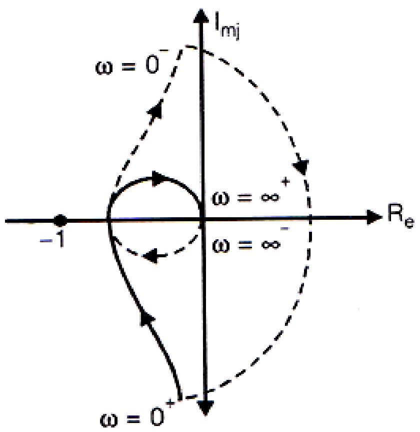
The double poles are present.

So, system gain decrease with -40 dB/ decade.

Q.12 (a)

Assuming no. of open loop poles in the RHS of s-plane=P=0 Complete nyquist plots.

1.



No. of encirclements = N=0

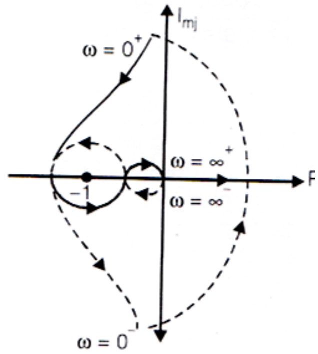
N=P-Z=0

$\Rightarrow$ 0-Z=0

$\Rightarrow$ Z=0

Hence system is stable.

2.



Two anti-clockwise encirclement

N=2

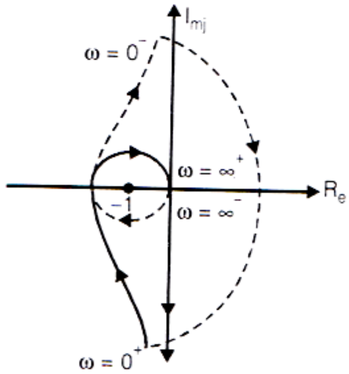
P-Z=N=2

$\Rightarrow$ 0-Z=2

Z=2

Hence system is unstable.

3.



Two clockwise encirclement of -1

Hence N=-2

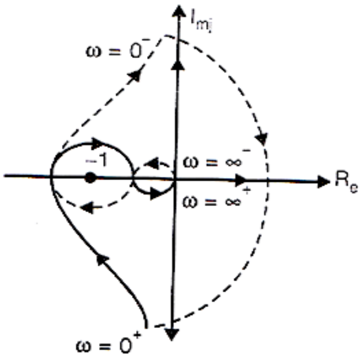
P-Z=N=-2

$\Rightarrow$ 0-Z=-2

Z=2

Hence the system is unstable.

4.



Two clockwise encirclement of -1
Hence N=-2
$\Rightarrow$ P-Z=N=-2
Z=2
Hence the system is unstable.
So, option (a) is correct.

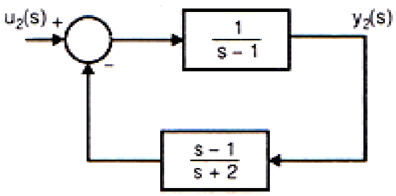
Q.13 (b)

$$G(s)=\frac{1}{s(s+1)(s+2)}$$

Put $s=j\omega$

$$G(j\omega)=\frac{1}{j\omega(1+j\omega)(2+j\omega)}=\frac{-j(1-j\omega)(2-j\omega)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$=\frac{-j(2-3j\omega-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}=\frac{-3\omega-j(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$



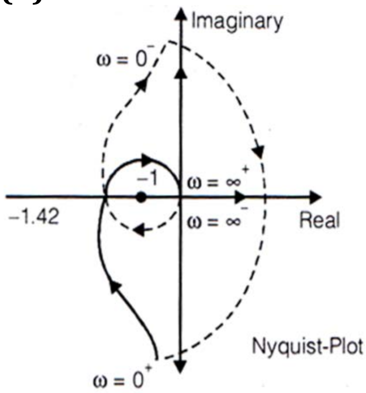
$$=\frac{-3}{(1+\omega^2)(4+\omega^2)}+\frac{j(\omega^2-2)}{\omega(1+\omega^2)(4+\omega^2)}$$

At $\omega\to 0, x\to -\frac{3}{4}, y\to -\infty$

Q.14 (c)

Initial slope is -40dB/ decade, it means there are double pole at origin.
Slope changes from -40dB/ decade to -20dB/ decade. It means there is a zero.
Slope changes from -20dB/ decade to 0dB/ decade at some other frequency. It means there is one more zero.
Therefore transfer function has two poles and two zeros.

Q.15 (d)



Two clockwise encirclement of 1+j0
$N=-2$
Open-loop system is stable
$\Rightarrow P=0$
$N=P-Z-2=0-Z$
Z=No. of closed loop poles in RHS of 8-planes.
Hence the system is unstable.

Q.16 (b)

Initial slope= -40 dB/dec.
Hence the system is type-2. So the corresponding term of the transfer function is $1/s^2$.
At ω=2rad/s, slope changes by -20 dB/dec. from -40 dB/dec to -60 dB/dec. Hence the corresponding term of the transfer function is $\frac{1}{\left(1+\frac{s}{2}\right)}$. At ω=5 rad/s, slope changes by 20 dB/dec from -60 dB/dec to -40 dB/dec. Hence, the corresponding term of the transfer function is $\left(1+\frac{s}{5}\right)$.
At ω=25the slope changes by -20 dB/dec. from -40 dB/dec to -60 dB/dec. Hence the corresponding term of the transfer function is $\left(\frac{1}{1+\frac{s}{25}}\right)$.

$$G(s)=\frac{k\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{2}\right)\left(1+\frac{s}{25}\right)}$$

At $\omega=0.1(\omega\le 2\text{rad/sec})$

Gain in

$dB=20\log k-40\log\omega$

$80=20\log k-40\log 0.1$

$20\log k=40$

$k=100$

$$G(s)=\frac{100\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{2}\right)\left(1+\frac{s}{25}\right)}$$

$$=\frac{1000(s+5)}{s^2(s+2)(s+25)}$$

Q.17 (d)

Open loop transfer function,

$$G(s)=\frac{e^{-0.1s}}{s}$$

Put $s=j\omega$

$$G(j\omega)=\frac{e^{-j0.1\omega}}{j\omega}$$

At phase crossover frequency (ω_{pc}), phase of OLTF is -180°

$$\angle G(j\omega)\big|_{\omega\setminus\omega_{pc}}=-180^\circ=-\pi-\frac{\pi}{2}$$

$-0.1\omega_{pc}=-\pi$

$$-0.1\omega_{pc}=-\frac{\pi}{2}$$

$\omega_{pc}=5\pi$

$\left|e^{-j0.1\omega}\right|$ is always 1 for any value of ω

$$|G(j\omega)|=\frac{1}{\omega}$$

Gain at ω_{pc} (phase-cross frequency)

$$|G(j\omega)|_{\omega_{pc=5}}=\frac{1}{5\pi}$$

$$\text{Gain margin } 20\log=\frac{1}{|G(j\omega_{pc})|}$$

$=20\log 5\pi=23.9\text{dB}$

Q.18 (a)

$$G(s)=\frac{1}{s(s+1)(s+2)}$$

Put $s=(j\omega)$

$$G(j\omega)=\frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$=\frac{1}{j\omega(1+j\omega)(2+j\omega)}\times\frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

$$=\frac{(2-j\omega-2j\omega-\omega^2)}{j\omega(1-\omega^2)(4+\omega^2)}$$

$$=\frac{(2-\omega^2)-3j\omega}{j\omega(1+\omega^2)(4+\omega^2)}$$

$$=\frac{-3}{(1+\omega^2)(4+\omega^2)}+j\frac{(\omega^2-2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$\text{Re}[G(j\omega)]\frac{-3}{(1+\omega^2)(4+\omega^2)}$$

and

$$\text{imj}[G(j\omega)]\frac{\omega^2-2}{\omega(1+\omega^2)(4+\omega^2)}$$

As $\omega\to 0, \text{Re}[G(j\omega)]\to-\frac{3}{4}$

and $\text{Imj}[G(j\omega)]\to-\infty$

As $\text{Re}[G(j\omega)]\to-0$,

And $\text{Imj}[G(j\omega)]\to+0$

at $\omega=\omega_{pc}$

Phase across frequency, the plot crosses negative real axis and imaginary part of G(jω) is zero.

$$\text{Imj}[G(j\omega_{pc})]=\frac{\omega^2{}_{pc}-2}{\omega_{pc}(1+\omega_{pc}^2)(4+\omega_{pc}^2)}=0$$

$\omega^2{}_{pc}=\sqrt{2}$ rad/ sec

So, plot crosses negative real axis at $\omega=\sqrt{2}$ rad/sec.

Therefore option (a) is correct.

Q.19 (a)

At gain across over frequency (ω_{gc}), magnitude of $G(j\omega)$ is 1.

$\left|G\left(j\omega_{gc}\right)\right| = 1$

Phase of $G(j\omega) = \angle G\left(j\omega_{gc}\right)$

$= -150^\circ$

Phase margin $= 180^\circ + \angle G\left(j\omega_{gc}\right)$

$= 180^\circ - 150^\circ = 30^\circ$

At phase cross frequency $\left(\omega_{pc}\right)$,

Phase of $G(j\omega)$ is

$-180^\circ, \angle G\left(j\omega_{pc}\right) = -180^\circ$

M = magnitude of $G(j\omega)$ at

$\left(\omega_{pc}\right) = \left|G\left(j\omega_{pc}\right)\right| = 0.5$

Gain margin $20\log\frac{1}{M}$

$= 20\log\frac{1}{0.5} = 6\text{dB}$

Q.20 (c)

For sinusoidal excitation

$s = j\omega$

$\therefore G(j\omega)$

$$= \frac{\left(-\omega^2+9\right)\left(j\omega+2\right)}{\left(j\omega+1\right)\left(j\omega+3\right)\left(j\omega+4\right)}$$

For zero steady –state output

$\left|G(j\omega)\right| = 0$

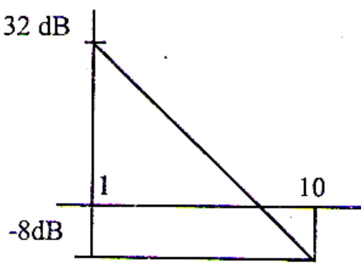
$$= \frac{\left(-w^2+9\right)\sqrt{w^2+4}}{\left(\sqrt{w^2+1}\right)\left(\sqrt{w^2+9}\right)\left(\sqrt{w^2+16}\right)}$$

For zero steady-state output

$\Rightarrow \quad \omega^n = 9$

$\Rightarrow \quad \omega = 3\text{rad/sec}$

Q.21 (b)



$\omega = 1$ to $\omega = 10$

Is 1 dec are change & change is (G) is 40 dB

$\therefore$ S lope is 40dB / dec

$\therefore$ There are 2poles is orign

So, $G(s) = \frac{K}{S^2}$

$|G|_{at\omega=1} = 32\text{dB}(\text{given})$

$\Rightarrow 2\log\left|\frac{k}{\omega^2}\right|_{\omega=1} = 32\text{dB}$

$\Rightarrow 20\log k = 32\text{dB} \Rightarrow k = 39.8$

$\therefore G = \frac{39.8}{S^2}$

Q.22 (a)

$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

If we convert it into time constants,

$$G(s) = \frac{5\times4\left[1+\frac{s}{4}\right]}{s[0.25]\left[1+\frac{s}{0.25}\right]25\left[1+\frac{4}{25}.s+\left[\frac{s}{5}\right]^2\right]}$$

$$G(s) = \frac{3.2\left[1+\frac{s}{4}\right]}{s\left[1+\frac{s}{0.25}\right]\left[1+\frac{4}{25}.s+\frac{s}{5}^2\right]}$$

Constant gain term is 3.2

$\omega_n = 5 \rightarrow$ highest corner frequency

Q.23 (c)

$$G(s) = k.\frac{(1+3s)}{(1+s)}$$

$$G(s) = \frac{3k.\left(s+\frac{1}{3}\right)}{(s+1)}$$

Here k=1

$\frac{1}{T} = \frac{1}{3} \Rightarrow \frac{1}{\alpha T} = 1$

$\omega_m = 1/\sqrt{3}; \alpha = 1/3$

$$G(s)\Big|_{\omega=1/\sqrt{3}} = \frac{\sqrt{4}}{\sqrt{4/3}} \Rightarrow \sqrt{3}$$

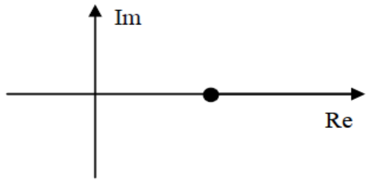
$G_m\big|_{\text{in dB}} = 20\log\sqrt{3} = 4.77\text{dB}$

$$\Phi_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right]$$

$$\alpha = 1/3 = \sin^{-1}\left[\frac{1-1/3}{1+1/3}\right] = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Phi_m = 30°$$

Q.24 (b)



$$G_1(s) = \frac{1}{s}; G_2(s) = s$$

$$G_1 \cdot G_2(s) = s \cdot \frac{1}{s} = 1$$

Q.25 (a)

Phase Crossover frequency

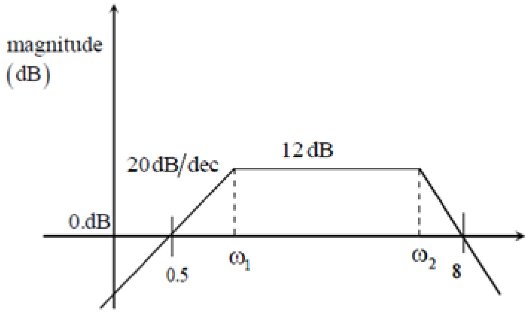
$\omega_{PC} : \underline{|GH}_{\omega=\omega_{PC}} = -180°$

$\underline{|GH} = -3\tan^{-1}\omega \Rightarrow 180° = -3\tan^{-1} W_{PC}$

$\Rightarrow W_{PC} = \tan 60° = \sqrt{3}$

Q.26 (a)

By looking to the plot we can say that since the initial slope is +20 there must be a zero on the origin



If we find ω_2 we can get the answer by eliminating options

$$\text{Slope} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$\Rightarrow -40 = \frac{0-12}{\log 8 - \log\omega_2}$$

$$\Rightarrow \log 8 - \log\omega_2 = \frac{12}{40}$$

$$\log\omega_2 = \log 8 - \frac{12}{40} \Rightarrow \omega_2 = 4$$

So one of the corner frequency is $\omega_2 = 4s$ at this frequency 2 poles should exist because the change in slope is -40db

From this we can say option A satisfies the condition

(i) A zero at origin

(ii) one of corner frequency 4H term will be $\left(1+\frac{s}{4}\right)$ having 2 poles 1

Q.27 (a)

$$GH = \frac{S+3}{S(S-3)}$$

$$|GH| = \frac{\left(\omega^2+a\right)^{1/2}}{\omega^2\left(\omega^2+a\right)^{1/2}} = \frac{1}{\omega^2}$$

$$\underline{|GH} = \left[\tan^{-1}\frac{\omega}{3}\right] - \left[180° + 180° - \tan^{-1}\frac{\omega}{3}\right]$$

$$= 2\tan^{-1}\frac{\omega}{3}$$

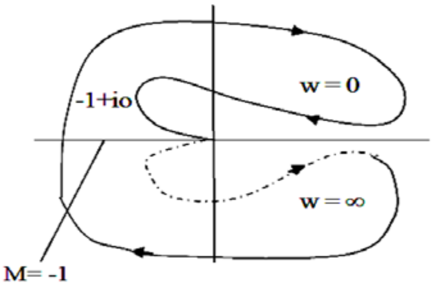
$$\rightarrow GH = \frac{1}{\omega^2}\underline{|2\tan^{-1}\frac{\omega}{3}}$$

At $\omega = 0, GH = \infty\underline{|0}$

At $\omega = \infty, GH = 0\underline{|180^0}$

At $\omega = 3, GH = \frac{1}{9}\underline{|90^0}$

So the plot start at 0° and goes to 180° through 90°. Since there are 2 poles on origin we will get 2 ∞ radius semicircle those will start where the mirror image ends and will terminate where the actual plot started in clockwise direction. So the plot will be



So the Nyquist plot of G(s) H(s)
Encircles — 1 + j0
Once in clockwise direction

GATE QUESTIONS(IN)

Q.1 A unity feedback system has the following open loop frequency response:

Ω|(rad /sec)|	2	3	4	5	6	8	10
$\|G(j\omega)\|$	7.5	4.8	3.15	2.25	1.70	1.00	0.64
$\angle G(j\omega)$	118^0	130^0	140^0	150^0	157^0	170^0	180^0

The gain and phase margin of the system are

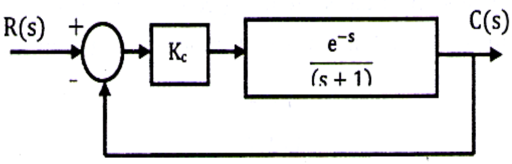
a) $0dB, -180°$ b) $3.88dB, -170°$

c) $0dB, 10°$ d) $3.88dB, 10°$

[GATE-2006]

Common Data for Question Q.2 & Q.3:

The following figure represents a proportional control scheme of a order system with transportation lag.



Q.2 The angular frequency in radians/s at which the loop phase lag becomes180° is

a)0.408 b)0.818

c)1.56 d)2.03

[GATE-2007]

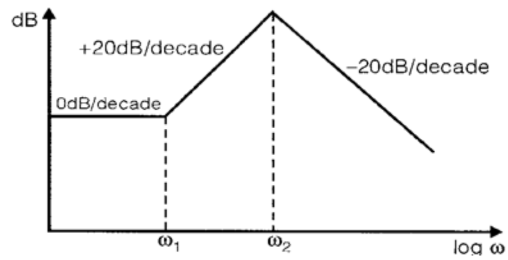
Q.3 The steady state error for a unit step input when the gain $K_c = 1$ is

a) $\frac{1}{4}$ b) $\frac{1}{2}$

c) 1 d) 2

[GATE-2007]

Q.4 The Bode asymptotic plot of a transfer function is given below, in the frequency range shown, the transfer function has



a)3 pole and 1 zero

b)1 pole and 2 zeros

c)2 poles and 1 zero

d)2 poles and 2 zeros

[GATE-2008]

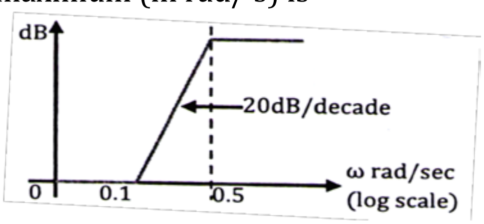
Q.5 A unity feedback control loop with an open transfer function of the form $\frac{K}{s(s+a)}$ has a gain crossover frequency of 1 rad/s and a phase margin of 60° if an element having a transfer function $\frac{s-\sqrt{3}}{s+\sqrt{3}}$ is inserted into the loop, the phase margin will become

a) 0° b) 30°

c) 45° d) 60°

[GATE-2009]

Q.6 The asymptotic Bode magnitude plot of a lead network with its pole and zero on the left half of the s-plane is shown in the adjoining figure. The frequency at which the phase angle of the network is maximum (in rad/ s) is



a) $\frac{3}{\sqrt{10}}$ b) $\frac{1}{\sqrt{20}}$

c) $\frac{1}{20}$ d) $\frac{1}{30}$

[GATE-2010]

Common data for question Q.7 & Q.8:
The open-loop transfer function of a unity negative feedback control system is given by $G(s)=\frac{k}{(s+5)^3}$

Q.7 The value of K for the phase margin of the system to be 45° is

a) $250\sqrt{5}$ b) $250\sqrt{2}$

c) $125\sqrt{5}$ d) $125\sqrt{2}$

[GATE-2011]

Q.8 The value of K for the damping ratio ζ to be 0.5, corresponding to the dominant closed loop complex conjugate pole pair is

a) 250 b) 125

c) 75 d) 50

[GATE-2011]

Q.9 The open loop transfer function of a unity negative feedback control system is given by $(s)=\frac{150}{s(s+9)(s+25)}$. The gain margin of the system is

a)10.8dB b)22.3 dB

c)34.1 dB d)45. 6 dB

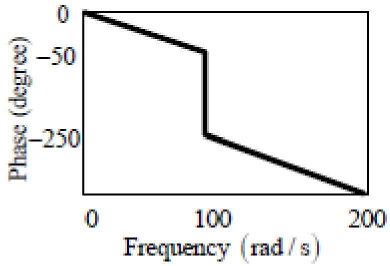
[GATE-2012]

Q.10 The loop transfer function of a feedback control system is given by $G(s)H(s)=\frac{1}{s(s+1)(9s+1)}$ Its phase crossover frequency (in rad/s), approximated to two decimal places, is________.

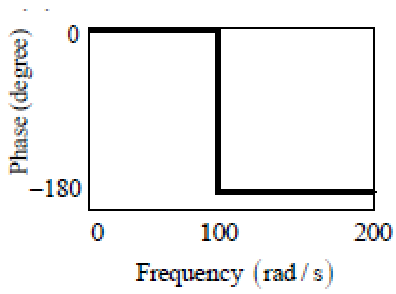
[GATE-2014]

Q.11 The approximate phase response of $\frac{100^2 e^{-0.01s}}{s^2+0.2s+100^2}$ is

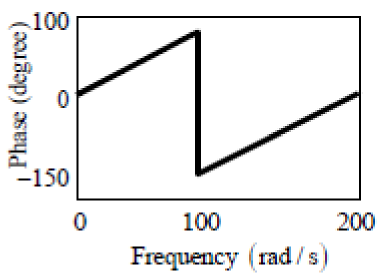
a)



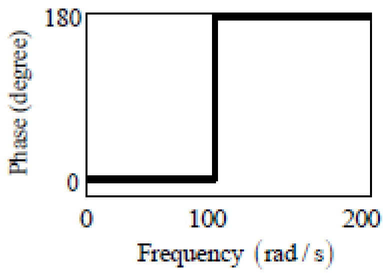
b)



c)



d)



[GATE-2018]

Q.12 An input p(t) = sin(t) is applied to the system $G(s)=\frac{s-1}{s+1}$. The corresponding steady state output is y(t) = $\sin(t+\phi)$, where the phase ϕ (in degrees), when restricted to $0^0 \le \phi \le 360^0$, is ____________.

[GATE-2018]

Q.13 Consider the transfer function $G(s)=\frac{2}{(s+1)(s+2)}$. The phase margin of G(s) in degree is __________.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(d)	(b)	(c)	(a)	(b)	(b)	(b)	(c)	0.33	(a)	90	180

EXPLANATIONS

Q.1 (d)

$\Omega_g = 8 \Rightarrow PM \Rightarrow 180° - 170° = 10°$

$\Omega_c = \frac{10rad}{sec} \Rightarrow GM$

$= 20\log_{10}\left(\frac{1}{0.4}\right) = 3.88dB$

Q.2 (d)

$GH(s) = \frac{K_c e^{-s}}{s+1}, GH(j\omega) = \frac{K_c e^{-j\omega}}{1+j\omega}$

$\angle GH(j\omega) = -\omega - \tan^{-1}(\omega)$

Phase lag $= \omega + \tan^{-1}(\omega) = \pi$ rad

$= 3.142rad$

Is satisfied only at

$\omega = 2.03rad/sec$

Q.3 (b)

For $K_c = 1$ and $H(s) = 1$

OLTF=$G(s) = \frac{e^{-s}}{s+1}$

For r(t)=R.u(t),R=1

Positional error constant =

$K_p = \underset{s\to 0}{Lt}\ \underset{s\to 0}{Lt}\ G(s) = 1$

$e_{ss} = \frac{R}{1+K_p} = \frac{1}{2}$

or

$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{s+1}{s+1+K_c e^{-s}}$

$E(s) = \frac{(s+1)}{s(s+1+e^{-s})}$

$e_{ss} = \underset{s\to 0}{Lt}\ \underset{s\to 0}{Lt}\ E(s) = \frac{1}{2}$

Q.4 (c)

Compare with Bode magnitude plot of standard transfer function.

Q.5 (a)

PM of $T(s) = 60°$

$= 180 + \left(-90° - \tan^{-1}\left(\frac{\omega}{a}\right)\right)$

$\Rightarrow a = \sqrt{3}$

PM of $T'(s) = \frac{s-\sqrt{3}}{s+\sqrt{3}} \cdot \frac{k}{s(s+\sigma)}$

$T'(s) = 180° + \angle T'(j\omega)_{\omega=1} = 0°$

Q.6 (b)

$\omega_m = \sqrt{\omega_1\omega_2} = \sqrt{0.1\times 0.5} = \frac{1}{\sqrt{20}}$

Q.7 (b)

PM of $45° = 180° - 3\tan^{-1}\left(\frac{\omega}{5}\right) =$

$45° \Rightarrow \tan^{-1}\left(\frac{\omega}{5}\right) = 1 \Rightarrow \omega = 5rad/sec$

$\therefore |G(J\omega)| = 1 \Rightarrow \left|\frac{K}{\left(\omega^2+5^2\right)^{3/2}}\right| = 1 \Rightarrow K$

$= 250\sqrt{2}$

Q.8 (b)

Q.9 (c)

G.M is evaluated using ω_{Pc}

$\phi = -90\tan^{-1}\left(\omega/9\right)$

$\tan^{-1}\left(\omega/25\right) = 180°[\because \omega = \omega_{pc}]$

A B

$\Rightarrow \tan^{-1}\left(\omega/9\right) + \tan^{-1}\left(\omega/25\right)$

$= 90°$ Taking tan on both sides

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan 90° = \infty$

$\Rightarrow 1 - \tan A \tan B = 0$

$\Rightarrow \omega^2 = 225$
$\Rightarrow \tan A \tan B = 0$
$\Rightarrow \omega = 15 r/s$

$$G = \frac{\Delta Q}{15\sqrt{225+81}\sqrt{225+625}}$$

$$\Rightarrow \text{G.M in dB} = 20\log\left[\frac{1}{4}\right] = 34dB$$

Q.10 (0.33)

Given $G(s)H(s) = \frac{1}{s(s+1)(9s+1)}$

Phase cross over frequency

$W_{PC} \Rightarrow |G(j\omega_{pc})H|G(j\omega_{pc}) = -180°$

$W_{PC} \Rightarrow |G(j\omega)H(j\omega) = -180°$

$-90° - \tan^{-1}(\omega) - \tan^{-1}(9\omega) = -180°$

$\Rightarrow \tan^{-1}(\omega) + \tan^{-1}(9\omega) = 90°$

$\Rightarrow \tan^{-1}\left[\frac{\omega + 9\omega}{1-9\omega^2}\right] = 90°$

$\Rightarrow 1-9\omega^2 = 0 \Rightarrow \omega = \frac{1}{3}$

$\omega = 0.33 r/s$

Q.11 (a)

$$G(s) = \frac{10^4 e^{-0.01s}}{s^2 + 0.2s + 100^2}$$

Put $s = j\omega$

$$G(j\omega) = \frac{10^4 e^{-0.01j\omega}}{-\omega^2 + 0.2j\omega + 10^4}$$

$$\angle G(j\omega) = -0.01\omega - \tan^{-1}\left(\frac{0.2\omega}{10^4 - \omega^2}\right)$$

$\omega = 0; \angle G(j\omega) = 0; \omega = 10; \angle G(j\omega) = -5.8^0$

$$\omega = 100; \angle G(j\omega) = -1 - \tan^{-2}\left(\frac{20}{0}\right)$$

$$= -\frac{180}{\pi} - 90^0 = -57.3 - 90^0 = -147.3^0$$

$\omega = 200$

$$\angle G(j\omega) = \frac{-360^0}{\pi} - \tan^{-1}\left(\frac{40}{-3\times 10^4}\right)$$

So, Phase decrease further. Option (a) satisfy.

Q.12 90

$$G(s) = \frac{s-1}{s+1} \quad ...(1)$$

$$y(t) = |G(j\omega)_{\omega=1}|\sin(t+\phi)$$

$$where\ \phi = \angle G(j\omega) = \frac{j\omega - 1}{j\omega + 1}$$

$$\angle G(j\omega) = 180 - \tan^{-1}(\omega) - \tan^{-1}(\omega)$$
$$= 180 - 2\tan^{-1}(\omega)$$

$$\phi = \angle G(j\omega)|_{\omega=1} = 180^0 - 90^0 = 90^0$$

$$|G(j\omega)|_{\omega=1} = 1$$

$$So,\ y(t) = 1\cdot\sin(t + 90^0)$$

$$\phi = 90^0$$

Q.13 180

$$|G(j\omega)| = \frac{2}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}} = 1$$

$(\omega^2+1)(\omega^2+4) = 4 \quad \therefore \omega = \omega_{gc}$

$\omega_{gc}^2\left[\omega_{gc}^2 + 5\right] = 0$

$\therefore \omega_{gc} = 0\ rad/\sec$

$$P\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) = 0^0$$

$P.M. = 180^0$

6 STATE VARIABLE ANALYSIS

6.1 INTRODUCTION

State space analysis is an excellent method for the design and analysis of control systems. The conventional and old method for the design and analysis of control systems is the transfer function method. The transfer function method for design and analysis had many drawbacks.

6.1.1 DRAWBACKS OF TRANSFER FUNCTION ANALYSIS

- Transfer function is defined under zero initial conditions.
- Transfer function approach can be applied only to linear time invariant systems.
- It does not give any idea about the internal state of the system.
- It cannot be applied to multiple input multiple output systems.
- It is comparatively difficult to perform transfer function analysis on computers.

Any way state variable analysis can be performed on any type systems and it is very easy to perform state variable analysis on computers. The most interesting feature of state space analysis is that the state variable we choose for describing the system need not be physical quantities related to the system. Variables that are not related to the physical quantities associated with the system can be also selected as the state variables. Even variables that are immeasurable or unobservable can be selected as state variables.

6.1.2 ADVANTAGES OF STATE VARIABLE ANALYSIS

- It can be applied to non linear system.
- It can be applied to time invariant systems.
- It can be applied to multiple input multiple output systems.
- Its gives idea about the internal state of the system.

6.2 STATE OF A SYSTEM

The state of a system is the minimum set of variables (state variables) whose knowledge at time $t = t_0$, along with the knowledge of the inputs at time $t \geq t_0$ completely describes the behavior of a dynamic system for a time $t \geq t_0$. State variable is a set of variables which fully describes a dynamic system at a given instant of time.

Consider a system having an inputs, b outputs and c state variables. Then,

Output variables

$= Y_1(t), Y_2(t), Y_3(t) \ldots\ldots\ldots\ldots Y_b(t)$

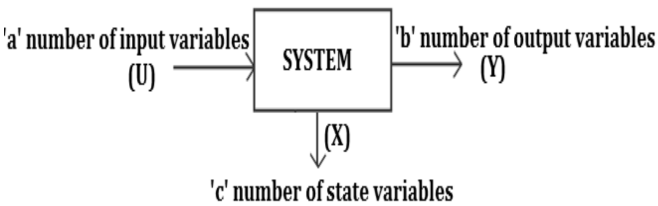
Input variables

$= U_1(t), U_2(t), U_3(t) \ldots\ldots\ldots\ldots U_a(t)$

State variables

$= X_1(t), X_2(t), X_3(t) \ldots\ldots\ldots\ldots X_c(t)$

Then the system can be represented as shown below.



6.2.1 STATE EQUATION BASED MODELING PROCEDURE

The complete system model for a linear time-invariant system consists of

1) A set of n state equations, defined in terms of the matrices A and B.

2) A set of output equations that relate any output variables of interest to the state variables and inputs, and expressed in terms of the C and D matrices.
The task of modeling the system is to derive the elements of the matrices, and to write the system model in the form:

$$x' = Ax + Bu$$

$$\frac{D}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$+\begin{bmatrix} b_{11} & \cdots & b_{1r} \\ b_{21} & \cdots & b_{2r} \\ \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & b_{nr} \end{bmatrix}\begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

$$y = Cx + Du$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$+\begin{bmatrix} d_{11} & \cdots & d_{1r} \\ d_{21} & \cdots & d_{2r} \\ \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & d_{mr} \end{bmatrix}\begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

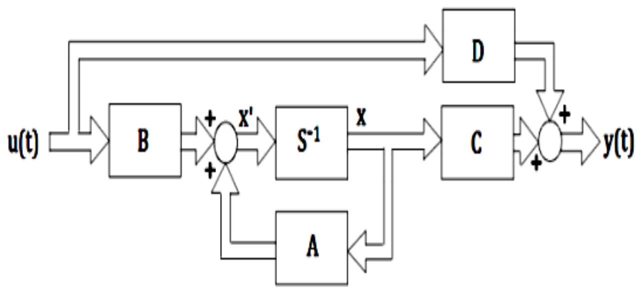
The matrices A and B are properties of the system and are determined by the system structure and elements. The output equation matrices C and D are determined by the particular choice of output variables. The overall modeling procedure developed in this chapter is based on the following steps:

1) Determination of the system order n and selection of a set of state variables from the linear graph system representation.
2) Generation of a set of state equations and the system A and B matrices using a well defined methodology. This step is also based on the linear graph system description.
3) Determination of a suitable set of output equations and derivation of the appropriate C and D matrices.

6.2.2 BLOCK DIAGRAM REPRESENTATION OF LINEAR SYSTEMS DESCRIBED BY STATE EQUATIONS

The matrix-based state equations express the derivatives of the state-variables explicitly in terms of the states themselves and the inputs. In this form, the state vector is expressed as the direct result of vector integration. The block dia. representation is shown in Fig. This general block diagram shows the matrix operations from input to output in terms of the A, B, C, D matrices, but does not show the path of individual variables.

- **Step 1:** Draw an integrator (s^{-1}) blocks, and assigns a state variable to the output of each block.
- **Step 2:** At the input to each block (which represents the derivative of its state variable) draw a summing element.
- **Step 3:** Use the state equations to connect the state variables and inputs to the summing elements through scaling operator blocks.
- **Step 4:** Expand the output equations and sum the state variables and inputs through a set of scaling operators to form the components of the output.



Example

Find the transfer function relating the output y(t) to the input u(t) for a system described by the first-order linear state and output equations:

$$\frac{dx}{dt} = ax(t) + bu(t)$$
$$y(t) = cx(t) + du(t)$$

Solution:

The Laplace transform of the state equation is $sX(s) = aX(s) + bU(s)$

Which may be rewritten with the state variable X(s) on the left-hand side:

$(s-a)X(s) = bU(s)$

Then dividing by (s−a), solve for the state variable:

$X(s) = \frac{b}{s-a}U(s)$ and substitute into the Laplace transform of the output equation $Y(s) = cX(s) + dU(s)$:

$$Y(s) = \left[\frac{bc}{s-a} + d\right]U(s)$$
$$= \frac{ds + (bc - ad)}{s-a}U(s)$$

The transfer function is:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{(ds + (bc - ad))}{(s-a)}$$

6.3 STATE MODEL FROM TRANSFER FUNCTION

Consider a transfer

$$T(s) = \frac{Y(s)}{U(S)} = \frac{s^2 + 3s + 3}{s^3 + 2s^2 + 3s + 1}$$

1) Write the transfer function in the form

$$T(s) = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{U(s)} = \frac{s^2 + 3s + 3}{1} \cdot \frac{1}{s^3 + 2s^2 + 3s + 1}$$

2) $\frac{X(s)}{U(S)} = \frac{1}{s^3+2s^2+3s+1}$

$$\Rightarrow s^3X(s) + 2s^2X(s) + 3sX(s) + X(s) = U(s)$$

From the above equation we can write the differential equation as

$$\frac{d^3x(t)}{dt^3} + 2\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + x(t) = u(t)$$

Put $x(t) = x_1$

$$\frac{dx(t)}{dt} = x_2 = x_1'$$
$$\frac{d^2x(t)}{dt^2} = x_3 = x_2'$$
$$\frac{d^3x(t)}{dt^3} = x_4 = x_3'$$

Now, $x_1' = x_2$

$x_2' = x_3$

and $x_3' = -2x_3 - 3x_2 - x_1 + u(t)$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3) $\frac{Y(s)}{X(s)} = \frac{s^2+3s+3}{1}$

$$\Rightarrow Y(s) = s^2X(s) + 3sX(s) + 3X(s)$$

From the above equation we can write the differential equation as

$$Y(t) = \frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 3x(t)$$

Put $x(t) = x_1$

$$\frac{dx(t)}{dt} = x_2$$
$$\frac{d^2x(t)}{dt^2} = x_3$$

$$Y = \begin{bmatrix} 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u(t)$$

$$C = \begin{bmatrix} 3 & 3 & 1 \end{bmatrix}$$
$$D = [0]$$

6.4 TRANSFER FUNCTION FROM STATE MODEL

Consider a state model derived for linear time invariant system as,

$x(t)' = Ax(t) + Bu(t)$ and

$y(t) = Cx(t) + Du(t)$

The transfer function of the above state model is,

$$T(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D$$

Where I is an Identity matrix

Example:

Consider a system having state model

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u \ \&$$

$$Y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solution:

We know that,

$$T.F. = C[sI - A]^{-1}B + D$$

$$A = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix}, \ B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} s+2 & +3 \\ -4 & s-2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\begin{bmatrix} s-2 & -3 \\ +4 & s+2 \end{bmatrix}}{s^2+8}$$

$$\therefore \ T.F. = \begin{bmatrix} 1 & 1 \end{bmatrix}\frac{\begin{bmatrix} s-2 & -3 \\ +4 & s+2 \end{bmatrix}}{s^2+8}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s-2+4 & -3+s+2 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2+8} = \frac{8s+1}{s^2+8}$$

Properties of state transition matrix:

1) $\varnothing(t) = e^{A\times 0} = I(\text{identity matrix})$

2) $\varnothing^{-1}(t) = \varnothing(-t)$

3) $\varnothing^{-1}(t_1 + t_2) = \varnothing(t_1)\times\varnothing(t_2)$

Example:

Find state transition matrix for $A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$$

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & +1 \\ -2 & s+3 \end{bmatrix}$$

$$\therefore [sI - A]^{-1} = \frac{\begin{bmatrix} s+3 & -1 \\ +2 & s \end{bmatrix}}{(s+1)(s+2)}$$

$$= \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{-1}{(s+1)(s+2)} \\ \frac{+2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

We know that, $e^{At} = \varnothing(t) = \mathcal{L}^{-1}[sI - A]^{-1}$

$$\therefore \varnothing(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & -e^{-t} + e^{-2t} \\ 2e^{-t} - 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

6.5 SOLUTION OF STATE EQUATION

Consider a constant matrix A & input control forces are zero, then the state equation will be $x(t)' = Ax(t)$

Such an equation is called as homogeneous equation & its solution is given by $x(t) = e^{At}x(0)$

Where $e^{At} = \varnothing(t)$ is called as state transition matrix &

$$e^{At} = \varnothing(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$

6.6 CONTROLLABILITY

A system is said to be controllable at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state to any other state in a finite interval of time.

Kalman's test for Controllability:

Consider a LTI system with state equation $x(t)' = Ax(t) + Bu(t)$. For this state equation a Q_c is defined as

$$Q_c = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}_{n\times n}$$

The system will be completely controllable if the rank of matrix Q_c is 'n' i.e. $|Q_c| \neq 0$.

6.7 OBSERVABILITY

A system is said to be observable at time t_0 if, with the system in state $x(t_0)$, it is possible to determine this state from the observation of the output over a finite interval of time.

Kalman's test for Observability:

Consider a LTI system with state equation $x(t)' = Ax(t) + Bu(t)$. For this state equation a Q_0 is defined as

$$Q_o = \begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T & \dots & (A^T)^{n-1}C^T \end{bmatrix}_{n\times n}$$

The system will be completely observable if the rank of matrix Q_o is 'n' i.e. $|Q_o| \neq 0$.

Example:

Check the controllability & observabilty of the system

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t) \text{ and}$$

$$y(t) = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solution:

For the given system

$$A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0]$$

$$Q_c = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|Q_c| = 0 - 1 = -1$$

$\neq 0$ hence the system is control lable

Now, $Q_o = \begin{bmatrix} C^T & A^TC^T \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$|Q_o| = 1 - 0 = 1$$

$\neq 0$; hence the system is observable

6.8 STABILITY OF THE SYSTEM

The transfer function of a system can be obtained from the state equations as

$$T(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D$$

$$= \frac{C \times Adj[sI - A] \times B}{\det[sI - A]} + D$$

$$Q\,[sI - A]^{-1} = \frac{Adj[sI - A]}{\det[sI - A]}$$

$$= \frac{C \times Adj[sI - A] \times B + D \times \det[sI - A]}{\det[sI - A]}$$

As the denominator of the transfer function is $\det[sI - A]$, the characteristics equation will be $|sI - A| = 0$

The roots of this characteristics equation will be the closed loop poles of the system & if any root is positive (on RHS of s-plane) system will be unstable.

Example:

Determine the stability of system $x' = Ax + Bu$

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Solution:

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & -2 \\ 0 & s-2 \end{bmatrix}$$

The characteristics equation is

$$|sI - A| = (s+1)(s-2) = 0$$

The roots of the characteristics equation are $s = 2 \; \& \; s = -1$

As on root (s = 2) lies on RHS of s-plane, system is unstable.

Example:

Find out state model of the following system. $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}+3\frac{dy}{dt}+y=u$

Solution:

The above equation can be written as

$s^3Y(s)+2s^2Y(s)+3sY(s)+Y(s)=U(s)$

The transfer function will be

$$T(s)=\frac{Y(s)}{U(S)}=\frac{1}{s^3+2s^2+3s+1}$$

It can also be written as

$$T(s)=\frac{Y(s)}{X(s)}.\frac{X(s)}{U(s)}=1.\ \frac{1}{s^3+2s^2+3s+1}$$

$$\frac{X(s)}{U(s)}=\frac{1}{s^3+2s^2+3s+1}$$

$\therefore\ s^3X(s)+2s^2X(s)+3sX(s)+X(s)=U(s)$

i.e. $\frac{d^3x}{dt^3}+2\frac{d^2x}{dt^2}+3\frac{dx}{dt}+x=u$

put $x=x_1$

$\frac{dx}{dt}=x_2=x_1'$

$\frac{d^2x}{dt^2}=x_3=x_2'$

$\frac{d^3x}{dt^3}=x_4=x_3'$

Now, $x_1'=x_2$

$x_2'=x_3$

$x_3'=x_4=-x_1-3x_2-2x_3+u$

$$\therefore\begin{bmatrix}x_1'\\x_2'\\x_3'\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-1&-3&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$\frac{Y(s)}{X(s)}=1\Rightarrow y(t)=x(t)$

$$\therefore\ y=\begin{bmatrix}1&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+[0]u$$

GATE QUESTIONS(EC)

Q.1 The transfer function Y(s)/U(s) of a system described by the state equations
$\dot{x}(t) = -2x(t) + 2u(t)$ and $y(t) = 0.5x(t)$
is
a) $0.5/(s-2)$ b) $1/(s-2)$
c) $\frac{0.5}{s+2}$ d) $\frac{1}{s+2}$
[GATE-2002]

Q.2 The zero-input response of a system given by the state –space equations
$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
is
a) $\begin{bmatrix} te^t \\ t \end{bmatrix}$ b) $\begin{bmatrix} e^t \\ t \end{bmatrix}$
c) $\begin{bmatrix} e^t \\ te^t \end{bmatrix}$ d) $\begin{bmatrix} t \\ te^t \end{bmatrix}$
[GATE-2003]

Q.3 The state variable equations of a system are:
1) $\dot{x}_2 = -3x_1 - x_2 + u$,
2) $\dot{x}_2 = 2x_1 y = x_1 + u$
The system is
a) controllable but not observable
b) observable but not controllable
c) neither controllable not observable
d) controllable and observable
[GATE-2004]

Q.4 Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the state transition matrix e^{At} is given by
a) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ b) $\begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix}$
c) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ d) $\begin{bmatrix} 0 & e^t \\ e^t & 0 \end{bmatrix}$
[GATE-2004]

Q.5 A linear system is equivalently represented by two sets of state equations. X=AX+BU and W=CW+DU The eigen values of the representations are also computed as [λ] and [μ] Which one of the following statements is true?
a) [λ]=[μ] and X=W
b) [λ]=[μ] and X≠W
c) [λ]≠[μ] and X=W
d) [λ]≠[μ] and X≠W
[GATE-2005]

Q.6 A linear system is described by the following state equation
$\dot{X}(t) = AX(t) + BU(t), A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
The state –transition matrix of the system is
a) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$
b) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$
c) $\begin{bmatrix} -\cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$
d) $\begin{bmatrix} \cos t & -\sin t \\ \cos t & \sin t \end{bmatrix}$
[GATE-2006]

Q.7 The state space representation of a separately excited DC servo motor dynamics is given as
$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di_a}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix}u$$

Where, ω is the speed of the motor, i_a is the armature current and u is the armature voltage. The transfer function $\frac{\omega(s)}{U(s)}$ of the motor is

a) $\frac{10}{s^2+11s+11}$ b) $\frac{1}{s^2+11s+11}$

c) $\frac{10s+10}{s^2+11s+11}$ d) $\frac{1}{s^2+s+1}$

[GATE-2007]

Statement for Linked Answer Questions Q.8 & Q.9:

Consider a linear system whose state space representation is $\dot{x}(t)=Ax(t)$. . If the initial state vector of the system is $(0)=\begin{bmatrix}1\\-2\end{bmatrix}$, then the system response is $x(t)=\begin{bmatrix}e^{-2t}\\-2e^{-2t}\end{bmatrix}$. If the initial state vector of the system changes to $x(0)=\begin{bmatrix}1\\-1\end{bmatrix}$ then the system response becomes $x(t)=\begin{bmatrix}e^{-t}\\-e^{-t}\end{bmatrix}$

Q.8 The Eigen-value and Eigen-vector pairs (λ_i, v_i) for the system are

a) $\left(-1,\begin{bmatrix}1\\1\end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix}1\\2\end{bmatrix}\right)$

b) $\left(-2,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(-1,\begin{bmatrix}1\\-2\end{bmatrix}\right)$

c) $\left(-1,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix}1\\-2\end{bmatrix}\right)$

d) $\left(-2,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(1,\begin{bmatrix}1\\-2\end{bmatrix}\right)$

[GATE-2007]

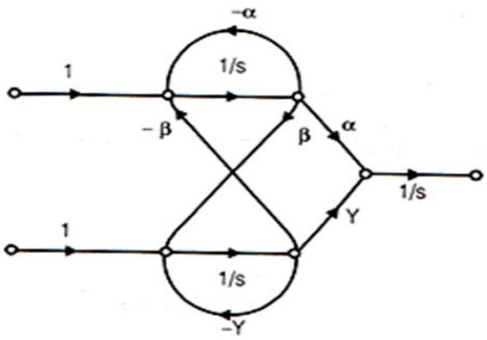
Q.9 The system matrix A is

a) $\begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$

c) $\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$ d) $\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}$

[GATE-2007]

Q.10 A signal flow graph of a system is given below.



The set of equations that correspond to this signal flow graph is

a) $\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}\beta & -\gamma & 0\\\gamma & \alpha & 0\\-\alpha & -\beta & 0\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{bmatrix}1 & 0\\0 & 0\\0 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

b) $\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}0 & \alpha & \gamma\\0 & -\alpha & -\gamma\\0 & \beta & -\beta\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{bmatrix}0 & 0\\0 & 1\\1 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

c) $\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}-\alpha & \beta & 0\\-\beta & -\gamma & 0\\\alpha & \gamma & 0\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

d) $\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}-\gamma & 0 & \beta\\\gamma & 0 & \alpha\\-\beta & 0 & -\alpha\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{bmatrix}0 & 1\\0 & 0\\1 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

[GATE-2008]

Q.11 Consider the system $\frac{dx}{dt}=Ax+Bu \text{with} A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \text{and} B=\begin{bmatrix}p\\q\end{bmatrix}$ where p and q are arbitrary real numbers. Which of the following statements about controllability of the system is true?

a) The system is completely state controllable for any nonzero values of p and q.

b) Only P=0 and q=0 result in controllability.

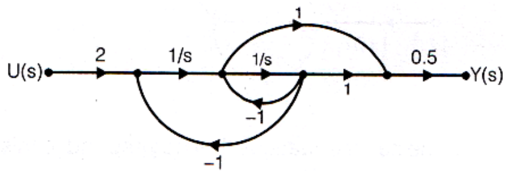
c) The system is uncontrollable for all values of p and q.

d) We cannot conclude about controllability from the given data

[GATE-2009]

Common Data for Questions Q.12 & Q.13:

The signal flow graph of a system is shown below:



Q.12 The state variable representation of the system can be

a) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

b) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x$

$y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x$

c) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

d) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

$y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

[GATE-2010]

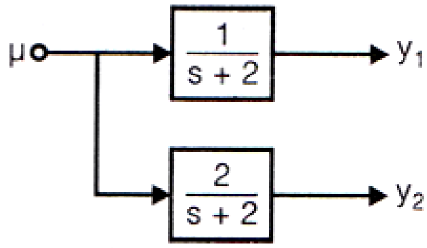
Q.13 The transfer function of the system is

a) $\frac{s+1}{s^2+1}$ b) $\frac{s-1}{s^2+1}$

c) $\frac{s+1}{s^2+s+1}$ d) $\frac{s-1}{s^2+s+1}$

[GATE-2010]

Q.14 The block diagram of a system with one input u and two outputs y_1 and y_2 is given below.



A state space model of the above system in terms of the state vector x and the output vector $y = [y_1 y_2]^T$ is

a) $\dot{x} = [2]x + [1]u; \quad y = [12]x$

b) $\dot{x} = [-2]x + [1]u; \quad y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x$

c) $\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u; y = [12]x$

d) $\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u; y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x$

[GATE-2011]

Q.15 The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

Where y is the output and u is the input. The system is controllable for

a) $a_1 \neq 0, a_2 \neq 0, a_3 \neq 0$

b) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

c) $a_1 = 0, a_2 \neq 0, a_3 = 0$

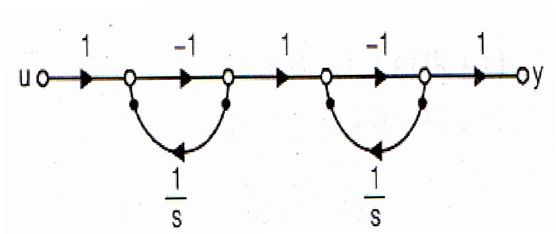
d) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

[GATE-2012]

Statement for Linked Answer Questions 16 & 17

The state diagram of a system is shown below is described by the state –variable equations:

$\dot{X} = AX + Bu; y = CX + Du$



Q.16 The state-variable equations of the system in the figure above are

a) $\dot{X} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

b) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

$y = [1 -1] X + u$

$y = [-1 -1] X + u$

c) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

d) $\dot{X} = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

$y = [-1 -1] X - u$

$y = [1 -1] X - u$

[GATE-2013]

Q.17 The state transition matrix e^{At} of the system shown in figure above is

a) $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$ b) $\begin{bmatrix} e^{-t} & 0 \\ -te^{-t} & e^{-t} \end{bmatrix}$

c) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-t} \end{bmatrix}$ d) $\begin{bmatrix} e^{-t} & -te^{-t} \\ 0 & e^{-t} \end{bmatrix}$

[GATE-2013]

Q.18 Consider the state space model system, as given below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} u;$$

$$y = [111] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The system is

a) controllable and observable

b) uncontrollable and observable

c) uncontrollable and unobservable

d) controllable and unobservable

[GATE-2014]

Q.19 An unforced liner time invariant (LTI) system is represented by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

If the initial conditions are $x_1(0) = 1$ and $x_2(0) = -1$, the solution of the state equation is

a) $x_1(t) = -1, x_2(t) = 2$

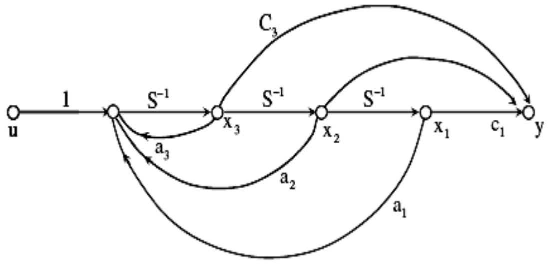
b) $x_1(t) = -e^{-t}, x_2(t) = 2e^{-t}$

c) $x_1(t) = e^{-t}, x_2(t) = -e^{-2t}$

d) $x_1(t) = -e^{-t}, x_2(t) = -2e^{-t}$

[GATE-2014]

Q.20 Consider the state space system expressed by the signal flow diagram shown in the figure.



The corresponding system is

a) always controllable

b) always observable

c) always stable

d) always unstable

[GATE-2014]

Q.21 The state equation of a second-order linear system is given by

$\dot{x}(t) = Ax(t)\ , x(0) = x_0$

For $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ and for

$x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x(t) = \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & +2e^{-2t} \end{bmatrix}$,

When $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, x(t) is

a) $\begin{bmatrix} -8e^{-t} & 11e^{-2t} \\ 8e^{-t} & 22e^{-2t} \end{bmatrix}$

b) $\begin{bmatrix} 11e^{-t} & -8e^{-2t} \\ -11e^{-t} & +16e^{-2t} \end{bmatrix}$

c) $\begin{bmatrix} 3e^{-t} & -5e^{-2t} \\ -3e^{-t} & +10e^{-2t} \end{bmatrix}$

d) $\begin{bmatrix} 5e^{-t} & -3e^{-2t} \\ -5e^{-t} & +6e^{-2t} \end{bmatrix}$

[GATE-2014]

Q.22 The state transition matrix $\phi(t)$ of a system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

a) $\begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}$
d) $\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

[GATE-2014]

Q.23 A second-order linear time-invariant system is described by the following state equations

$$\frac{d}{dt}\text{-}x_1(t) + 2x_1(t) = 3u(t)$$

$$\frac{d}{dt}\text{-}x_2(t) + x_2(t) = u(t)$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and u(t) denotes the input. If the output $c(t) = x_1(t)$, then the system is

a) controllable but not observable
b) observable but not controllable
c) both controllable and observable
d) neither controllable nor observable

[GATE-2016]

Q.24 The state equation and the output equation of a control system are given below:

$$\dot{x} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} x + \begin{pmatrix} 2 \\ 0 \end{pmatrix} u$$

$$y = [1.5 \quad 0.625]x$$

Then transfer function representation of the system is

a) $\dfrac{3s+5}{s^2+4s+6}$

b) $\dfrac{3s-1.875}{s^2+4s+6}$

c) $\dfrac{4s+1.5}{s^2+4s+6}$

d) $\dfrac{6s+5}{s^2+4s+6}$

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(c)	(d)	(b)	(b)	(a)	(a)	(a)	(d)	(d)	(c)	(b)	(c)	(b)	(d)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**						
(a)	(a)	(b)	(c)	(a)	(b)	(d)	(a)	(a)						

EXPLANATIONS

Q.1 (d)

$\dot{x}(t) = -2x(t) + 2u(t)$ ...(i)

$y(t) = 0.5x(t)$ ...(ii)

From (i), Taking Laplace transform of (i)

$sX(s) = -2X(s) + 2U(s)$

$X(s)[s+2]=2U(s)$

$\Rightarrow X(s) = \frac{2U(s)}{(s+2)}$

Taking Laplace transform of (ii)

$Y(s) = 0.5X(s)$

$Y(s) = \frac{0.5\times 2U(s)}{s+2}$

$\therefore \frac{Y(s)}{U(s)} = \frac{1}{(s+2)}$

Q.2 (c)

$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$

$(sI - A)^{-1} = \frac{\begin{bmatrix} s-1 & 0 \\ +1 & s-1 \end{bmatrix}}{(s-1)^2}$

$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{+1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$

$L^{-1}[sI - A]^{-1} = e^{At} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$

$x(t) = e^{At}[x(t_o)]$

$= \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$

Q.3 (d)

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$

$y = [10]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$

$Q_c = [B\ AB]$

$AB = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$\therefore Q_c = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \neq 0$

$\therefore$ Controllable

$Q_0 = \left[C^T A^T C^T\right]$

$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

$A^T C^T = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$\therefore Q_0 = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} \neq 0$

$\therefore$ observable

Q.4 (b)

$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$[sI - A] = \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}$

$e^{At} = [sI - A]^{-1}$

$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{bmatrix} = \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix}$

Q.5 (b)

Eigen values of $A = [\lambda]$

Eigen values of $W = [\mu]$

The eigen value of a system are always unique

So, $[\lambda]=[\mu]$

But a system can be represented by different state models having different set of variables.

X=W

X≠W

Both are possible conditions.

Q.6 **(a)**

$$\Phi(t)=L^{-1}[sI-A]^{-1}$$

$$=L^{-1}\left[\begin{bmatrix} s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix} 0 & 1\\ -1 & 0\end{bmatrix}\right]^{-1}$$

$$=L^{-1}\begin{bmatrix} s & -1\\ 1 & s\end{bmatrix}^{-1}$$

$$=L^{-1}\begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1}\\ \frac{-1}{s^2+1} & \frac{s}{s^2+1}\end{bmatrix}\begin{bmatrix} D\\ =s^2+1\end{bmatrix}$$

$$=\begin{bmatrix} \cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$$

Q.7 **(a)**

$$\begin{bmatrix} \frac{d\omega}{dt}\\ \frac{di_a}{dt}\end{bmatrix}=\begin{bmatrix} -1 & 1\\ -1 & -10\end{bmatrix}\begin{bmatrix} \omega\\ i_a\end{bmatrix}+\begin{bmatrix} 0\\ 10\end{bmatrix}u$$

$$\Rightarrow \frac{d\omega}{dt}=-\omega+i_a \qquad \text{...(i)}$$

$$\Rightarrow \frac{di_a}{dt}=-\omega-10i_a+10u \quad \text{...(ii)}$$

Taking Laplace transform (i) &(ii)

$$\Rightarrow s\omega(s)=-\omega(s)+I_a(s)$$

$$\Rightarrow (s+1)\omega(s)=I_a(s) \qquad \text{...(iii)}$$

$$\Rightarrow sI_a(s)=-\omega(s)-10I_a(s)+10U(s)$$

$$\Rightarrow \omega(s)=(-10-s)I_a(s)+10U(s)$$

$$=(-10-s)(s+1)\omega(s)+10U(s)$$

$$=-\left[s^2+11s+10\right]\omega(s)+10U(s)$$

$$\Rightarrow [s^2+11s+11]\omega(s)+10U(s)$$

$$\Rightarrow \frac{\omega(s)}{U(s)}=\frac{10}{(s^2+11s+11)}$$

Q.8 **(a)**

Sum of the Eigen value =Trace of the principle diagonal matrix

Sum $=-3$. Only potion (a) satisfies both conditions.

Q.9 **(d)**

Multiplication of the eigen value=determinant of the matrix

Therefore from options it seems determinant should be ± 2. Only option (d) satisfies as det=2

Q.11 **(c)**

$$A=\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}$$

$$B=\begin{bmatrix} p\\ q\end{bmatrix}$$

For controllability condition is

$$Q_c=[B, AB, \ldots. A^{n-1}B]\neq 0$$

$$AB=\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} p\\ q\end{bmatrix}$$

$$=\begin{bmatrix} p+0\\ 0+q\end{bmatrix}=\begin{bmatrix} p\\ q\end{bmatrix}$$

So, $\quad Q_c=\begin{bmatrix} p & p\\ q & q\end{bmatrix}=0$

So, the system is uncontrollable for all values of p and q.

Q.12 **(b)**

Q.13 **(c)**

Forward path gain,

$$p_1=2\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.5)=\frac{1}{s^2}$$

$$p_2=2\left(\frac{1}{s}\right)(1)(0.5)=\frac{1}{s}$$

$$\Delta_1=1$$

$$\Delta_2=1$$

$$\Delta=1-\left\{-\frac{1}{s}-\frac{1}{s^2}\right\}$$

$$= 1 + \frac{1}{s} + \frac{1}{s^2}$$

Transfer function of the system,

$$\frac{Y(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{\frac{1}{s^2} + \frac{1}{s}}{1 + \frac{1}{s} + \frac{1}{s^2}} = \frac{s+1}{s^2+s+1}$$

Q.14 (b)

$$\frac{Y_1(s)}{U(s)} = \frac{1}{s+2} \quad \frac{Y_2(s)}{U(s)} = \frac{2}{s+2}$$

$$\frac{Y_1(s)X_1(s)}{X_1(s)U(s)} = \frac{1}{s+2}$$

$$\frac{Y_2(s)X_2(s)}{X_2(s)U(s)} = \frac{2}{s+2}$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s+2} \text{ and } \frac{Y_1(s)}{X_1(s)} = 1$$

$$\frac{X_2(s)}{U(s)} = \frac{1}{s+2} \text{ and } \frac{Y_2(s)}{U(s)} = 2$$

$$sX_1(s) + 2X_1(s) = U(s)$$

And, $Y_1(s) = X_1(s)$

$$sX_2(s) + 2X_2(s) = U(s)$$

And, $Y_2(s) = 2X_2(s)$

$$\dot{x}_1(t) + 2x_2(t) = U(t)$$

And, $y_1(t) = x_1(t)$

$$\dot{x}_2(t) + 2x_2(t) = U(t)$$

And, $y_2(t) = 2x_2(t)$

From Questions

$$y = [y_1 y_2]^T = [12]^1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\dot{x}_1(t) = -2x_1(t) = U(t)$$

$$\dot{x}_2(t) = -2x_2(t) + U(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

Or $\dot{x} = [-2]x + [1]u$

Only option (b) is satisfied

Q.15 (d)

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0]^T$$

For system to be controllable, the metric Q_c must be nonsingular.

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = [0 \quad a_2 \quad 0]^T$$

$$A^2 = \begin{bmatrix} 0 & 0 & a_1a_2 \\ a_2a_3 & 0 & 0 \\ 0 & a_1a_3 & 0 \end{bmatrix}$$

$$A^2B = [a_1a_2 \quad 0 \quad 0]^2$$

$$Q_c = \begin{bmatrix} 0 & 0 & a_1a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

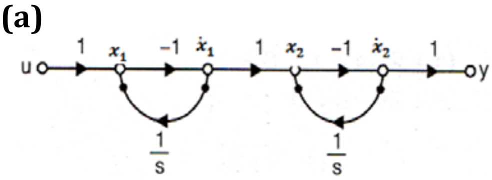
$$|Q_c| = -a_1a_2^2$$

For Q_c to be nonsingular

$$|Q_c| \neq 0$$

$\therefore$ $a_1 \neq 0$ and $a_2 \neq 0$ and $a_3 \epsilon R$

Q.16 (a)



So, $\dot{x}_1 = -x_1 - u$

$$\dot{x}_2 = -(x_2 + \dot{x}_1) = -(x_2 - x_1 - u)$$

$$\dot{x}_2 = x_1 - x_2 + u$$

$$y = \dot{x}_2$$

$$y = x_1 - x_2 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

Q.17 (a)

$$A=\begin{bmatrix}-1 & 0\\ 1 & -1\end{bmatrix}$$

$$sI-A=\begin{bmatrix}s+1 & 0\\ -1 & s+1\end{bmatrix}$$

$$[sI-1]^{-1}$$

$$=\frac{1}{(s+1)\times(s+1)}\begin{bmatrix}s+1 & 0\\ 1 & s+1\end{bmatrix}$$

$$[sI-1]^{-1}=\begin{bmatrix}\frac{1}{s+1} & 0\\ \frac{1}{(s+1)^2} & \frac{1}{s+1}\end{bmatrix}$$

$$\Phi(t)=e^{At}=L^{-1}\{(s-1)^{-1}\}$$

$$e^{At}=\begin{bmatrix}e^{-t} & 0\\ te^{-t} & e^{-t}\end{bmatrix}$$

Q.18 (b)

From the given state model,

$$A=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 0\\ 0 & 0 & -2\end{bmatrix} B=\begin{bmatrix}0\\ 4\\ 0\end{bmatrix} c=[111]$$

Controllable: Q_c = c= [B AB A^2B]

if |Qc|≠0 controllable

$$Q_c=\begin{bmatrix}0 & 4 & -8\\ 4 & -4 & 4\\ 0 & 0 & 0\end{bmatrix}\Rightarrow |Q_c=0$$

∴ uncontrollable

$$\text{Observable : } Q_0=\begin{bmatrix}C\\ CA\\ CA^2\end{bmatrix}$$

If $|Q_{c|}\neq 0\rightarrow$ observable

$$Q_c=\begin{bmatrix}1 & 1 & 0\\ -1 & 0 & -2\\ 1 & -1 & 4\end{bmatrix}\Rightarrow |Q_c|=1$$

∴ Observable.

The system is uncontrollable and observable

Q.19 (c)

Solution of state equation of

X (t) = L^{-1} [SI –A^{-1}] X (0)

$$X(0)=\begin{bmatrix}1\\ -1\end{bmatrix} A=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$$

$$[SI-A]^{-1}=\begin{bmatrix}S+1 & 0\\ 0 & S+2\end{bmatrix}^{-1}$$

$$=\frac{1}{(S+1)(S+2)}\begin{bmatrix}S+2 & 0\\ 0 & S+1\end{bmatrix}$$

$$[SI-A]^{-1}=\begin{bmatrix}\frac{1}{S+1} & 0\\ 0 & \frac{1}{S+2}\end{bmatrix}$$

$$L^{-1}[SI-A]^{-1}=\begin{bmatrix}L^{-1}\left[\frac{1}{S+1}\right] & 0\\ 0 & L^{-1}\left[\frac{1}{S+2}\right]\end{bmatrix}$$

$$L^{-1}\left[(SI-A)^{-1}\right]=\begin{bmatrix}e^{-t} & 0\\ 0 & e^{-2t}\end{bmatrix}$$

$$\begin{bmatrix}X_1 & (t)\\ X_2 & (t)\end{bmatrix}=\begin{bmatrix}e^{-t} & 0\\ 0 & e^{-2t}\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$\begin{bmatrix}X_1 & (t)\\ X_2 & (t)\end{bmatrix}=\begin{bmatrix}-e^{t}\\ -e^{-2t}\end{bmatrix}$$

$$\therefore \frac{X_1(t)=e^{-t}}{X_2(t)=-e^{-2t}}$$

Q.20 (a)

From the given signal flow graph, the state model is

$$\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ a_3 & a_2 & a_1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$Y=[C_1 \quad C_2 \quad C_3]\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}$$

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ a_3 & a_2 & a_1\end{bmatrix}$$

$$B=\begin{bmatrix}0\\0\\1\end{bmatrix};C=[C_1 \quad C_2 \quad C_3]$$

Controllability:

$$Q_c=\begin{bmatrix}B & AB & A^2B\end{bmatrix}$$

$$Q_c=\begin{bmatrix}0 & 0 & 1\\0 & 1 & a_1\\1 & a_1 & a_2+a_1^2\end{bmatrix}$$

$$|Q_c|=1\neq 0$$

Observability

$$Q_0\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}\Rightarrow$$

$$\begin{bmatrix}C_1 & C_1 & C_3\\a_3c_3 & c_1+a_2c_3 & c_2+a_1c_3\\c_2a_3+c_3(a_1a_3) & a_2c_2+c_3(a_1a_2+a_3) & c_1+a_1c_2+c_3(a_1^2+a_2)\end{bmatrix}$$

$|Q_0| \Rightarrow$

depends on $a_1,a_2,a_3 \& c_1 \& c_2 \& c_3$

It is always controllable

Q.21 (b)

Apply linearity principle,

$$\begin{bmatrix}3\\5\end{bmatrix}=a\begin{bmatrix}1\\-1\end{bmatrix}+b\begin{bmatrix}0\\1\end{bmatrix}s \quad a=3;\ b=8$$

$$\Rightarrow x(t)=3\begin{bmatrix}e^{-t}\\-e^{-t}\end{bmatrix}+\begin{bmatrix}e^{-t} & -e^{-2t}\\-e^{-t} & +2e^{-2t}\end{bmatrix}$$

$$\Rightarrow x(t)=\begin{bmatrix}11e^{-t} & -8e^{-2t}\\-11e^{-t} & +16e^{-2t}\end{bmatrix}$$

Q.22 (d)

Given state model,

$$\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}$$

$$A=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$

state transition matrix

$$\phi(t)\Rightarrow L^{-1}\left[(SI-A)^{-1}\right]$$

$$[SI-A]^{-1}=\begin{bmatrix}s & -1\\0 & s\end{bmatrix}^{-1}=\ \Rightarrow\frac{1}{s^2}\begin{bmatrix}s & 1\\0 & s\end{bmatrix}$$

$$\phi(t)=L^{-1}\begin{bmatrix}1/s & 1/s^2\\0 & 1/s\end{bmatrix}$$

$$\phi(t)=\begin{bmatrix}1 & t\\0 & 1\end{bmatrix}$$

Q.23 (a)

The set of equation of the system are

$$\rightarrow\frac{dx_1(t)}{dt}+2x_1(t)=3u(t)\Rightarrow$$

$$x_1(t)=-2x_1(t)+0x_2(t)+3u(t)$$

$$\frac{dx_2(t)}{dt}+x_2(t)=u(t)\Rightarrow$$

$$x_2(t)=0x_1(t)-x_2(t)+u(t)$$

$$C(t)=x_1(t)+0x_2(t)$$

→ we can frame the n state of the system as

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}3\\1\end{bmatrix}4$$

$$Y=[0 \quad 1]\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

→A matrix is $\begin{bmatrix}-2 & 0\\0 & -1\end{bmatrix}$

B matrix is $\begin{bmatrix}3\\1\end{bmatrix}$

C matrix is $[1 \quad 0]$

→ for control ability determinant of $|B\ AB|\neq 0$

$$\begin{bmatrix}3 & -6\\1 & -1\end{bmatrix}=-3+6\neq 0 \quad \text{so controllable}$$

→ For observability determinant of

$$C\begin{bmatrix}C\\Ca\end{bmatrix}\neq 0$$

$$\rightarrow\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix}=0 \text{ so not observable}$$

→Final controllable but not observable

Q.24 (a)

From the given state space representation of the system, we can find matrices as,

$$(A)=\begin{pmatrix}-4 & -1.5\\ 4 & 0\end{pmatrix}\ (B)=\begin{pmatrix}2\\0\end{pmatrix}$$

$$[C]=[1.5 \quad 0.625]$$

We can find the transfer function using

$$T(s)=C\left[(sI-A)^{-1}.B\right] \qquad \text{(i)}$$

$$[sI-A]=\begin{pmatrix}s & 0\\ 0 & s\end{pmatrix}-\begin{pmatrix}-4 & -1.5\\ 4 & 0\end{pmatrix}$$

$$=\begin{pmatrix}s+4 & 1.5\\ -4 & s\end{pmatrix}$$

$$|sI-A|=s(s+4)-(-4)\times 1.5=s^2+4s+6$$

$$\text{Adj}[sI-A]=\begin{pmatrix}s & -1.5\\ 4 & s+4\end{pmatrix}$$

Obtained by interchanging principle diagonal elements and changing signs of other elements

Hence, $[sI-A]^{-1}=\dfrac{\text{Adj}[sI-A]}{|sI-A|}$

$$=\begin{pmatrix}\dfrac{s}{s^2+4s+6} & \dfrac{-1.5}{s^2+4s+6}\\ \dfrac{4}{s^2+4s+6} & \dfrac{s+4}{s^2+4s+6}\end{pmatrix}$$

$$[sI-A]^{-1}.B=\begin{pmatrix}\dfrac{s}{s^2+4s+6} & \dfrac{-1.5}{s^2+4s+6}\\ \dfrac{4}{s^2+4s+6} & \dfrac{s+4}{s^2+4s+6}\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}$$

$$[sI-A]^{-1}.B=\begin{pmatrix}\dfrac{2s}{s^2+4s+6}\\ \dfrac{8}{s^2+4s+6}\end{pmatrix}$$

Substituting values of $[sI-A]^{-1}.B$ and *C* in equation (i),

$$T(s)=[1.5 \quad 0.625]\begin{pmatrix}\dfrac{2s}{s^2+4s+6}\\ \dfrac{8}{s^2+4s+6}\end{pmatrix}$$

$$T(s)=\left[\frac{3s}{s^2+4s+6}+\frac{5}{s^2+4s+6}\right]$$

$$[T(s)]_{1x1}=\left[\frac{3s}{s^2+4s+6}\right]_{1x1}$$

GATE QUESTIONS(EE)

Q.1 Given the homogeneous state-space equation $\dot{X} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x$ the steady state value of $X_{ss} = \lim_{x\to\infty} x(t)$, given the initial state value of $x(t) = [10 -10]^T$, is

a) $X_{ss} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ b) $X_{ss} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

c) $X_{ss} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$ d) $X_{ss} = \begin{bmatrix} \infty \\ \infty \end{bmatrix}$

[GATE-2001]

Q.2 The state transition matrix for the system $\dot{X} = AX$ with initial state is

a) $(s|-A)^{-1}$

b) $e^{At}X(O)$

c) Laplace inverse of $\left[(s|-A)^{-1}\right]$

d) Laplace inverse of $\left[(s|-A)^{-1} X(O)\right]$

[GATE-2002]

Q.3 For the system $\dot{X} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$; $y = [4 \quad 0] X$ with u as unit impulse and with zero initial state, the output y becomes

a) $2e^{2t}$ b) $4e^{2t}$

c) $2e^{4t}$ d) $4e^{4t}$

[GATE-2002]

Q.4 For the system $X = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, which of the following statement is true?

a) The system is controllable but unstable.

b) The system is uncontrollable and unstable.

c) The system is controllable and stable.

d) The system is uncontrollable and stable.

[GATE-2002]

Q.5 A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. The state of the system at the end of 1 second is given by

a) $\begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$ b) $\begin{bmatrix} 0.135 \\ 0.368 \end{bmatrix}$

c) $\begin{bmatrix} 0.271 \\ 0.736 \end{bmatrix}$ d) $\begin{bmatrix} 0.135 \\ 1.100 \end{bmatrix}$

[GATE-2003]

Q.6 The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt^2} + \frac{B}{J}\frac{d\omega}{dt} + \frac{K^2}{LJ}\omega = \frac{K}{LJ}V_a$$

The above equation may be organized in the state-space form as follows

$$\begin{bmatrix} \frac{d^2\omega}{dt^2} \\ \frac{d\omega}{dt} \end{bmatrix} = P \begin{bmatrix} \frac{d\omega}{dt} \\ \omega \end{bmatrix} + QV_a$$

where the P matrix is given by

a) $\begin{bmatrix} -\frac{B}{J} & -\frac{K^2}{LJ} \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} -\frac{K^2}{LJ} & -\frac{B}{J} \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 \\ -\frac{K^2}{LJ} & -\frac{B}{J} \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ -\frac{B}{J} & -\frac{K^2}{LJ} \end{bmatrix}$

[GATE-2003]

Q.7 The state variable description of a linear autonomous system is, X=AX, where X is the two dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The roots of the characteristic equation are

a) -2 and $+2$ b) $-j2$ and $+j2$

c) -2 and -2 d) $+2$ and $+2$

[GATE-2004]

Statement for common data question Q.8 and Q.9:

A state variable system

$$X(t) = \begin{bmatrix} s & 1 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t),$$

With initial condition

$X(0) = [-1 \;\; 3]^T$ and the unit step input u(t) has

Q.8 The state transition matrix

a) $\begin{bmatrix} 1 & \frac{1}{3}\left(1-e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$

b) $\begin{bmatrix} 1 & \frac{1}{3}\left(e^{-t}-e^{-3t}\right) \\ 0 & e^{-t} \end{bmatrix}$

c) $\begin{bmatrix} 1 & \frac{1}{3}\left(e^{-t}-e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$

d) $\begin{bmatrix} 1 & \left(1-e^{-t}\right) \\ 0 & e^{-t} \end{bmatrix}$

[GATE-2005]

Q.9 The state transition equation

a) $X(t) = \begin{bmatrix} t-e^{-t} \\ e^{-t} \end{bmatrix}$ b) $X(t) = \begin{bmatrix} t-e^{-t} \\ 3e^{-3t} \end{bmatrix}$

c) $X(t) = \begin{bmatrix} t-e^{-3t} \\ 3e^{-3t} \end{bmatrix}$ d) $X(t) = \begin{bmatrix} t-e^{-3t} \\ e^{-t} \end{bmatrix}$

[GATE-2005]

Q.10 For a system with the transfer function $H(s) = \frac{3(s-2)}{s^2+4s^2-2s+1}$, the matrix A in the state space form $X = Ax + Bu$ is equal to

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & -4 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

[GATE-2006]

Statement for Linked Answer Questions Q.11 and Q.12:

The sate space equation of a described by x=Ax+Bu, y= Cx where x is where x is state vector, u is input, y is output and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} C = [1 \;\; 1]$$

Q.11 The transfer function G(s) of this system will be

a) $\frac{s}{(s+2)}$ b) $\frac{s+1}{s(s-2)}$

c) $\frac{s}{(s-2)}$ d) $\frac{1}{s(s+2)}$

[GATE-2008]

Q.12 A unity feedback is provided to the above system G(s) to make it a closed loop system as shown in figure.

For a unit step input r(t), the steady state error in the input will be

a)0 b) 1

c)2 d) ∞

[GATE-2008]

Statement for common data Question Q.13 and Q.14:

A system is described by the following state and output equations

$$=\frac{dx_1(t)}{dt}-3x_1(t)+x_2(t)+2u(t)$$

$$\frac{dx_2(t)}{dt}=-2x_2(t)+u(t)$$

$y(t)=x_1(t)$, when u(t) is the input and y(t) is the output.

Q.13 The system transfer function is

a) $\frac{s+2}{s^2+5s-6}$ b) $\frac{s+3}{s^2+5s+6}$

c) $\frac{2s+5}{s^2+5s+6}$ d) $\frac{2s-5}{s^2+5s-6}$

[GATE-2009]

Q.14 The state-transition matrix of the above system is

a) $\begin{bmatrix} e^{-3t} & 0 \\ e^{-2t}+e^{-3t} & e^{-2t} \end{bmatrix}$ b) $\begin{bmatrix} e^{-3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

c) $\begin{bmatrix} e^{-3t} & e^{-2t}+e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$ d) $\begin{bmatrix} e^{3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

[GATE-2009]

Q.15 The system $\dot{X}=AX+BU$ and $A=\begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

a) stable and controllable
b) stable but uncontrollable
c) unstable but controllable
d) unstable and uncontrollable

[GATE-2010]

Q.16 The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix}=\begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}u$$

$$y=\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

Where y is the output and u is the input. The system is controllable for

a) $a_1 \neq 0, a_2=0, a_3 \neq 0$
b) $a_1=0, a_2 \neq 0, a_3 \neq 0$
c) $a_1=0, a_2 \neq 0, a_3=0$
d) $a_1 \neq 0, a_2 \neq 0, a_3=0$

[GATE-2012]

Common Data for Questions Q.17 and Q.18

The state variable formulation of a system is given as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix}=\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u, x_1(0)$$

$=0, x_2(0)=0$ And $y=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Q.17 The system is

a) Controllable but not observable
b) Not controllable but observable
c) Both controllable and observable
d) Both not controllable and not observable

[GATE-2013]

Q.18 The response y(t) to a unit step input is

a) $\frac{1}{2}-\frac{1}{2}e^{-2t}$ b) $1-\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-t}$

c) $e^{-2t}-e^{-t}$ d) $1-e^{-t}$

[GATE-2013]

Q.19 The state transition matrix for the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u \text{ is}$$

a) $\begin{bmatrix} e^t & 0 \\ e^t & e^t \end{bmatrix}$ b) $\begin{bmatrix} e^t & 0 \\ t^2e^t & e^t \end{bmatrix}$

c) $\begin{bmatrix} e^t & 0 \\ -te^t & e^t \end{bmatrix}$ d) $\begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$

[GATE-2014]

Q.20 Consider the system described by following state space equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u;$$

$$y = [1, 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

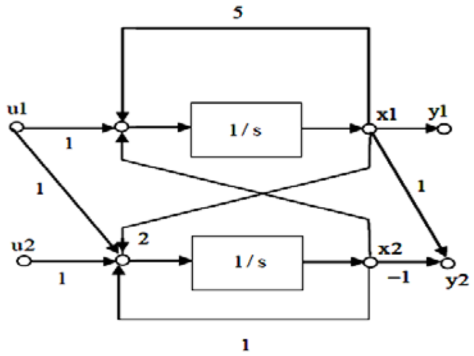
If μ is unit step input, then the steady state error of the system is

a) 0 b) 1/2

c) 2/3 d) 1

[GATE-2014]

Q.21 In the signal flow diagram given in the figure, u_1 and u_2 are possible inputs whereas y_1 and y_2 are possible outputs. When would the SISO system derived from this diagram be controllable and observable?



a) When u_1 is the only input and y_1 is the only output

b) When u_2 is the only input and y_1 is the only output

c) When u_1 is the only input and y_2 is the only output

d) When u_2 is the only input and y_2 is the only output

[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(a)	(c)	(b)	(b)	(a)	(a)	(a)	(a)	(c)	(b)	(d)	(a)	(c)	(b)
15	**16**	**17**	**18**	**19**	**20**	**21**							
(c)	(d)	(a)	(a)	(c)	(a)	(b)							

EXPLANATIONS

Q.1 (a)

$$[sI - A]^{-1}$$

$$= \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}$$

$$e^{At} = \mathcal{L}^{-1}[sI - A]^{-1}$$

$$\begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

$$x(t) = e^{At}X(O)$$

$$\begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}\begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\therefore\ x(t) = \begin{bmatrix} 20e^{-3t} & -10e^{-2t} \\ -10e^{-2t} & \end{bmatrix}$$

$$\therefore\ X_{ss} = \lim_{t \to \infty} x(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Q.2 (c)

$$e^{At} = \mathcal{L}^{-1}[sI - A]^{-1}$$

Q.3 (b)

$$X(t) = e^{At}X(O) + \int_0^t e^{A(t-\tau)}BU(\tau)d\tau$$

$$\therefore x(t) = 0(\text{given})$$

$$\therefore x(t) = O$$

$$+\int_0^t \begin{bmatrix} e^{2(t-\tau)} & 0 \\ 0 & e^{4(t-\tau)} \end{bmatrix}.\begin{bmatrix} 1 \\ 1 \end{bmatrix}.\delta(t)dt$$

$$= \int_0^t \begin{bmatrix} e^{2(t-\tau)} \\ e^{4(t-\tau)} \end{bmatrix}.\delta(t)\,dt = \begin{bmatrix} e^{2t} \\ e^{4t} \end{bmatrix}$$

Output $y(t) = [4 \quad 0].x(t)$

$$= [4 \quad 0].\begin{bmatrix} e^{2t} \\ e^{4t} \end{bmatrix}$$

Hence, $y(t) = 4e^{2t}$

Q.4 (b)

$$|Q_c| = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ Uncontrollable. Characteristic equation:

$$|sI - A| = 0$$

$$\begin{vmatrix} s-2 & -3 \\ 0 & s-5 \end{vmatrix} = 0$$

$$s^2 - 7s + 13 = 0$$

$\Rightarrow$ eigen values,

$s = 3.5 \pm j0.866$

i.e. roots lies on right side of s-plane.

$\Rightarrow$ unstable.

Q.5 (a)

State transition matrix

$$\phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}.$$

Initial conditions, $x(O) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Zero input response is given by

$$X(t) = \phi(t)x(O)$$

$$= \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ 3e^{-t} \end{bmatrix}$$

State of the system at $t = 1\ s$

$$x(t)\big|_{t=1} = \begin{bmatrix} 2e^{-2} \\ 3e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$$

Q.6 (a)

Let $x_1 = \frac{d\omega}{dt}$ and $x_2 = \omega$

$\dot{x}_2 = x_1$

$$\frac{d^2\omega}{dt^2} + \frac{B}{J}\frac{d\omega}{dt} + \frac{K^2}{LJ}\omega = \frac{K}{LJ}V_a$$

$$\frac{d^2\omega}{dt^2} = -\frac{B}{J}\frac{d\omega}{dt} - \frac{K^2}{LJ}\omega + \frac{K}{\omega}V_a$$

$$\Rightarrow \dot{x}_1 = -\frac{B}{J}x_1 - \frac{K^2}{LJ}x_2 + \frac{K}{LJ}V_a$$

$\dot{x}_2 = x_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = P\begin{bmatrix}x_1\\ x_2\end{bmatrix} + QV_a$$

Where

$$P = \begin{bmatrix}-\frac{B}{J} & -\frac{K^2}{LJ}\\ 1 & 0\end{bmatrix} \text{ and } Q = \begin{bmatrix}\frac{K}{LJ}\\ 0\end{bmatrix}$$

Q.7 (a)

System a matrix $= A = \begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$

$$s|-A = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$$

$$= \begin{bmatrix}s & -2\\ -2 & s\end{bmatrix}$$

Characteristic equation

$\Rightarrow |s|-A| = 0$

$$\begin{vmatrix}s & -2\\ -2 & s\end{vmatrix} = 0$$

$\Rightarrow s^2 - 4 = 0$

Roots of the characteristic equation are -2 and +2.

Q.8 (a)

$$\dot{x}(t) = \begin{bmatrix}0 & 1\\ 0 & -3\end{bmatrix}x(t) + \begin{bmatrix}1\\ 0\end{bmatrix}V(t) \quad ...(i)$$

$$\dot{x}(t) = Ax(t) + Bu(t) \quad ...(ii)$$

Comparing eq. (i) and (ii), we get

A=System matrix

$$= \begin{bmatrix}0 & 1\\ 0 & -3\end{bmatrix} \& B = \begin{bmatrix}1\\ 0\end{bmatrix}$$

$$[s|-A] = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 1\\ 0 & -3\end{bmatrix}$$

$$= \begin{bmatrix}s & -1\\ s & s+3\end{bmatrix}$$

$$[s|-A]^{-1} = \frac{\begin{bmatrix}s+3 & 1\\ 0 & s\end{bmatrix}}{s(s+3) - 0x(-1)}$$

$$= \begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+3)}\\ 0 & \frac{1}{s+3}\end{bmatrix}$$

$$[s|-A]^{-1} = \begin{bmatrix}\frac{1}{s} & \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right)\\ 0 & \frac{1}{s+3}\end{bmatrix}$$

State transition matrix

$$= \mathcal{L}^{-1}\left[\left(s|-A\right)^{-1}\right]$$

$$\phi(t) = \begin{bmatrix}1 & \frac{1}{3}\left(1-e^{-3t}\right)\\ 0 & e^{-3t}\end{bmatrix}$$

Q.9 (c)

ZIR (zero input response)

$$= \phi(t) \times X(O) = \begin{bmatrix}1 & \frac{1}{3}(1-e^{-3t})\\ 0 & e^{-3t}\end{bmatrix}\begin{bmatrix}-1\\ 3\end{bmatrix}$$

$$= \begin{bmatrix}-1+1-e^{-3t}\\ 3e^{-3t}\end{bmatrix}$$

$$= \begin{bmatrix}-e^{-3t}\\ 3e^{-3t}\end{bmatrix}$$

ZSR (zero state response)

$$= \mathcal{L}^{-1}\left[\left(s|-A\right)^{-1} BU(s)\right]$$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+3)}\\ 0 & \frac{1}{s+3}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\frac{1}{s}\right\}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} 1/s^2 \\ 0 \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix}$$

∴State transition equation

$$= ZIR + ZSR = \begin{bmatrix} -e^{-3t} \\ 3e^{-3t} \end{bmatrix} + \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

Q.10 (b)

$$H(s) = \frac{Y(s)}{U(s)} = \frac{3(s-2)}{s^2 + 4s^2 - 2s + 1}$$

$$\frac{Y(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)}$$

$$= 3(s-2)\left(\frac{1}{s^3 + 4s^2 - 2s + 1}\right)$$

Let $\frac{X_1(s)}{U(s)} = \frac{1}{s^3 + 4s^2 - 2s + 1}$

$$S^3X_1(s) + 4s^2x_1(s) - 2sx_1(s) + x_1(s) = u(s)$$

Replacing s by $\frac{d}{dt}$

$$\frac{d^3x_1}{dt^2} + 4\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} + x_1 = u(t) \text{ ...(i)}$$

Let $\frac{dx_1}{dt} = x_2 = \dot{x}_1$

$$\frac{d^2x_1}{dt} = \dot{x}_2 = x_3$$

Replacing eq. (i)

$$\dot{x}_3 + 4x_3 - 2x_2 + x_1 = u(t)$$

$$\dot{x}_3 = -x_1 + 2x_2 - 4x_3 + u(t)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -x_1 + 2x_2 - 4x_3 + u(t)$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(t)$$

So, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

Q.11 (d)

$$\dot{X} = Ax + Bu$$

and $y = cx$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \& D = 0$$

$$[S| - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[s| - A]^{-1} = \frac{1}{s(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}$$

Transfer function

$$= C[s| - A]^{-1}B + D$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix}\frac{\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s(s+2)}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix}\frac{\begin{bmatrix} 1 \\ s \end{bmatrix}}{s(s+2)} = \frac{1}{s(s+2)}$$

Q.12 (a)

$$G(s) = \frac{1}{s(s+2)}$$

and $H(s) = 1$

$$r(t) = u(t)$$

$$\Rightarrow \quad R(s) = \frac{1}{s}$$

$$\text{Error } = E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$= \frac{1/s}{1 + \frac{1}{s(s+2)}}$$

$$\Rightarrow \quad E(s) = \frac{s+2}{s(s+2)+1}$$

Steady state error, using final value theorem

$$e_{ss} = \underset{s \to 0}{\text{Lt}}\, sE(s)$$

$$= \lim_{s \to 0} \frac{s(s+2)}{s(s+2)+1} = 0$$

$$= \frac{2s+5}{(s+2)(s+3)} = \frac{2s+5}{s^2+5s+6}$$

Q.13 (c)

Selecting $X_1(t)$ and $X_2(t)$ as state variables.

$$\dot{X}_1(t) = \frac{dx_1(t)}{dt}$$

$$= -3x_1(t) + x_2(t) + 2u(t)$$

$$\dot{X}_2(t) = \frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$\dot{X} = AX + BU$$

So, $A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$y(t) = x_1(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y = CX + DU$$

So, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ & $D = 0$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

Transfer function

$$= C[sI - A]^{-1} B + D$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0$$

$$= \frac{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2(s+2)+1 \\ s+3 \end{bmatrix}}{(s+2)(s+3)}$$

Q.14 (b)

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

State transition matrix

$$= \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

Q.15 (c)

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix} \& B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & -2 \\ 0 & s-2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s-2 & 2 \\ 0 & s+1 \end{bmatrix}}{(s+1)(s-2)} \quad \text{...(i)}$$

Transfer function $= C[sI - A]^{-1} B$

So denominator of eq. (i) gives

Poles of the system

$$(s+1)(s-2) = 0$$

$$s = -1 \& 2$$

One pole lies in RHS of s-plane.

Hence, the so, system is unstable.
For controllability, is Q_c defined as

$$Q_c = [B \;\; : AB]$$

$$AB = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$Q_c = [B \;\; : AB]$$

$$= \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$$

$$|Q_c| \neq 0$$

Hence the system is controllable.

Q.16 (d)

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [1 \;\; 0 \;\; 0]^T$$

For system to be controllable, the metric Q_c must be nonsingular.

$$Q_c = [B \;\; AB \;\; A^2B]$$

$$AB = [0 \;\; a_2 \;\; 0]^T$$

$$A^2 = \begin{bmatrix} 0 & 0 & a_1a_2 \\ a_2a_3 & 0 & 0 \\ 0 & a_1a_3 & 0 \end{bmatrix}$$

$$A^2B = [a_1a_2 \;\; 0 \;\; 0]^2$$

$$Q_c = \begin{bmatrix} 0 & 0 & a_1a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$|Q_c| = -a_1a_2^2$$

For Q_c to be nonsingular

$$|Q_c| \neq 0$$

$\therefore\; a_1 \neq 0$ and $a_2 \neq 0$ and $a_3 \epsilon R$

Q.17 (a)

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

For controllability, $|B : AB| \neq 0$

or $\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1$

$-(-2) = 1 \neq 0$

The system is controllable.

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A^TC^T = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

For observability, $|C^T : A^TC^T| \neq 0$

or $\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix} = 0$

The system is not observable.

Q.18 (a)

$$[S| - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}$$

$$[s| - A]^{-1} = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$[S| - A]^{-1}[B]$$

$$= \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+2} \\ \frac{1}{s+1} \end{bmatrix}$$

$$C[s| - A]^{-1}[B]$$

$$= [1 \;\; 0]\begin{bmatrix} \frac{1}{s+2} \\ \frac{1}{s+1} \end{bmatrix} = \frac{1}{s+2}$$

$$G(s) = \frac{1}{s+2}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$$Y(s)=\frac{1}{s(s+2)}=\frac{1}{2}\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$Y(t)=\frac{1}{2}\left(1-e^{-2t}\right)$$

$$=\frac{1}{2}-\frac{1}{2}e^{-2t}$$

$$e_{ss}=\frac{1}{1+\infty}$$

$$e_{ss}=0$$

Q.21 (b)

Q.19 (c)

Give $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}1\\ 1\end{bmatrix}$

$$[SI-A]^{-1}=\left[\begin{pmatrix}s & 0\\ 0 & s\end{pmatrix}-\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\right]^{-1}$$

$$[SI-A]^{-1}=\begin{bmatrix}\frac{1}{(s-1)} & 0\\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)}\end{bmatrix}$$

The state transition matrix

$$e^{At}=L^{-1}[SI-A]^{-1}$$

$$e^{At}=\begin{bmatrix}e^t & 0\\ te^t & e^t\end{bmatrix}$$

Q.20 (a)

Transfer function $\Rightarrow C[SI—A1]^{-1}.B$

$$=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}S & -1\\ 1 & (s+1)\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}$$

Transfer function $=\frac{1}{s^2+s+1}$

$$\frac{G(s)}{1+G(s)}=\frac{1}{s^2+s+1}$$

$$\Rightarrow G(s)=\frac{1}{s^2+s}$$

Steady state error for unit step

$$e_{ss}=\frac{A}{1+K_p}$$

$$e_{ss}=\frac{1}{1+\lim_{s\to 0}G(s)}$$

$$e_{ss}=\frac{1}{1+\lim_{s\to 0}\frac{1}{s^2+s}}$$

GATE QUESTIONS(IN)

Q.1 The state-variable representation of a plant is given by $\dot{x} = Ax + Bu, y = Cx$
Where the state, u is the input and y is the output. Assuming zero initial conditions, the impulse response of the plant is given by

a) $\exp(At)$

b) $\int \exp[A(t-\tau)]Bu(\tau)d\tau$

c) $C\exp(At)B$

d) $C\int \exp[A(t-\tau)]Bu(\tau)d\tau$

[GATE-2006]

Q.2 The state space representation of a system is given by $\dot{x}=\begin{bmatrix}0 & 1\\ 0 & -3\end{bmatrix}X+\begin{bmatrix}1\\0\end{bmatrix}u, y=\begin{bmatrix}1 & 0\end{bmatrix}x$. The transfer function $\frac{Y(s)}{U(s)}$ of the system will be

a) $\frac{1}{S}$ b) $\frac{1}{s(s+3)}$

c) $\frac{1}{(s+3)}$ d) $\frac{1}{S^2}$

[GATE-2008]

Q.3 A linear time invariant single –input single output system has state space middle given by
$\frac{dx}{dt} = Fx + Gu; y = Hx$
where
$F=\begin{bmatrix}0 & 1\\ -4 & -2\end{bmatrix};G=\begin{bmatrix}1\\0\end{bmatrix};H\begin{bmatrix}1 & 0\end{bmatrix}$
Here, x is the state vector, u is the input, and y is the output. The damping ratio of the system is

a)0.25 b)0.5

c)1 d)2

[GATE-2009]

Q.4 The transfer function of the system described by the state- space equations.
$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-4 & -1\\ -3 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u, y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ is

a) $\frac{s}{s^2+5s+1}$ b) $\frac{2s}{s^2+5s+1}$

c) $\frac{3s}{s^2+5s+1}$ d) $\frac{4s}{s^2+5s+1}$

[GATE-2011]

Q.5 A system is represented in state-space as $\dot{X} = AX + Bu$, , where = A $\begin{bmatrix}1 & 2\\ \alpha & 6\end{bmatrix}$ and $B=\begin{bmatrix}1\\1\end{bmatrix}$. The value of α for which the system is not controllable is _____.

[GATE-2015]

ANSWER KEY:

1	2	3	4	5
(d)	(a)	(b)	(a)	-3

EXPLANATIONS

Q.1 (d)

$x_0(t) = L^{-1}(C(SI-A)^{-1}B.U(S))$

$\Rightarrow x_0(t) = C\left(e^{At} * Bu(t)\right)$

$x_0(t) = C\int e^{A(t-\tau)}.Bu(\tau)d\tau$

Q.2 (a)

$G(s) = C(SI-A)^{-1}B;(SI-A)$

$= \begin{bmatrix} S & -1 \\ 0 & (s+3) \end{bmatrix}$

$\Rightarrow (SI-A)^{-1} = \frac{1}{S(S+3)}\begin{bmatrix} S+3 & 1 \\ 0 & 3 \end{bmatrix}$

$G(s) = \frac{1}{S(S+3)}\left(\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} S+3 & 1 \\ 0 & S \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

$= \frac{1}{S(S+3)}\left(\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} S+3 \\ 0 \end{bmatrix}\right)$

$\Rightarrow G(s) = \frac{1}{S}$

Q.3 (b)

$(SI-A) = \begin{bmatrix} 3 & -1 \\ 4 & S+2 \end{bmatrix}$

$\Rightarrow (SI-A)^{-1}$

$= \frac{1}{(s^2+2s+4)}\begin{bmatrix} (s+2) & 1 \\ -4 & S \end{bmatrix}$

$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix}\frac{1}{(S^2+2S+4)}\begin{bmatrix} (s+2) & 1 \\ -4 & S \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{1}{(S^2+2S+4)}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ S \end{bmatrix} = \frac{1}{(S^2+2S+4)}$

$\Rightarrow \zeta = \frac{1}{2}$

Q.4 (a)

$T(s) = C(SI-A)^{-1}B + D$

$= \begin{bmatrix} 1 & 0 \end{bmatrix}\frac{1}{(S^2+5S+1)}\begin{bmatrix} s+2 & -1 \\ -3 & S+4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{(S^2+5S+1)}$

Q.5 (-3)

For a system to be uncontrollable, its controllability determinant should be equal to zero.

$Q_c = |BAB| = 0$

$AB = \begin{bmatrix} 1 & 2 \\ \alpha & 6 \end{bmatrix}_{2\times2}\begin{bmatrix} 1 \\ +1 \end{bmatrix}_{2\times1} = \begin{bmatrix} 3 \\ \alpha+6 \end{bmatrix}$

$Q_c = |BAB| \rightarrow \begin{vmatrix} 1 & 3 \\ 1 & \alpha+6 \end{vmatrix} = 0$

$\Rightarrow \alpha+6-3 = 0 \Rightarrow \alpha = -3$

7 CONTROLLERS

7.1 INTRODUCTION

In control theory a controller is a device which monitors and physically alters the operating conditions of a given dynamic system. Typical applications of controllers are to hold settings for temperature, pressure, flow or speed. The following six basic control actions are very common among industrial analog controllers:

1) Two-position or on-off
2) Proportional
3) Proportional-plus integral
4) Proportional-plus derivative
5) Proportional plus integral plus derivative

7.2 TWO-POSITION OR ON-OFF CONTROLLERS

An on-off controller is the simplest form of temperature control device. The output from the device is either on or off, with no middle state. An on-off controller will switch the output only when the temperature crosses the set point. For heating control, the output is on when the temperature is below the set point, and off above set point. Two-position or on-off control is relatively simple and inexpensive and, for this reason, is very widely used in both industrial and domestic control systems.

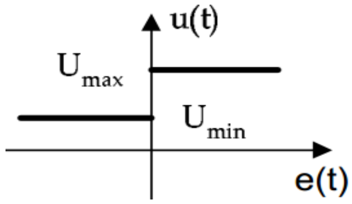
On-off controller algorithm is defined as:

$$u(t) = \begin{cases} U_{max} & \text{if } e(t) > 0 \text{(ON state)} \\ U_{min} & \text{if } e(t) < 0 \text{(OFF state)} \end{cases}$$

Where:

e (t) – control error (for unit feedback)

u (t) – control signal (controller output).



7.2.1 DISADVANTAGE

An inadequacy in this way of control is that control signal oscillates which may cause control variable to oscillate around desired value. Sometimes there is no remedy for this problem.

e.g. if level of liquid in tank is controlled using valve with only two possible states (open or closed) the level will always oscillates around desired value.

7.3 PROPORTIONAL CONTROLLER

Proportional action is the simplest and most commonly encountered of all continuous control modes. In this type of action, the controller produces an output signal which is proportional to the error. Hence, the greater the magnitude of the error, the larger is the corrective action applied.

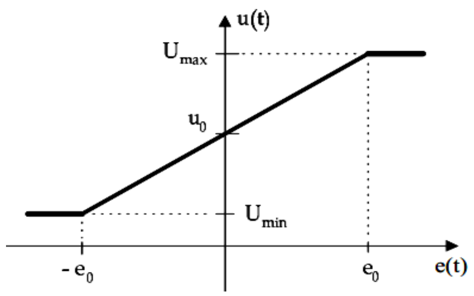
P controller control algorithm is given as:

$$u(t) = \begin{cases} U_{max} & \text{for } e(t) > e_o \\ u_o + K_p e(t) & \text{for } -e_o < e(t) < e_o \\ U_{min} & \text{for } e(t) < -e_o \end{cases}$$

Where:

u_o – Amplitude of control signal when control error is equal 0

K_p– P controller gain for P mode nominal area $e(t)<|e_o|$



Many industrial controllers have defined a proportional band (PB) instead of gain:

PB = $\frac{100}{K_P}\%$

It should be noted that for $K_p = 1$ a proportional band is equal PB = 100%.

P controller can eliminate forced oscillations caused by use of on-off controller. However, a second problem arises. There exists now a steady state error. A relationship between control signal and error inside area $e(t) < |e_o|$ is given as:

$u(t) = u_o + K_p e(t)$

Steady state error is then:

$e(t) = \frac{u(t) - u_o}{K_p}$

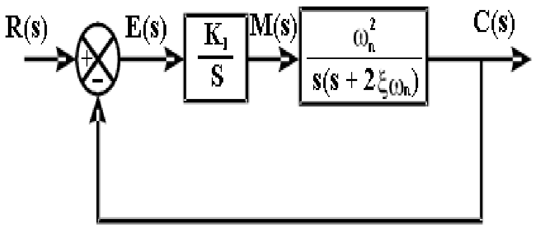
i.e. $e(t) \propto \frac{1}{K_p}$

Proportional controller can stabilize only 1st order unstable process. In general it can be said that P controller cannot stabilize higher order processes. Changing controller gain K can change closed loop dynamics. A large controller gain will result in control system with:

a) smaller steady state error, i.e. better reference following
b) faster dynamics, i.e. broader signal frequency band of the closed loop system and larger sensitivity with respect to measuring noise
c) smaller gain and phase margin

7.4 INTEGRAL CONTROLLER

In integral controller the signal driving the controlled system is derived by integrating the error in the system. An integral control is sometimes called reset control. The transfer function of the controller is



$$\frac{M(s)}{E(s)} = \frac{K_i}{s}$$

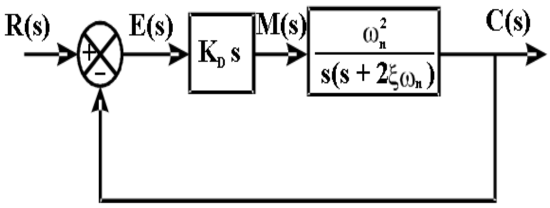
and $m(t) = K_i \int_{-\infty}^{t} e(t)\,dt$

7.4.1 EFFECTS

1) It increases type and order by '1'
2) Integral controller improves the steady state response
3) Steady state error reduces
4) Makes the system lesser stable
5) Speed of response reduces

7.5 DERIVATIVE CONTROLLERS

In derivative controller the signal driving the controlled system is derived by differentiating the error in the system. The transfer function of the controller is



$$\frac{M(s)}{E(s)} = K_D s \text{ and } m(t) = K_D \frac{de(t)}{dt}$$

The o/p of controller can be predicted on the basis of knowledge of the slope of error. Therefore a derivative controller is anticipatory in nature giving output in advance.

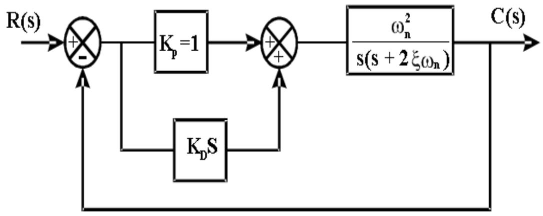
7.5.1 EFFECTS

1) It improves transient response
2) Not used in isolation.

3) Adding derivative action can restrict the overshoots on controlled responses.

7.6 PD CONTROLLER

A controller in the forward path, which changes the controller output corresponding to proportional plus derivative of the error signal is called PD controller.



The transfer function of a PD controller is

$$\frac{M(s)}{E(s)} = K_P + K_D s \text{ and}$$

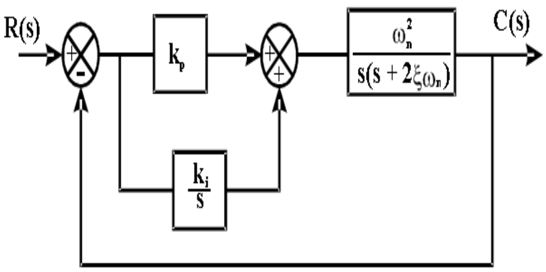
$$m(t) = K_P e(t) + K_D \frac{de(t)}{dt}$$

7.6.1 EFFECTS

1) Transient response is improved
2) It increases damping ratio
3) Type of the system remains unchanged
4) It reduces peak overshoot
5) It reduces settling time
6) Bandwidth increases

7.7 PI CONTROLLER

A controller in the forward path, which changes the controller output corresponding to the proportional plus integral of the error signal is called PI controller.



The transfer function of a PI controller is

$$\frac{M(s)}{E(s)} = K_P + \frac{K_i}{s}$$

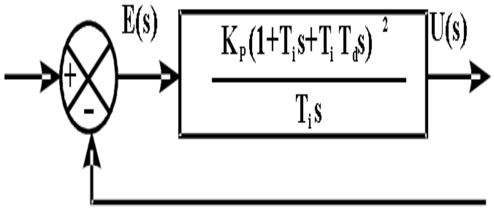
and $m(t) = K_P e(t) + K_i \int_{-\infty}^{t} e(t)\,dt$

7.7.1 EFFECTS

1) It increases the order & type of the system
2) Reduces steady state error
3) Bandwidth is reduced

7.8 PID CONTROLLERS

The combination of proportional control action, integral control action, and derivative control action is termed proportional – plus – integral – plus – derivative control action. This combined action has the advantages of each of the three individual control actions.



The equation of a controller with this combined action is given by

$$m(t) = K_P e(t) + \frac{K_P}{T_i} \int_{-\infty}^{t} e(t)\,dt + K_P T_D \frac{de(t)}{dt}$$

Or the transfer function is

$$\frac{U(s)}{E(s)} = K_P \left(1 + \frac{1}{T_i s} + T_D s\right)$$

Where, K_P is the proportional gain

$T_i = \frac{1}{K_i}$ is the integral time

$T_D = K_D$ is the derivative time

7.8.1 EFFECTS

It improves both steady state as well as transient response.

8 COMPENSATORS

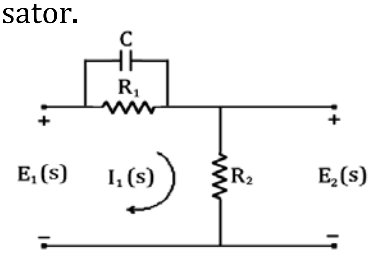
8.1 INTRODUCTION

Because of the prolonged use of the system the parameters of the system can change and output of the system may start deviating from the desired value. The system can be either replaced by a new system or it can be provided with a unit called compensators.

The compensators are used to improve the performance of the system during runtime. In this chapter, we shall discuss electric network realization of basic compensators and their frequency characteristics.

8.2 LEAD COMPENSATOR

The circuit shows a typical lead derivative compensator.



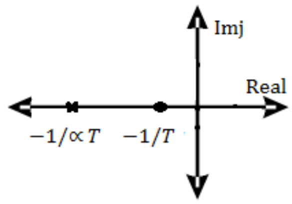
The designation lead applied to this network is based on the steady-state sinusoidal response. The sinusoidal response E_2 with a sinusoidal input E_1 (Initial conditions are considered to be zero) is

$$E_2(s) = \frac{s + 1/R_1C}{s + (R_2 + R_1)/R_1R_2C} E_1(s)$$

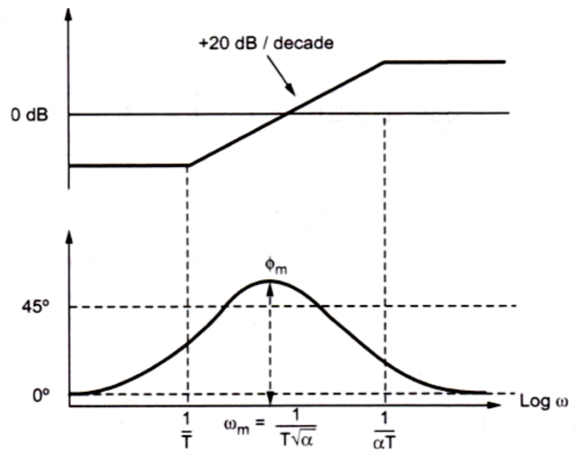
$$\therefore \frac{E_2(s)}{E_1(s)} = \frac{s + 1/T}{s + 1/\alpha T}$$

Where, $T = R_1C$ and $\alpha = R_2/(R_2 + R_1) < 1$

Now, a lead compensator has a zero at $s = -1/T$ & pole at $s = -1/\alpha T$.



8.2.1 BODE PLOT FOR LEAD COMPENSATOR



- For drawing Bode plot the corner frequencies are $\omega_{c1} = 1/T$ & $\omega_{c2} = 1/\alpha T$.
- The maximum phase lead occurs at a frequency ω_m which is geometric mean of the two corner frequencies.

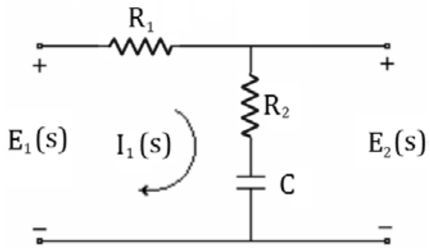
$$\omega_m = \sqrt{\frac{1}{\alpha T} \cdot \frac{1}{T}} = \frac{1}{T\sqrt{\alpha}}$$

- The maximum phase lead at frequency ω_m is given by $\varnothing_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right]$
- A phase lead compensator shifts the gain crossover frequency to higher values where the desired phase margin is acceptable therefore it is effective when the slope of the uncompensated system near the gain crossover frequency is low.

8.3

8.3 LAG COMPENSATOR

The circuit shown is a typical lag or integral compensator.



The output signal is proportional to the sum of the input signal and its integral. The designation lag applied to this network is based on the steady-state sinusoidal response. The sinusoidal response E_2 with a sinusoidal input E_1 (Initial conditions are considered to be zero) is

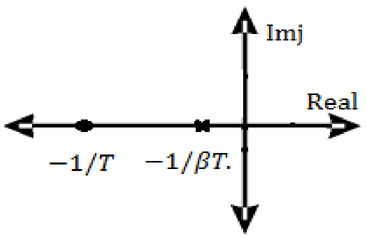
$$E_2(s)=\frac{1+R_2Cs}{1+(R_2+R_1)Cs}E_1(s)$$

$$\therefore \frac{E_2(s)}{E_1(s)}=\left(\frac{R_2}{R_1+R_2}\right)\frac{s+1/R_2C}{s+1/(R_2+R_1)C}$$

$$=\frac{1}{\beta}\cdot\frac{s+1/T}{s+1/\beta T}$$

Where, $T=R_2C$ and $\beta=(R_2+R_1)/R_2>1$

Now, the lag compensator has zero at $s=-1/T$ & pole at $s=-1/\beta T$.

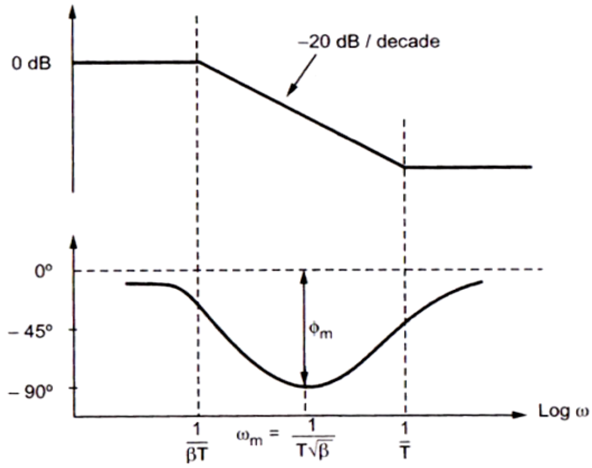


8.3.1 BODE PLOT FOR LAG COMPENSATOR

- For drawing Bode plot the corner frequencies are $\omega_{c1}=1/\beta T$ & $\omega_{c2}=1/T$.
- The maximum phase lag occurs at a frequency ω_m which is geometric mean of the two corner frequencies.

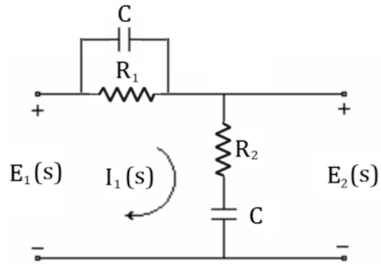
$$\omega_m=\sqrt{\frac{1}{\beta T}\cdot\frac{1}{T}}=\frac{1}{T\sqrt{\beta}}$$

- The maximum phase lead at frequency ω_m is given by $\varnothing_m=\sin^{-1}\left[\frac{1-\beta}{1+\beta}\right]$
- A phase lag compensator shifts the gain crossover frequency to lower values where the desired phase margin is acceptable therefore it is effective when the slope of the uncompensated system near the gain crossover frequency is high.



8.4 LAG-LEAD COMPENSATOR

A lag-lead compensator is a combination of a lag compensator and a lead compensator. The lag section has one real pole and one real zero with the pole to the right of zero. The lead section also has one real pole and one real zero but the zero is to be the right of the pole. The general form of this compensator is



The transfer function of the lag-lead compensator from the figure is

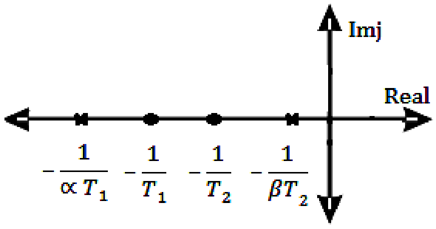
$$\frac{E_2(s)}{E_1(s)}=\frac{(1+T_1s)(1+T_2s)}{(1+\alpha T_1s)(1+\beta T_2s)}$$

Where, $T_1=R_1C_1$, $T_2=R_2C_2$

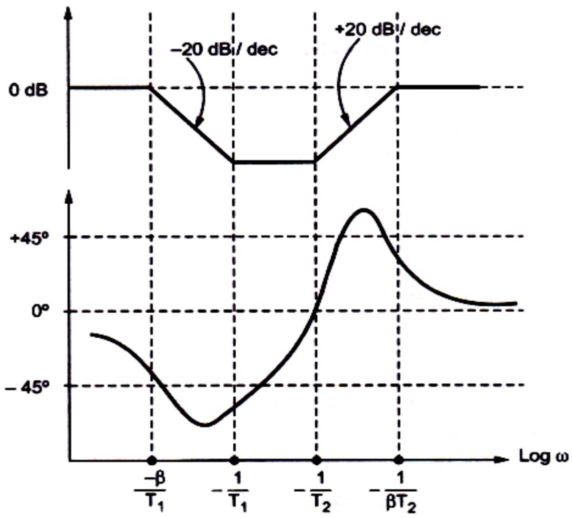
$\beta=(R_2+R_1)/R_2$

$\alpha = R_2 / (R_2 + R_1)$

The pole zero plot is



8.4.1 BODE PLOT LAG-LEAD COMPENSATOR



Note:

1) Lag-lead network introduces both steady state and transient response improvement of the system.
2) For lead lag compensator the position of compensating poles and zeros of lag lead compensator can be interchanged.

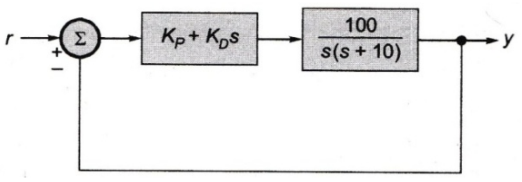
GATE QUESTIONS(EC)

Q.1 A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has
a) a higher type number
b) reduced damping
c) higher noise amplification
d) larger transient overshoot
[GATE-2003]

Q.2 A double integrator plant, $G(s) = \frac{K}{s^2}$, H(s) = 1 is to be compensated to achieve the damping ratio $\zeta = 0.5$, and an undamped natural frequency, $\omega_n = 5 rad/s$. . Which one of the following compensator $G_e(s)$ will be suitable?
a) $\frac{s+3}{s+9.9}$ b) $\frac{s+9.9}{s+3}$
c) $\frac{s-6}{s+8.33}$ d) $\frac{s+6}{s}$
[GATE-2005]

Q.3 The transfer function of a phase-lead compensator is given by
$G_e(s) = \frac{1+3Ts}{1+Ts}$ where T > 0
The maximum phase-shift provided by such a compensator is
a) $\pi/2$ b) $\pi/3$
c) $\pi/4$ d) $\pi/6$
[GATE-2006]

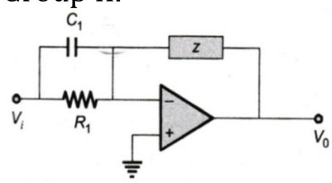
Q.4 A control system with a PD controller is shown in the figure. If the velocity error constant $K_V = 1000$ & the damping ratio $\zeta = 0.5$, then the values of K_P and K_D are



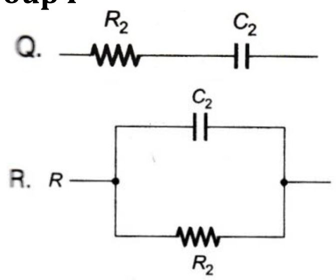
a) $K_P = 100, K_D = 0.09$
b) $K_P = 100, K_D = 0.9$
c) $K_P = 10, K_D = 0.09$
d) $K_P = 10, K_D = 0.9$
[GATE-2007]

Q.5 The open-loop transfer function of a plant is given as $G(s) = \frac{1}{s^2 - 1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is
a) $\frac{10(s-1)}{s+2}$ b) $\frac{10(s+4)}{s+2}$
c) $\frac{10(s+2)}{s+10}$ d) $\frac{10(s+4)}{(s+1)}$
[GATE-2007]

Q.6 Group I gives two possible choices for the impedance Z in the diagram. The circuit elements in Z satisfy the condition $R_2C_2 > R_1C_1$. The transfer function $\frac{V_0}{V_i}$ represents a king of controller. Match the impedances in Group I with the types of controllers in Group II.



Group I



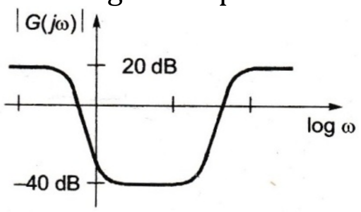
Group II

1. PID controller
2. Lead compensator
3. Lag compensator

a) Q-1, R-2 b) Q-1, R-3
c) Q-2, R-3 d) Q-3, R-2

[GATE-2008]

Q.7 The magnitude plot of a rational transfer function G(s) with real coefficients is shown below. Which of the following compensators has such a magnitude plot?



a) Lead compensator
b) Lag compensator
c) PID compensator
d) Lead-lag compensator

[GATE-2009]

Q.8 A unity negative feedback closed loop system has a plant with the transfer function $G(s)=\frac{1}{s^2+2s+2}$ and a controller $G_c(s)$ in the feed forward path. For a unit step input, the transfer function of the controller that gives minimum steady state error is

a) $G_c(s)=\frac{s+1}{s+2}$

b) $G_c(s)=\frac{s+2}{s+1}$

c) $G_c(s)=\frac{(s+1)(s+4)}{(s+2)(s+3)}$

d) $G_c(s)=1+\frac{2}{s}+3s$

[GATE-2010]

Statement of linked answer Q.9 and Q.10
The transfer function of a compensator is given as $G_c(s)=\frac{s+a}{s+b}$

Q.9 $G_c(s)$ is a lead compensator if

a) a=1,b=2 b) a=3,b=2
c) a = -3,b = -1 d) a = 3,b = 1

[GATE-2012]

Q.10 The phase of the above lead compensator is maximum at

a) $\sqrt{2}\,rad/s$ b) $\sqrt{3}\,rad/s$
c) $\sqrt{6}\,rad/s$ d) $1/\sqrt{3}\,rad/s$

[GATE-2012]

Q.11 A lead compensator network includes a parallel combination of R and C in the feed-forward path. If the transfer function of the compensator is

$G_c(s)=\frac{s+2}{s+4}$, the value of RC is ____

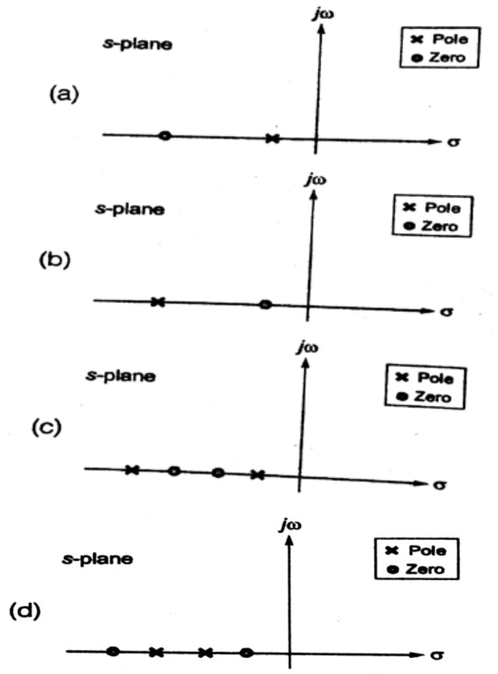
[GATE-2015]

Q.12 The transfer function of a first order controller is given as $G_c(s)=\frac{K(s+a)}{s+b}$ where, K, a and b are positive real numbers. The condition for this controller to act as a phase lead compensator is

a) a < b b) a > b
c) K < ab d) K > ab

[GATE-2015]

Q.13 Which of the following can be the pole-zero configuration of a phase-lag controller (lag compensator)?



[GATE-2017-01]

Q.14 Which of the following statements is incorrect?

a) Lead compensator is used to reduce the setting time.

b) Lag compensator is used to reduce the steady state error.

c) Lead compensator may increase the order of a system.

d) Lag compensator always stabilizes an unstable system.

[GATE-2017-02]

ANSWER KEY:

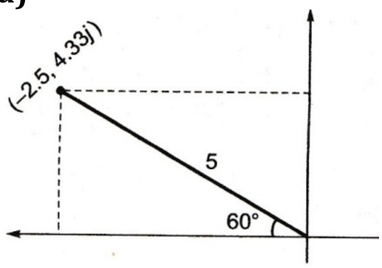
1	2	3	4	5	6	7	8	9	10	11	12	13	14
(c)	(a)	(d)	(b)	(c)	(b)	(d)	(d)	(a)	(a)	(0.5)	(a)	(a)	(d)

EXPLANATIONS

Q.1 (c)

B.W. increases and SNR decreases. System becomes more prone to noise.

Q.2 (a)



$\xi = 0.5$

$\cos^{-1} 0.5 = 60°$

$\therefore\ \theta = 60°$

$\angle G(s) = \left.\frac{K}{s^2}\right|_{s=-2.5+4.33j}$ v

$= -2\tan^{-1}\frac{4.33}{-2.5}; 120°$

$\therefore$ For compensated system $\angle = 180 - 120; 60°$

(b) & (d) are lag network & for compensating lag network K/s^2, a lead network is required

$\therefore$ Putting $s = -2.5 + 4.33j$ in (a) gives

$$\frac{K(s+3)}{s^2(s+9.9)} = \frac{0.5 + j4.33}{7.4 + j4.33} = 53°; 60°$$

$\therefore$ (a) is the correct answer.

Q.3 (d)

Max phase shift

$\phi_m = \angle G_e(s)$

$\phi = \tan^{-1} 3\omega T - \tan^{-1}\omega T$

For maximum phase shift

$\frac{d\phi}{dt} = 0$

$$\Rightarrow \frac{3T}{1+(3T\omega)^2} = \frac{T}{1+(T\omega)^2}$$

$3[1+(T\omega)^2] = 1 + (3T\omega)^2$

$3 + 3T^2\omega^2 = 1 + 9T^2\omega^2$

$2 = 6(\omega T)^2$

$(\omega T)^2 = \frac{1}{3}$

$\omega T = \frac{1}{\sqrt{3}}$

$\phi_{max} = \tan^{-1} 3\times\frac{1}{\sqrt{3}} - \tan^{-1}\frac{1}{\sqrt{3}}$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

Q.4 (b)

$K_v = \lim_{s\to 0} sG(s)H(s)$

$1000 = \lim_{s\to 0} s \times \frac{(K_P + K_D s)100}{s(s+10)}$

$\Rightarrow K_P = 100$

Now characteristics eq.

$(1 + G(s)H(s)) = 0$

$1 + \frac{(K_P + K_D s)100}{s(s+10)} = 0$

Putting $K_P = 100$

$s^2 + 10s + 10^4 + 100K_D s = 0$

$s^2 + (10 + 100K_D)s + 10^4 = 0$

Comparing with standard second order eq.

i.e. $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

So $\omega_n = 100; 2\xi\omega_n = 10 + 100K_D$

Given $\xi = 0.5; 2\times 0.5\times 100$

$= 10 + 100K_D$

$K_D = 0.9$

Q.5 (c)

Lead compensator is required for stability.

Q.6 **(b)**

$$\frac{V_i(R_1C_1s+1)1}{R_1} = -\frac{V_0}{Z}$$

$$\Rightarrow \frac{V_0}{V_i} = -\frac{Z(R_1C_1s+1)}{R_1}$$

In case of Q, $Z = \frac{R_2C_2s+1}{C_2s+1}$

In case of R, $Z = \frac{R_2}{R_2C_2s+1}$

Considering Q,

$$\frac{V_0}{V_i} = -\frac{(R_1C_1s+1)}{R_1} \cdot \frac{(R_2C_2s+1)}{C_2s}$$

Considering R,

$$\frac{V_0}{V_i} = -\frac{(R_1C_1s+1)}{R_1} \cdot \frac{R_2}{(R_2C_2s+1)}$$

Q Given that $R_2C_2 > R_1C_1$

$\therefore$ Considering R, controller is log compensator.

and considering Q, Controller is PID controller

Q.7 **(d)**

Q.8 **(d)**

Steady state error,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)G_c(s)}$$

$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s}}{1+G(s)G_c(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+G(s)G_c(s)}$$

Taking, $G_c(s) = \frac{s+1}{s+2}, e_{ss} = \frac{2}{3}$

Taking, $G_c(s) = \frac{s+2}{s+1}, e_{ss} = \frac{1}{3}$

Taking, $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}, e_{ss} = \frac{3}{5}$

Taking, $G_c(s) = 1 + \frac{2}{s} + 3s, e_{ss} = 0$

Q.9 **(a)**

$$G_c(s) = \frac{s+a}{s+b}$$

$$G_c(j\omega) = \frac{j\omega+a}{j\omega+b}$$

$$\angle G_c(j\omega) = \tan^{-1}\frac{\omega}{a} - \tan^{-1}\frac{\omega}{b}$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{a} - \frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right)$$

For $G_c(s)$ to be a lead compensator

$$\angle G_c(j\omega) > 0$$

$$\frac{\omega}{a} > \frac{\omega}{b}$$

$\Rightarrow \quad b > a$

Option (a) satisfies the above equation.

Q.10 **(a)**

For phase to be maximum

$$\frac{d\phi}{dt} = 0$$

$$\Rightarrow \left(1+\frac{\omega^2}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right) - \left(\frac{\omega}{a} - \frac{\omega}{b}\right)\left(\frac{2\omega}{ab}\right) = 0$$

$$\left(1+\frac{\omega^2}{2}\right)\left(\frac{1}{1} - \frac{1}{2}\right) = \left(\frac{\omega}{1} - \frac{\omega}{2}\right)\left(\frac{2\omega}{2}\right)$$

$$\frac{1}{2} + \frac{\omega^2}{4} = \frac{\omega^2}{2}$$

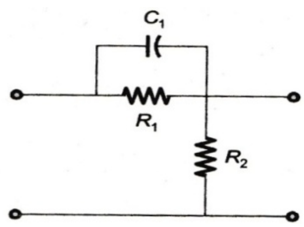
$$\Rightarrow \quad \frac{\omega^2}{4} = \frac{1}{2}$$

$$\Rightarrow \quad \omega = \sqrt{2}\,\text{rad/sec.}$$

Q.11 **(0.5)**

$$G_C(s) = \frac{s+2}{s+4} \qquad \ldots (i)$$

For lead compensator



Transfer function $=\dfrac{1+s\tau}{1+\alpha s\tau}$ ... (ii)

Where,

τ = Lead time constant = R_1C

and $\alpha=\dfrac{R_2}{R_1+R_2}$

Comparing equation (i) and (ii), we get

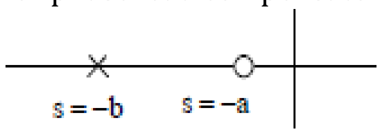
$\tau=\dfrac{1}{2}$ and $\alpha\tau=\dfrac{1}{4}$

or $\alpha=\dfrac{1}{2}$

$\therefore$ RC time constant = 0.5

Q.12 (a)

For phase lead compensator



a < b

GATE QUESTIONS(EE)

Q.1 A lead compensator used for a closed loop controller has the following transfer function $\frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$

For such a lead compensator

a) $a < b$ b) $b < a$

c) $a > Kb$ d) $a < Kb$

[GATE-2003]

Q.2 The system $900/s(s+1)(s+9)$ is to be such that its gain-crossover frequency becomes same as its uncompensated phase crossover frequency and provides a 45° phase margin. To achieve this, one may use

a) a lag compensator that provides an attenuation of 20dBand a phase lag of 45° at the frequency of $3\sqrt{3}$ rad/s

b) a lead compensator that provides an amplification of 20dBand a phase lead of 45° at the frequency of 3 rad/s

c) a lag-lead compensator that provides an amplification of 20dB and a phase lag of 45° at the frequency of $\sqrt{3}$ rad/s

d) a lag-lead compensator that provides an attenuation of 20dB and a phase lead of 45° at the frequency of 3 rad/s

[GATE-2007]

Q.3 The transfer function of two compensators are given below:

$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$ Which one of the following statements is correct?

a) C_1 is lead compensator and C_2 is a lag compensator

b) C_1 is lag compensator and C_2 is a lead compensator

c) Both C_1 and C_2 are lead compensator

d) Both C_1 and C_2 are lag compensator

[GATE-2008]

Q.4 The second order dynamic system $\frac{dX}{dt} = PX + Qu$, $Y = RX$ has the matrices P, Q and R as follows:

$P = \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix}$ $Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $R = [01]$ The system has the following controllability and observability properties:

a) Controllable and observable

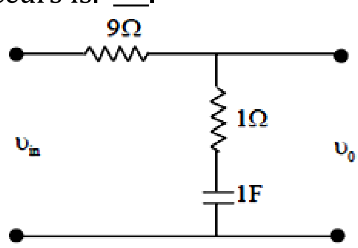
b) Not controllable but observable

c) Controllable but not observable

d) Not controllable and not observable

[GATE-2014]

Q.5 For the network shown in the figure below, the frequency (in rad/s) at which the maximum phase lag occurs is. __.



[GATE-2016]

ANSWER KEY:

1	2	3	4	5
(a)	(d)	(a)	(c)	0.316

EXPLANATIONS

Q.1 (a)

Transfer function $\frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$

Zero of TF $= -a$
Pole of TF $= -b$
For a lead-compensator, the zero is nearer to origin as compared to pole, hence the effect of zero is dominant, therefore, the lead-compensator when introduced in series with forward path of the transfer function the phase shift is increased.
So, from pole-zero configuration of the compensator $a < b$

Q.2 (d)
Let uncompensated system

$$T(s) = \frac{900}{s(s+1)(s+9)}$$

Phase crossover frequency of uncompensated system $= (\omega_{pc})$, at this frequency phase of $T(j\omega)$ is -180°

Put $s = j\omega$ in $T(s)$

$$T(j\omega) = \frac{900}{j\omega(j\omega+1)(j\omega+9)}$$

$$\angle T(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{9}\right)$$

at $(\omega_{pc})_1, \angle T(j\omega) = 180^\circ$

$$-180^\circ = -90^\circ - \tan^{-1}(\omega_{pc})_1$$

$$-\tan^{-1}\left(\frac{(\omega_{pc})_1}{9}\right) - 90^\circ$$

$$= \tan^{-1}\frac{(\omega_{pc})_1 + \frac{(\omega_{pc})_1}{9}}{1 - \frac{(\omega_{pc})_1^2}{9}}$$

$$\Rightarrow \quad 1 - \frac{(\omega_{pc})_1^2}{9} = 0$$

$$\Rightarrow \quad (\omega_{pc})_1 = 3\text{rad/sec}$$

Gain cross frequency of compensated system, $(\omega_{pc})_2$

Phase cross frequency of uncompensated system, $(\omega_{pc})_1$

$$\Rightarrow (\omega_{gc})_2 = (\omega_{gc})_1 = 3\text{rad/sec}$$

Phase-margin

$$= 180^\circ + \angle T(j\omega)\big|_{\omega=(\omega_{gc})_2}$$

$$\Rightarrow 45^\circ = 180^\circ + \angle T(j\omega)\big|_{\omega=(\omega_{gc})_2}$$

At $(\omega_{gc})_2 = \frac{3\text{rad}}{\text{sec}}$,

phase angle of $\angle T(j\omega)$ is -135° and phase of uncompensated system is -180° at 3rad/sec Therefore, the compensator provides phase lead of 45° at the frequency of 3 rad/sec.
Let XdB is the gain provided by the compensator, so at gain cross frequency.

$$|T(j\omega)|_{com} = 1 \text{ or } 0 \text{ dB.}$$

Gain of uncompensated system

$$|T(j\omega)|_{un-com} = \frac{100}{\omega\sqrt{1+\omega^2}\sqrt{1+\left(\frac{\omega}{9}\right)^2}}$$

$|T(j\omega)|_{un-com}$ in dB

$$= 40 - 20\log\omega - 20\log\sqrt{1+\omega^2}$$

$$-20\log\sqrt{1+\left(\frac{\omega}{9}\right)^2}$$

Gain of compensated system

$$|T(j\omega)|_{com} = X + |T(j\omega)_{un-com}|$$

$|T(j\omega)|_{com}$ must be zero at gain cross frequency $(\omega_{gc})_2$

$$\left|T\left(j\omega_{gc}\right)_2\right|_{com} = X + 40 - 20\log\left(\omega_{gc}\right)_2$$

$$-20\log\sqrt{1+\left(\omega_{gc}\right)_2^2}$$

$$-20\log\sqrt{1+\frac{\left(\omega_{gc}\right)^2}{9^2}} = 0$$

$$X + 40 - 20\log 3 - 20\log\sqrt{1+3^2}$$

$$-20\log\sqrt{1+\left(\frac{3}{9}\right)^2} = 0$$

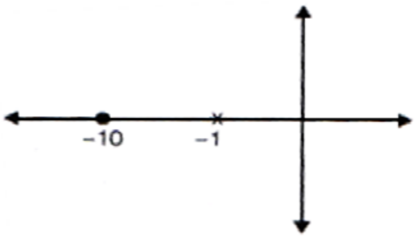
$$X = -20\text{dB}$$

So, the compensator provides an attenuation of 20 dB.
Hence option (d) is correct.

Q.3 **(a)**

$$C_1 = \frac{10(s+1)}{(s+10)}$$

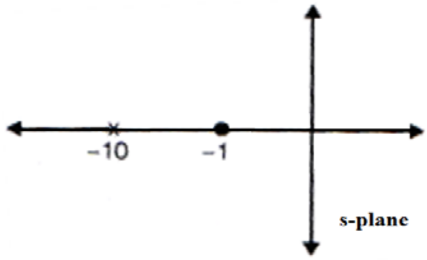
zero at s = -1
pole at s = -10



As zero is closer origin, zero dominates pole.
Hence C_1 is lead compensator.

$$C_2 = \frac{s+10}{10(s+1)}$$

zero at s=-10
pole at s=-1



As pole is closer origin, pole dominates zero.
Hence C_2 is lag compensator.
For phase to be maximum

Q.4 **(c)**

Given $P = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

For controllability:

$$Q_c = [Q\ PQ] \Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$|Q_c| \neq 0 \quad \therefore$ controllable

For observability

$$Q_0 = \begin{bmatrix} R^T & P^T & R^T \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & -3 \end{bmatrix}$$

$$|Q_0| = 0$$

$\therefore$ Not observable

Q.5 **(0.316)**

The given circuit is standard lag compensator
Whose Transfer function

$$G(s)\frac{1+\frac{1}{s}}{a+1+\frac{1}{s}} = \frac{s+1}{1+10s} = \frac{1+\tau s}{1+\alpha\tau s}$$

So $\tau = 1, \alpha = 10$ the frequency at which maximum phase lag happen

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}} = \frac{1}{\sqrt{10}} = 0.316 \text{ rad/sec}$$

GATE QUESTIONS(IN)

Q.1 The transfer function of a position servo system is given as $G(s)=\frac{1}{s(s+1)}$. A first order compensator is designed in a unity feedback configuration so that the poles of the compensated system are placed at $-1\pm j1$ and -4. The transfer function of the compensated system is

a) $\frac{s+3}{2(s+5)}$ b) $\frac{2s+3}{s+5}$

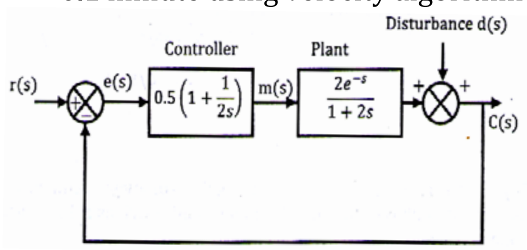
c) $\frac{5(s+1.6)}{s+5}$ d) $\frac{3(2s+3)}{s+4}$

[GATE-2006]

Common Data Questions Q.2, Q.3 and Q.4:

The following figure describes the block diagram of a closed loop process control system. The unit of time is given in minute

Q.2 The digital implementation of the controller with a sampling time of 0.1 minute using velocity algorithm is



a) $m(k)=0.5\left[e(k)+0.5\sum_{i=1}^{k}e(k)\right]$

b) $m(k)=2.0\left[e(k)+2.0\sum_{i=1}^{k-1}e(k-1)\right]$

c) $m(k)-m(k-1)=0.5\left[e(k)-0.85e(k-1)\right]$

d) $m(k)-m(k-1)=0.5\left[1.05e(k)-e(k-1)\right]$

[GATE-2006]

Q.3 Suppose a disturbance signal $d(t)=\sin 0.2t$ units is applied .Then at steady state, the amplitude of the output e(t) due to the effect of disturbance alone is

a) 0.129 unit b) 0.40 unit

c) 0.529 unit d) 2.102 unit

[GATE-2006]

Q.4 The control action recommended for reducing the effect of disturbance at the output (provided that the disturbance signal is measurable) is

a) cascade control

b) P-D control

c) ratio control

d)feedback –feed forward control

[GATE-2006]

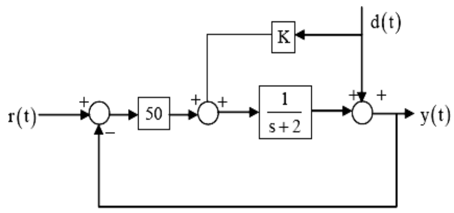
Q.5 A plant with a transfer function $\frac{2}{s(s+3)}$ is controlled by a PI controller with K_p=1 and $K_i\geq 0$ in a unity feedback configuration .The lowest value of K_i that ensures zero steady state error for a step change in the reference input is

a) 0 b) 1/3

c) 1/2 d) 1

[GATE-2009]

Q.6 Consider the control system shown in figure with feed forward action for rejection of a measurable disturbance d(t). The value of K for which the disturbance response at the output y(t) is zero mean, is



a) 1 b) -1

c) 2 d) -2

[GATE-2014]

ANSWER KEY:

1	2	3	4	5	6
(c)	(a)	(b)	(b)	(a)	(d)

EXPLANATIONS

Q.1 (c)

Let the T.F of the compensator be $G_c(s)$

CLTF of the compensated system

$$= \frac{G(s)G_c(s)}{1+G(s)G_c(s)} = \frac{\frac{G_c(s)}{s(s+1)}}{1+\frac{G_c(s)}{s(s+1)}} = \frac{G_c(s)}{s(s+1)G_c(s)}$$

Poles of the compensated system re given as $s=-1\pm j1, s=-4$

Let $G_c(s) = \frac{K_c(s+b)}{(s+a)}$

Characteristic equation

$$s(s+1)(s+a)+K_c(s+b)$$

$$=\left[(s+1)^2+1^2\right](s+4)s^3+(1+a)s^2$$

$$+(a+K_c)s+K_c b=(s^2+2s+2)(s+4)$$

$$=s^3+6s^2+10s+81+a$$

$$=6,(a+K_c)=10, K_c b=8$$

$$a=5, K_c=5, b=\frac{8}{5}=1.6$$

$$G_c(s)=\frac{5(s+1.6)}{(s+5)}$$

Q.2 (a)

Given

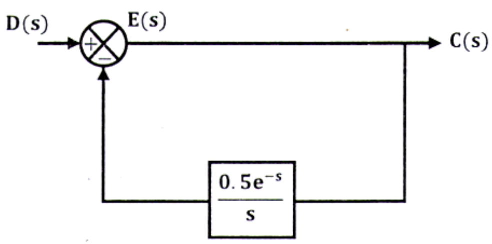
$$m(s)=e(s)\left\{0.5\left(1+\frac{1}{2s}\right)\right\}$$

$$=0.5\left\{e(s)+\frac{0.5}{s}e(s)\right\}$$

As, Laplace transform of $\int i(t)\,dt = \frac{I(s)}{s}$

Q.3 (b)

The given diagram can be reduced as



Transfer function

$$H(s)=\frac{C(s)}{D(s)}=\frac{E(s)}{D(s)}=\frac{1}{1+\frac{0.5e^{-5}}{s}}$$

$$\frac{E(s)}{D(s)}=\frac{s}{0.5+0.5s}=\frac{2s}{(s+1)}$$

$$E(s)=\frac{2s}{(s+1)}D(s)$$

$$d(t)=\sin 0.2t \Rightarrow \omega=0.2$$

For getting multiplying factor A

$$A=\left|\frac{2s}{s+1}\right|_{\omega=0.2}$$

$$A = \left|\frac{2j\omega}{j\omega+1}\right| = \frac{2\times 0.2}{\sqrt{1+(0.2)^2}} = 0.3922$$

A=0.40

So, amplitude of output e(t)

$= A \times \text{initial amplitude}$

$= 0.4 \times 1 = 0.4$

Q.4 **(b)**

(PD controller can be used)

Q.5 **(a)**

$$\frac{E(s)}{R(s)} = \frac{1}{1+\left(1+\frac{ki}{s}\right)\left(\frac{2}{s(s+3)}\right)}$$

$$= \frac{S^2(s+3)}{S^2(s+3)+2(s+ki)}$$

$$e_{ss} = \lim_{S\to 0} s.E(s) \lim_{S\to 0} s.\frac{1}{s}.\frac{s^2(s+3)}{s^2(s+3)+2(s+ki)}$$

$= 0 \forall ki \geq 0$

Q.6 **(d)**

$$Y(s)\left[1+\frac{50}{(s+2)}\right] = D(s)\left[\frac{s+2+K}{s+2}\right] + \frac{50}{(s+2)}R(s)$$

$$Y(s)D(s) + \frac{1}{(s+2)}\left[K.d(s)+50(R(s)-Y(S))\right]$$

$$Y(s)D(s)\left[\frac{s+2+K}{s+2}\right] + \frac{50}{(s+52)}R(s)$$

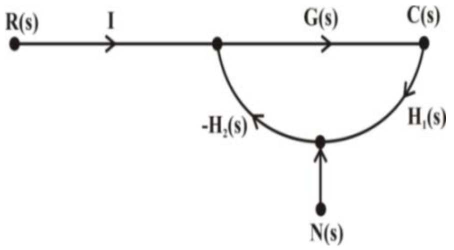
i.e., s+2+K=0

$\Rightarrow K+2=0$

$\Rightarrow K=-2$

ASSIGNMENT QUESTIONS

Q.1 A closed-loop system is shown in the given figure. The noise transfer function $\frac{C_n(s)}{N(s)}[C_n(s) =$ output corresponding to noise input N(s) is approximately



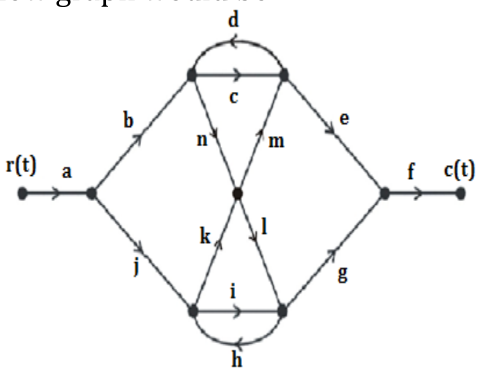
a) $\frac{1}{G(s)H_1(s)}$ for $|G_1(s)H_1(s)H_2(s)|<<1$

b) $-\frac{1}{H_1(s)}$ for $|G_1(s)H_1(s)H_2(s)|>>1$

c) $-\frac{1}{H_1(s)H_2(s)}$ for $|G_1(s)H_1(s)H_2(s)|>>1$

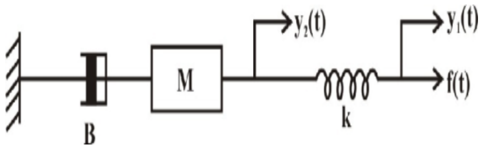
d) $\frac{1}{G(s)H_1(s)H_2(s)}$ for $|G_1(s)H_1(s)H_2(s)|<<1$

Q.2 A signal flow graph is shown in the given figure. The number of forward paths M and the number of individual loops P for this signal flow graph would be



a) M =4 and P = 4 b) M=6 and P=4
c) M=4 and P=6 d) M=6 and P=6

Q.3 The mechanical system is shown in the given figure



The system is described as

a) $M\frac{d^2y_1(t)}{dt^2}+B\frac{d\,y_1(t)}{dt}=k\left[y_2(t)-y_1(t)\right]$

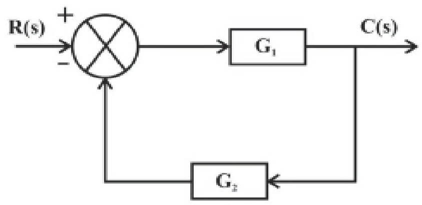
b) $M\frac{d^2y_2(t)}{dt^2}+B\frac{d\,y_2(t)}{dt}=k\left[y_2(t)-y_1(t)\right]$

c) $M\frac{d^2y_1(t)}{dt^2}+B\frac{dy_1(t)}{dt}=k\left[y_1(t)-y_2(t)\right]$

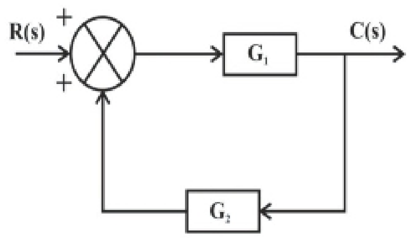
d) $M\frac{d^2y_2(t)}{dt^2}+B\frac{dy_2(t)}{dt}=k\left[y_1(t)-y_2(t)\right]$

Q.4 Consider the following block diagrams:

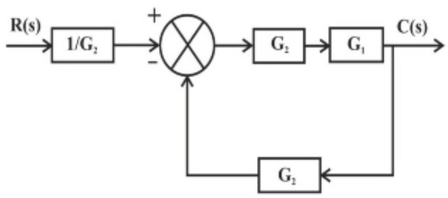
a)



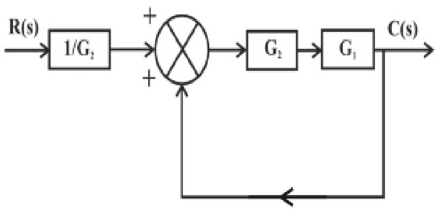
b)



c)



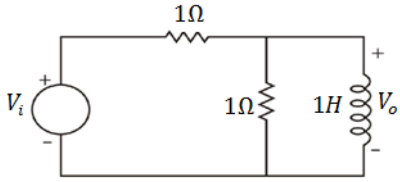
d)



Which of these block diagrams can be reduced to transfer function $\frac{C(s)}{R(s)} = \frac{G_1}{1 - G_1G_2}$?

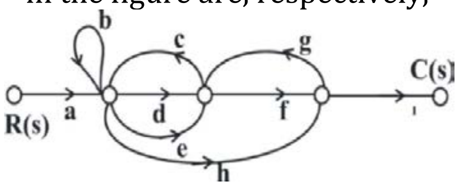
a) 1 and 3 b) 2 and 4
c) 1 and 4 d) 2 and 3

Q.5 Select the correct transfer function $v_0(s)/v_i(s)$ from the following, for the given network.



a) $\frac{2}{s(s+1)}$ b) $\frac{s}{(s+2)}$
c) $\frac{s}{(2s+1)}$ d) $\frac{2s}{(s+1)}$

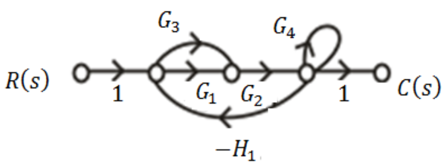
Q.6 The number of forward paths and the number of non-touching loop pairs for the signal flow graph given in the figure are, respectively,



a) 1, 3 b) 3, 2
c) 3, 1 d) 2, 4

Q.7 Which one of the following effects in the system is NOT caused by negative feedback?
a) Reduction in gain
b) Increase in bandwidth
c) Increase in distortion
d) Reduction in output impedance

Q.8 The gain $\frac{C(s)}{R(s)}$ of the signal flow graph shown is



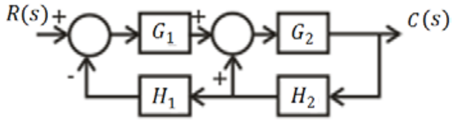
a) $\frac{G_1G_2 + G_2G_3}{1 + G_1G_2H_1 + G_2G_3H_1 + G_4}$

b) $\frac{G_1G_2 + G_2G_3}{1 + G_1G_3H_1 + G_2G_3H_1 - G_4}$

c) $\frac{G_1G_3 + G_2G_3}{1 + G_1G_3H_1 + G_2G_3H_1 + G_4}$

d) $\frac{G_1G_3 + G_2G_3}{1 + G_1G_3H_1 + G_2G_3H_1 - G_4}$

Q.9 The overall gain $\frac{C(s)}{R(s)}$ of the block diagram shown is



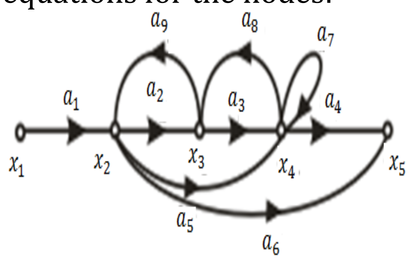
a) $\frac{G_1G_2}{1 - G_1G_2H_1H_2}$

b) $\frac{G_1G_2}{1 - G_2H_2 - G_1G_2H_1}$

c) $\frac{G_1G_2}{1 - G_2H_2 + G_1G_2H_1H_2}$

d) $\frac{G_1G_2}{1 - G_1G_2H_1 - G_1G_2H_2}$

Q.10 The signal flow graph for a certain feedback control system is shown:
Now consider the following set of equations for the nodes:

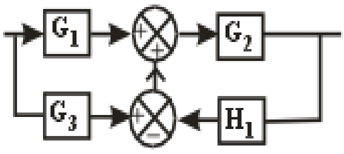


1. $x_2 = a_1x_1 + a_9x_3$
2. $x_3 = a_2x_2 + a_8x_4$
3. $x_4 = a_3x_3 + a_5x_2$
4. $x_5 = a_4x_4 + a_6x_2$

Which of the above equations are correct?

a) 1, 2 and 3 b) 1, 3 and 4
c) 2, 3 and 4 d) 1, 2 and 4

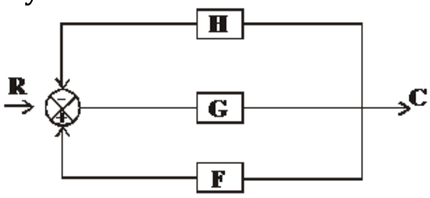
Q.11 Which is the overall transfer function of the block diagram given?



a) $\dfrac{G_1G_2+G_2G_3}{1+G_2H_1}$ b) $\dfrac{G_1G_3+G_2G_3}{1+G_3H_1}$

c) $G_1G_2+G_2G_3$ d) $\dfrac{G_1G_3+G_2G_3}{1+G_2G_3H_1}$

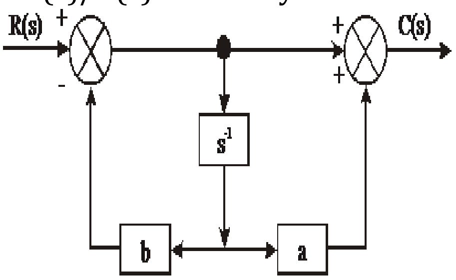
Q.12 For the feedback system shown in the figure above, which one of the following expresses the input-output relation C/R of the overall system?



a) $\dfrac{G}{1-FG+GH}$ b) $\dfrac{G}{1+FG-GH}$

c) $\dfrac{FG}{1+FGH}$ d) $\dfrac{GH}{1-FGH}$

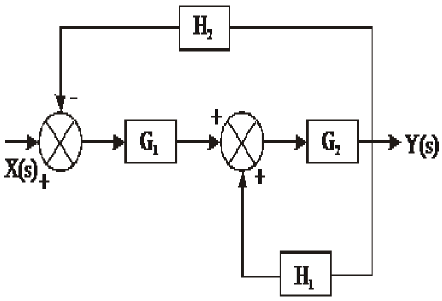
Q.13 The block diagram for a particular control system is shown in the figure. What is the transfer function C(s)/R(s) for this system?



a) $\dfrac{s+a}{s-b}$ b) $\dfrac{s+a}{s+b}$

c) $\dfrac{s-b}{s+a}$ d) $\dfrac{s+b}{s-a}$

Q.14 Which one of the following is the transfer function $\dfrac{Y(s)}{X(s)}$ for the block diagram given?



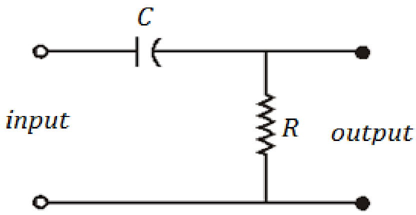
a) $\dfrac{G_1G_2}{1+H_2G_1G_2-H_1G_2}$

b) $\dfrac{G_1G_2}{1-H_2G_1G_2+H_1G_2}$

c) $\dfrac{H_1G_1G_2}{1-H_2G_1G_2+H_1G_2}$

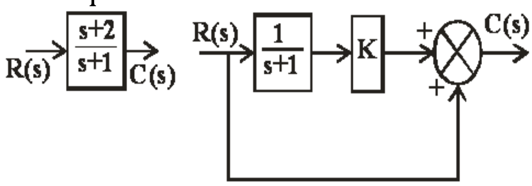
d) $\dfrac{H_1G_1G_2}{1+H_2G_1G_2-H_1G_2}$

Q.15 The transfer function for the diagram shown above is given by which one of the following?



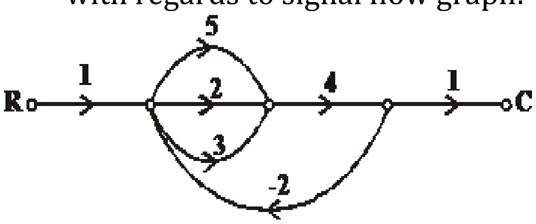
a) 1/(1 + sRC) b) sRC/(1 + sRC)
c) sRC/(1 – sRC) d) 1 + sRC

Q.16 For what value of K, are the two block diagrams as shown above equivalent?



a) 1 b) 2
c) (s + 1) d) (s + 2)

Q.17 Consider the following statements with regards to signal flow graph:

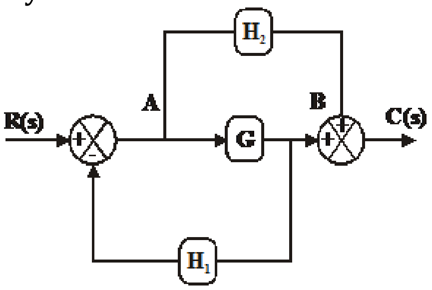


1. The number of loops are 3.
2. The number of loops are 2.
3. The number of forward paths are 3.
4. $\frac{C}{R}$ ratio is $\frac{40}{81}$
5. $\frac{C}{R}$ ratio is $\frac{28}{81}$

Which of these statements are correct?

a) 1, 3, 4 and 5 b) 1, 3 and 4
c) 2, 3 and 4 d) 3, 4 and 5

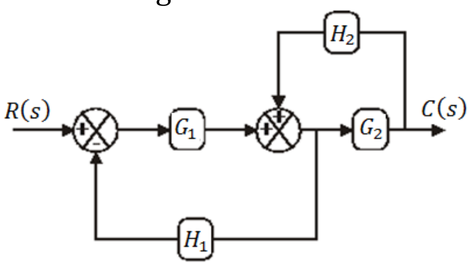
Q.18 The transfer function $\frac{C(s)}{R(s)}$ for the system shown above is



a) $\frac{G+H_1}{1+GH_2}$ b) $\frac{G+H_2}{1+GH_1}$

c) $\frac{H_2}{1+GH_2}$ d) $\frac{GH_2}{1+GH_1}$

Q.19 The system transfer function for the block diagram shown is



a) $\frac{G_1G_2}{1-G_2H_2+G_1H_1}$

b) $\frac{G_1G_2}{1-H_1G_1+G_2H_1}$

c) $\frac{G_1G_2H_1}{1+G_2H_1+G_1H_1}$

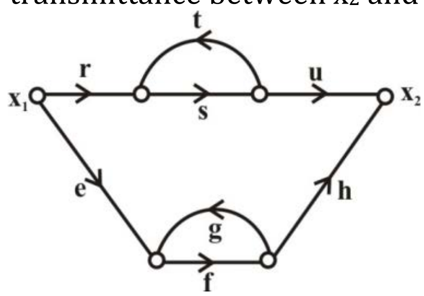
d) $\frac{G_1G_2H_1}{1+G_2H_2+G_1H_1}$

Q.20 Consider the following statements:
1. The effect of feedback is to reduce the system error.
2. Feedback increases the gain of the system in one frequency range but decreases in another.
3. Feedback can cause a system that is originally stable to become unstable.

Which of these statements are correct?

a) 1, 2 and 3 b) 1 and 2
c) 2 and 3 d) 1 and 3

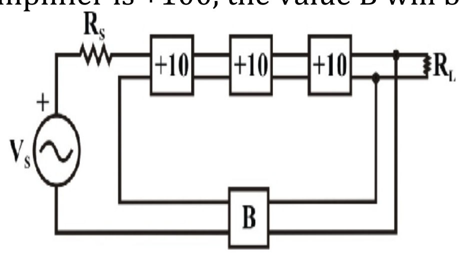
Q.21 For the signal flow diagram shown in the given figure, the transmittance between x_2 and x_1 is



a) $\frac{rsu}{1-st}+\frac{efh}{1-fg}$ b) $\frac{rsu}{1-fg}+\frac{efh}{1-st}$

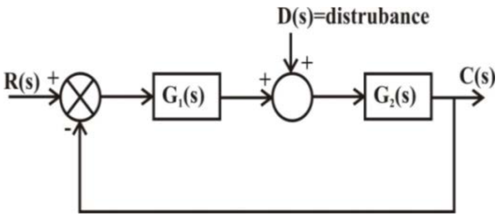
c) $\frac{efh}{1-ru}+\frac{rsu}{1-eh}$ d) $\frac{rst}{1-eh}+\frac{rsu}{1-st}$

Q.22 Consider the following amplifier with negative feedback:
If the closed-loop gain of the amplifier is +100, the value B will be



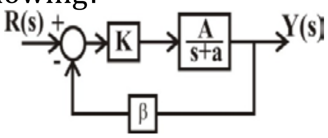
a) -9×10^{-3} b) $+9\times10^{-3}$
c) -11×10^{-3} d) $+11\times10^{-3}$

Q.23 For the given system, how can the steady state error produced by step disturbance be reduced?



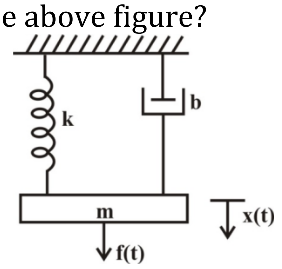
a) By increasing dc gain of $G_1(s)$ $G_2(s)$
b) By increasing dc gain G_2 (s)
c) By increasing dc gain of G_1 (s)
d) By removing the feedback

Q.24 For the system given below, the feedback does not reduce the closed-loop sensitivity due to variation of which one of the following?



a) K b) A
c) $K\alpha$ d) β

Q.25 Which one of the following represents the linear mathematical model of the physical system shown in the above figure?



a) $m\frac{d^2x(t)}{dt^2}+b\frac{dx(t)}{dt}+kx(t)=f(t)$

b) $m\frac{d^2x(t)}{dt^2}+b\frac{dx(t)}{dt}+kx(t)=0$

c) $m\frac{d^2x(t)}{dt^2}+b\frac{dx(t)}{dt}+kx(t)+f(t)=0$

d) $m\frac{d^2x(t)}{dt^2}+b\frac{dx(t)}{dt}-kx(t)-f(t)$

Q.26 A linear time invariant system, initially at rest when subjected to a unit step input gave to a response $c(t)=te^{-t}(t\geq 0)$. the transfer function of the system is

a) $\frac{s}{(s+1)^2}$ b) $\frac{1}{s(s+1)^2}$

c) $\frac{1}{(s+1)^2}$ d) $\frac{1}{s(s+1)}$

Q.27 In closed loop control system, what is the sensitivity of the gain of the overall system, M to the variation in G?

a) $\frac{1}{1+G(s)H(s)}$ b) $\frac{1}{1+G(s)}$

c) $\frac{G(s)}{1+G(s)H(s)}$ d) $\frac{G(s)}{1+G(s)}$

Q.28 Which one of the following is the transfer function of a linear system whose output is t^2e^{-t} for a unit step input?

a) $\frac{s}{(s+1)^3}$ b) $\frac{2s}{(s+1)^3}$

c) $\frac{1}{s^2(s+1)}$ d) $\frac{2}{s(s+1)^2}$

Q.29 Which of the following are the characteristics of closed-loop system?

1. It does not compensate for disturbances
2. It reduces the sensitivity of plant parameter variations
3. It does not involve output measurements
4. It has the ability to control the system transient response.

Select the correct answer using the codes given below:

a) 1 and 4 b) 2 and 4
c) 1 and 3 d) 2 and 3

Q.30 The unit step response of a particular control system is given by $c(t)=1-10e^{-t}$. Then its transfer function is

a) $\frac{10}{s+1}$ b) $\frac{s-9}{s+1}$

c) $\frac{1-9s}{s+1}$ d) $\frac{1-9s}{s(s+1)}$

Q.31 $[-a \pm jb]$ are the complex conjugate roots of the characteristics equation of a second order system. Its damping coefficient and natural frequency will be respectively

a) $\frac{b}{\sqrt{a^2+b^2}}$ and $\sqrt{a^2+b^2}$

b) $\frac{b}{\sqrt{a^2+b^2}}$ and a^2+b^2

c) $\frac{a}{\sqrt{a^2+b^2}}$ and $\sqrt{a^2+b^2}$

d) $\frac{a}{\sqrt{a^2+b^2}}$ and a^2+b^2

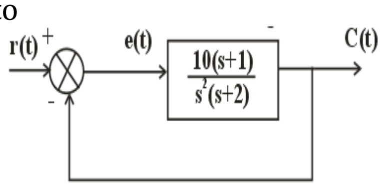
Q.32 A unity feedback control system has a forward path transfer function $G(s)=\frac{10(1+4s)}{s^2(1+s)}$

If the system is subjected to an input $r(t)=1+\ t\ +\frac{t^2}{2}(t \geq 0)$, the steady-state error of the system will be

a) zero b) 0.1

c) 10 d) infinity

Q.33 In the system shown in the given figure, r(t)=1+2t ($t \geq 0$). The steady-state value of the error e(t) is equal to



a)Zero b)2/10

c)10/2 d)infinity

Q.34 A second order control system is defined by the following differential equation:

$$4\frac{d^2c(t)}{dt^2}+8\frac{dc(t)}{dt}+16c(t)=16u(t)$$

The damping ratio and natural frequency for this system are respectively.

a) 0.25 and 2 rad/s

b) 0.50 and 2 rad/s

c) 0.25 and 4 rad/s

d) 0.50 and 4 rad/s

Q.35 The open loop transfer function of a unity feedback system is given by $\frac{K}{s(s+1)}$. if the value of gain K is such that the system is critically damped, the closed loop poles of the system will lie at

a) -0.5 and -0.5 b) $\pm$j0.5

c) 0 and -1 d) 0.5 $\pm$j 0.5

Q.36 Given the transfer function $G(s)=\frac{121}{s^2+13.2s+121}$ of a system. Which of the following characteristics does it have?

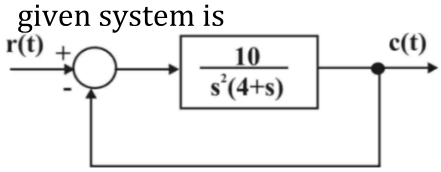
a) Overdamped and settling time 1.1

b) Underdamped and settling time 0.6s

c) Critically damped and settling time 0.8s

d) Underdamped and settling time 0.707s

Q.37 The steady state error resulting from an input r(t) = 2 + 3t + 4t² for given system is



a) 2.4 b) 4.0

c) Zero d) 3.2

Q.38 The unit impulse response of a second order system is $1/6e^{-0.8t}$ sin (0.6t). Then the natural frequency and damping ratio of the system are respectively?

a) 1 and 0.6 b) 1 and 0.8

c) 2 and 0.4 d) 2 and 0.3

Q.39 A second order control system has $M(j\omega)\ =\frac{100}{100-\omega^2+10\sqrt{2}j\omega}$.It M_p(peak magnitude) is

a) 0.5 b) 1
c) $\sqrt{2}$ d) 2

Q.40 The open-loop transfer function for unity feedback system is given by $\frac{5(1+0.1s)}{s(1+5s)(1+20s)}$

Consider the following statements:

1. The steady-state error for a step input of magnitude 10 is equal to zero.
2. The steady-state error for a ramp input of magnitude 10 is 2.
3. The steady-state error for an acceleration input of magnitude 10 is infinite.

Which of the statements given above are correct?

a) Only 1 and 2 b) Only 1 and
c) Only 2 and 3 d) 1, 2 and 3

Q.41 For a second order system, natural frequency of oscillation is 10 rad/s and damping ratio is 0.1. What is the 2% settling time?

a) 40s b) 10s
c) 0.4s d) 4s

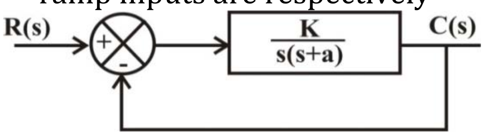
Q.42 For a unity feedback control system with forward path transfer function $G(s) = \frac{K}{s+5}$, what is error transfer function $W_e(s)$ used for determination of error coefficients?

a) $\frac{K}{s+5}$ b) $\frac{K}{s+K+5}$
c) $\frac{s+5}{s+K+5}$ d) $\frac{K(s+5)}{s+K+5}$

Q.43 The unit step response of a system is $[1 - e^{-t}(1 + t)]\, u(t)$. What is the nature of the system in turn of stability?

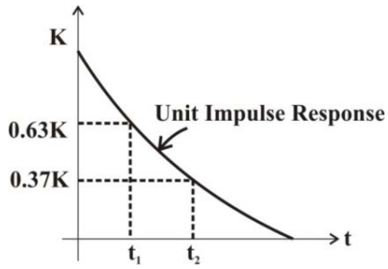
a) Unstable b) Stable
c) Critically stable d) Oscillatory

Q.44 Consider the unity feedback system as shown. The sensitivity of the steady state error to change in parameter K and parameter a with ramp inputs are respectively



a) 1, -1 b) -1, 1
c) 1, 0 d) 0, 1

Q.45 The unit impulse response of a system having transfer function $\frac{K}{s+\alpha}$ is shown. The value of α is



a) t_1 b) $\frac{1}{t_1}$
c) t_2 d) $\frac{1}{t_2}$

Q.46 Match List I (system G(s)) with List II (Nature of Response) and select the correct answer using the codes given below the lists:

List I (System G(s))

A. $\frac{400}{s^2+12s+400}$

B. $\frac{900}{s^2+90s+900}$

C. $\frac{225}{s^2+30s+225}$

D. $\frac{625}{s^2+0s+625}$

List II (Nature of Response)

1. Undamped
2. Critically damped
3. Underdamped
4. Overdamped

Codes:

	A	B	C	D
a)	3	1	2	4
b)	2	4	3	1
c)	3	4	2	1
d)	2	1	3	4

Q.47 Which one of the following expresses the time at which second peak in step response occurs for a second order system?

a) $\frac{\pi}{\omega_n\sqrt{1-\varsigma^2}}$ b) $\frac{2\pi}{\omega_n\sqrt{1-\varsigma^2}}$

c) $\frac{3\pi}{\omega_n\sqrt{1-\varsigma^2}}$ d) $\frac{\pi}{\sqrt{1-\varsigma^2}}$

Q.48 What is the value of k for a unity feedback system with G(s) = $\frac{k}{s(1+s)}$ to have a peak overshoot of 50%?

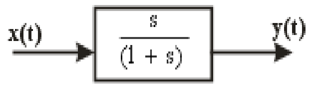
a) 0.53 b) 5.3
c) 0.6 d) 0.047

Q.49 The unit step response of a second order system is $1 - e^{-5t} - 5t\, e^{-5t}$
Consider the following statements:
1. The undamped natural frequency 5 rad/s.
2. The damping ratio is 1.
3. The impulse response is $25\, t\, e^{-5t}$

Which of the statements given above is/are correct?
a) Only 1 and 2 b) Only 2 and 3
c) Only 1 and 3 d) 1, 2 and 3

Q.50 The damping ratio and natural frequency of a second order system are 0.6 and 2 rad/s respectively.
Which one of the following combinations gives the correct values of peak and settling time, respectively for the unit step response of the system?
a) 3.33s and 1.95s
b) 1.95s and 3.33s
c) 1.95s and 1.5s
d) 1.5s and 1.95s

Q.51 Consider the following system shown in the diagram:
In the system shown in the below diagram x(t) = sin t.
What will be the response y(t) in the steady state?



a) $\sin(t-45°/\sqrt{2})$ b) $\sin(t+45^o)/\sqrt{2}$
c) $\sqrt{2}e^{-t}\sin t$ d) $\sin t - \cos t$

Q.52 A unity feedback control system has a forward loop transfer function as $\frac{e-Ts}{[s(s+1)]}$. Its phase value will be zero at frequency ω_1. Which one of the following equations should be satisfied by ω_1?
a) $\omega_1 = \cot(T\omega_1)$ b) $\omega_1 = \tan(T\omega_1)$
c) $T\omega_1 = \cot(\omega_1)$ d) $T\omega_1 = \tan(\omega_1$

Q.53 The open loop transfer function of a unity feedback control system is given by G(s) = $\frac{k}{s(s+1)}$. If gain k is increased to infinity, then damping ratio will tend to become
a) zero b) 0.707
c) Unity d) Infinite

Q.54 The impulse response of a second order under-damped system starting from rest is given by: $C(t) = 12.5\, e^{-6t} \sin 8t;\ t \geq 0$.
What are the values of natural frequency and damping factor of the system, respectively?
a) 10 units and 0.6
b) 10 units and 0.8
c) 8 units and 0.6
d) 8 units and 0.8

Q.55 Which one of the following is the most likely reason for large overshoot in a control system?
a) High gain in a system
b) Presence of dead time delay in a system
c) High positive correcting torque
d) High retarding torque

Q.56 In the time domain analysis of feedback control systems which one pair of the following is not correctly matched?

a) Under damped: Minimizes the effect of nonlinearities
b) Dominant: Transients die out more rapidly
c) Far away poles to the left half of s-plane: Transients die out more rapidly
d) A pole near to the left of dominant: Magnitude of transient is small complex poles and near a zero

Q.57 A second order system has a natural frequency of oscillations of 3 rad/sec and damping ratio of 0.5. What are the value of resonant frequency and resonant peak of the system?
a) 1.5 rad/sec and 1.16
b) 1.16 rad/sec and 1.5
c) 1.16 rad/sec and 2.1
d) 2.1 rad/sec and 1.16

Q.58 Consider the following:
1. Rise time
2. Setting time
3. Delay time
4. Peak time
What is the correct sequence of the time domain specifications of a second order system in the ascending order of the values?
a) 2 – 4 – 1 – 3 b) 3 – 4 – 1 – 2
c) 2 – 1 – 4 – 3 d) 3 – 1 – 4 – 2

Q.59 A unity feedback system with open loop transfer function of $\frac{20}{s(s+5)}$ is excited by a unity step input. How much time will be required for the response to settle within 2% of final desired value?
a) 0.25 sec b) 1.60 sec
c) 2.40 sec d) 4.00 sec

Q.60 Given a unity feedback system with $G(s) = \frac{K}{s(s+4)}$, the value of K for damping ratio of 0.5 is
a) 1 b) 16
c) 4 d) 2

Q.61 The open-loop transfer function G(s) of a unity feedback control system is $\frac{1}{s(s+1)}$. The system is subjected to an input r(t) sin t. The steady state error will be
a) zero b) 1
c) $\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)$ d) $\sqrt{2}\sin\left(t+\frac{\pi}{4}\right)$

Q.62 A third-order system is approximated to an equivalent second order system. The rise time of this approximated lower order system will be
a) Same as original system for any input.
b) Smaller than the original system for any input.
c) Larger than the original system for any input
d) Large or smaller depending on the input

Q.63 A system has a single pole at origin. Its impulse response will be
a) Constant
b) Ramp
c) Decaying exponential
d) Oscillatory

Q.64 Which one of the following statements is correct?
A second order system is critically damped when the roots of its characteristic equation are
a) Negative, real and unequal
b) Complex conjugates
c) Negative, real and equal
d) Positive, real and equal

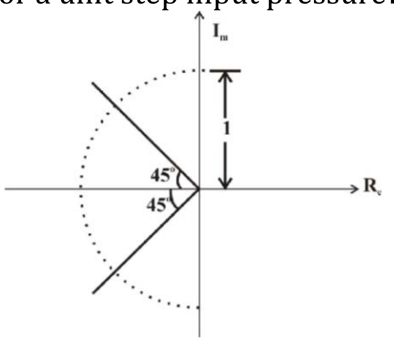
Q.65 An underdamped second order system with negative damping will have the two roots
a) On the negative real axis as real roots.

b) On the left hand side of complex plane as complex roots.
c) On the right hand side of complex plane as complex conjugates
d) On the positive real axis as real roots

Q.66 How can the steady-state error in a system be reduced?
a) By decreasing the type of system
b) By increasing system gain
c) By decreasing the static error constant
d) By increasing the input

Q.67 A control system whose step response is $-0.5(1 + e^{-2t})$ is cascaded to another control block whose impulse response is e^{-t}. What is the transfer function of the cascaded combination?
a) $\frac{1}{(s+1)(s+2)}$ b) $\frac{1}{s(s+1)}$
c) $\frac{1}{s(s+2)}$ d) $\frac{0.5}{(s+1)(s+2)}$

Q.68 A diaphragm type pressure sensor has two poles as shown in the figure above. The zeros are at infinity. What is its steady state deformation for a unit step input pressure?

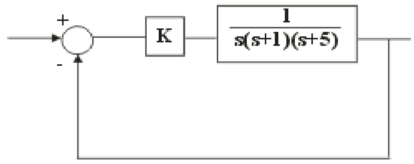


a) 0.25 b) 0.5
c) 0.707 d) 1

Q.69 What is the value of the damping ratio of a second order system when the value of the resonant peak is unity?
a) $\sqrt{2}$ b) Unity
c) $1/\sqrt{2}$ d) Zero

Q.70 Consider a second order all-pole transfer function model, if the desired settling time (5%) is 0.60 sec and the desired damping ratio 0.707, where should the poles be located in s-plane?
a) $-5 \pm j4\sqrt{2}$ b) $-5 \pm j5$
c) $-4 \pm j5\sqrt{2}$ d) $-4 \pm j7$

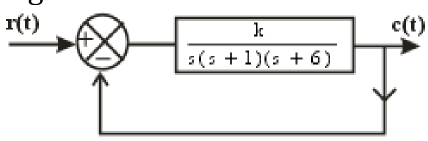
Q.71 The closed loop system shown above becomes marginally stable if the constant K is chosen to be



a) 10 b) 20
c) 30 d) 40

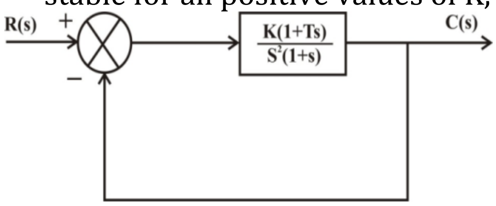
Q.72 The characteristic equation of a system is given by $3s^4 + 10s^3 + 5s^2 + 2 = 0$. This system is
a) Stable
b) Marginally stable
c) Unstable
d) Neither (a) (b) nor (c)

Q.73 For which of the following values of k, the feedback system shown in the figure is stable?



a) k > 0 b) k < 0
c) 0 < k < 42 d) 0 < k < 60

Q.74 A feedback control system is shown in the given figure. The system is stable for all positive values of K, If



a) T = 0 b) T <0
c) T > 1 d) 0<T<1

Q.75 Consider the following equation
$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$
How many roots does this equation have in the right half of s-plane?
a) One b) Two
c) Three d) Four

Q.76 If the poles of a system lie on the imaginary axis, the system will be:
a) Stable
b) Conditionally stable
c) Marginally stable
d) Unstable

Q.77 The characteristic equation of a feedback control system in given by:
$s^3 + 6s^2 + 9s + 4 = 0$
What is the number of roots in the left-half of the s-plane?
a) Three b) Two
c) One d) Zero

Q.78 The system having characteristic equation: $s^4 + 2s^3 + 3s^2 + 2s + k = 0$ is to be used as an oscillator. What are the values of k and the frequency of oscillation ω?
a) k = 1 and ω = 1 r/s
b) k = 1 and ω = 2 r/s
c) k = 2 and ω = 1r/s
d) k =2 and ω = 2 r/s

Q.79 What is the range of K for which the open loop transfer function G(s) = $\frac{K}{s^2(s+a)}$ represents an unstable closed loop system?
a) K > 0 only b) K = 0 only
c) K < 0 only d) $-\infty < K < \infty$

Q.80 The closed loop transfer function of a control system is $\frac{K}{s(s+1)(s+5)+K}$.
What is the frequency of the sustained oscillations for marginally stable condition?
a) $\sqrt{5}$ rad / s b) $\sqrt{6}$ rad / s
c) 5 rad / s d) 6 rad / s

Q.81 The characteristic equation of a control system is given as:
$s^4 + 8s^3 + 24s^2 + 32s + K = 0$
What is the value of K for which the system is unstable?
a) 10 b) 20
c) 60 d) 100

Q.82 The characteristics equation for a third-order system is:
$q(s) = a_0s^3 + a_1s^2 + a_2s + a_3 = 0$.
For the third-order system to be stable, besides that all the coefficients have to be positive, which one of the following has to be satisfied as a necessary and sufficient condition?
a) $a_0a_1 \ge a_2a_3$ b) $a_1a_2 \ge a_0a_3$
c) $a_2a_3 \ge a_1a_0$ d) $a_0a_3 \ge a_1a_2$

Q.83 Which one of the following is correct?
A unity feedback system with forward path transfer function G(s) = $\frac{K}{s(1+sT_1)(1+sT_2)}$ is stable provided the value of K is given by
a) $K < \frac{T_1+T_2}{T_1T_2}$ b) $K < \frac{T_1T_2}{T_1+T_2}$
c) $K > \frac{T_1+T_2}{T_1T_2}$ d) $K > \frac{T_1T_2}{T_1+T_2}$

Q.84 The open-loop transfer function of unity feedback control system is
$G(s) = \frac{K}{s(s+a)(s+b)}, 0 < a \le b$
The system is stable if
a) $0 < K < \frac{(a+b)}{ab}$ b) $0 < K < \frac{ab}{(a+b)}$
c) $0 < K < ab(a+b)$ d) $0 < K < \frac{a}{b}(a+b)$

Q.85 The characteristic equation of a system is given as $s^3 + 25s^2 + 10s + 50$

=0. What is the number of roots in the right half s-plane and on the jω axis, respectively?

a) 1, 1 b) 0, 0
c) 2, 1 d) 1, 2

Q.86 The given characteristic polynomial $s^4 + s^2 + 2s^2 + 2s + 3 = 0$ has

a) Zero root in RHS of s-plane
b) One root in RHS of s-plane
c) Two roots in RHS of s-plane
d) Three roots in RHS of s-plane

Q.87 Which one of the following characteristic equations can result in the stable operation of the feedback system?

a) $s^3 + 4s^2 + s - 6 = 0$
b) $s^3 - s^2 + 5s + 6 = 0$
c) $s^3 + 4s^2 + 10s + 11 = 0$
d) $s^4 + s^3 + 2s^2 + 4s + 6 = 0$

Q.88 The characteristic equation of a control system is given by $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. The number of the roots of the equation which lie on the imaginary axis of s-plane is

a) Zero b) 2
c) 4 d) 6

Q.89 Consider the unity feedback system with $G(s) = \dfrac{K}{(s^2+2s+2)(s+2)}$. The system is marginally stable. What is the radian frequency of oscillation?

a) $\sqrt{2}$ b) $\sqrt{3}$
c) $\sqrt{5}$ d) $\sqrt{6}$

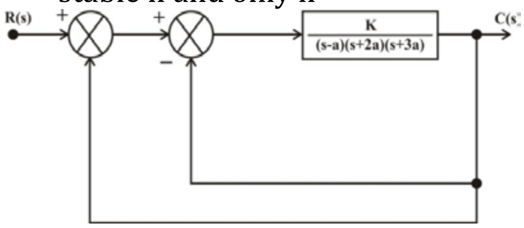
Q.90 The open – loop transfer function of a unity feedback control system is given by G(s) = K e^{-Ts} where K and T are variables and are greater than zero. The stability of the closed-loop system depends on

a) K only b) Both K and T
c) T only d) Neither K nor T

Q.91 Which of the following may result in instability problem?

a) Large error b) High selectivity
c) High gain d) Noise

Q.92 For the block diagram shown in the given figure, the limiting values of K for stability of inner loop is found to be X<K<Y. the overall system will be stable if and only if



a) 4X<K<4Y b) 2X<K<2Y
c) X<K<Y d) $\dfrac{X}{2} < K < \dfrac{Y}{2}$

Q.93 The loop transfer function of a system is given by

$$G(s)H(s) = \frac{K(s+10)^2(s+100)}{s(s+25)}$$

The number of loci terminating at infinity is

a) 0 b) 1
c) 2 d) 3

Q.94 A control system has G(s)H(s)= K/[s(s + 4) (s^2 + 4s + 20)](0< K < ∞). What is the number of breakaway points in the root locus diagram?

a) One b) Two
c) Three d) Zero

Q.95 The characteristic equation of a control system is given by s(s + 4)(s^2 + 2s + 2) + k(s + 1) = 0. What are the angles of the asymptotes for the root loci for $k \geq 0$?

a) 60°, 180°, 300° b) 0°, 180°, 300°
c) 120°, 180°, 240° d) 0°, 120°, 240°

Q.96 The open loop transfer function of a feedback system has m poles and n zeros $(m > n)$

Consider the following statements

1. The number of separate root loci is m.
2. The number of separate root loci is n.
3. The number of root loci approaching infinity is (m – n)
4. The number of root loci approaching infinity is (m + n)

Which of the statements given above are correct?

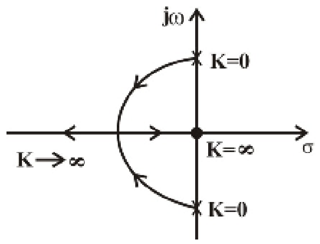
a) 1 and 4 b) 1 and 3
c) 2 and 3 d) 2 and 4

Q.97 The characteristic equation of a feedback control system is given by $s^3 + 5s^2+(K+6)s+K=0$. In the root loci diagram, the asymptotes of the root loci for large 'K' meet at a point in the s-plane whose coordinates are

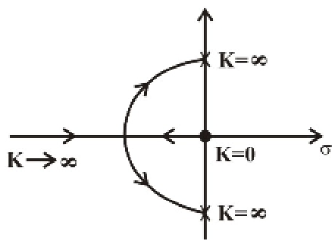
a) (2, 0) b) (-1, 0)
c) (-2, 0) d) (-3, 0)

Q.98 The characteristic equation of a linear control system is $s^2+5Ks+10=0$. The root –loci of the system for $0<K<\infty$ is

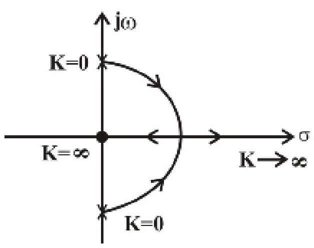
a)



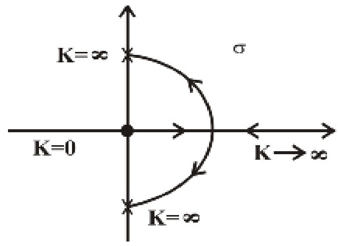
b)



c)



d)



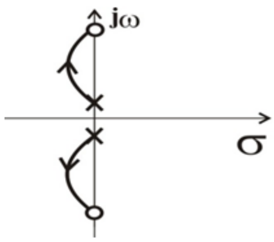
Q.99 The open loop transfer function of a closed loop control system is given as:

$G(s)H(s) = \dfrac{K(s+2)}{s(s+1)(s+4)^2}$. What are the number of asymptotes and the centroid of the asymptotes of the root-loci of closed loop system?

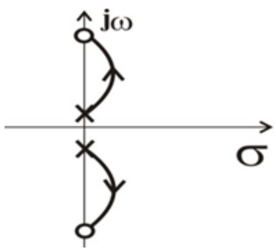
a) $-3;\left(\frac{7}{3},0\right)$ b) $-2;(2,0)$
c) $3;\left(\frac{-7}{3},0\right)$ d) $2;(-2,0)$

Q.100 Loop transfer function of unity feedback system is $G(s) = \dfrac{K(s^2+64)}{s(s^2+16)}$. The correct root locus diagram for the system is

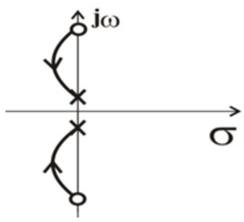
a)



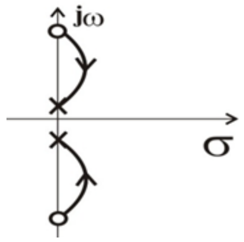
b)



c)



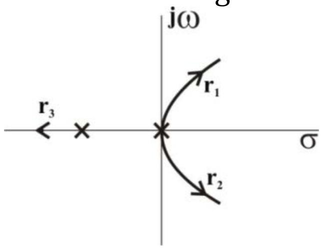
d)



Q.101 An open loop transfer functions is given by $G(s)H(s)=\frac{k(s+1)}{s(s+2)(s^2+2s+2)}$. It has

a) One zero at infinity
b) Two zeros at infinity
c) Three zeros at infinity
d) Four zeros at infinity

Q.102 Which of the following is the open loop transfer function of the root loci shown in the figure?



a) $\frac{K}{s(s+T_1)^2}$ b) $\frac{K}{(s+T_1)(s+T_2)^2}$
c) $\frac{K}{(s+T)^3}$ d) $\frac{K}{s^2(sT_1+1)}$

Q.103 A control system has $G(s)H(s)=\frac{K(s+1)}{s(s+3)(s+4)}$
Root locus of the system can lie on the real axis

a) Between s = -1 and s = -3
b) Between s = 0 and s = -4
c) Between s = -3 and s = -4
d) Towards left of s = -4

Q.104 The characteristic equation of a control system is
$s^5+15s^4+85s^3+225s^2+247s+120=0$
What are the number of roots of the equation which lie to the left of the line s + 1 = 0?

a) 2 b) 3
c) 4 d) 5

Q.105 For a given unity feedback system with $G(s)=\frac{k(s+3)}{s(s+1)(s+2)(s+5)}$, what is the real axis intercept for root locus asymptotes?

a) 2/3 b) 1/4
c) –5/3 d) –3/2

Q.106 How many number of branches the root loci of the equation s(s + 2)(s + 3) + K(s + 1) = 0 have?

a) Zero b) One
c) Two d) Three

Q.107 The addition of open loop zero pulls the root-loci towards:

a) The left and therefore system becomes more stable
b) The right and therefore system becomes unstable
c) Imaginary axis and therefore system becomes marginally stable
d) The left and therefore system becomes unstable

Q.108 The characteristic equation of a control system is given as:
$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+2)}=0$$
For large values of s, the root loci for K ≥ 0 are asymptotic to asymptotes, where do the asymptotes intersect on the real axis?

a) $\frac{5}{3}$ b) $\frac{2}{3}$
c) $-\frac{5}{3}$ d) $\frac{4}{3}$

Q.109 Consider the equation $s^2+2s+2+K(s+2)=0$. Where do the roots of this equation break on the root loci plot?

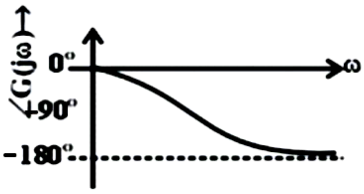
a) – 3.414 b) – 2.414
c) – 1.414 d) – 0.414

Q.110 A system has fourteen poles and two zeros. The slope of its highest

frequency asymptote in its magnitude plot is
a)-40 dB/decade
b)-240 dB/decade
c)-280 dB/decade
d)-320 dB/decade

Q.111 The phase angle of the system $G(s) = \frac{s+5}{s^2+4s+9}$ varies between
a) 0^0 and 90^0 b) 0^0 and -90^0
c) 0^0 and -180^0 d) -90^0 and -180^0

Q.112 The Bode phase angle plot of a system is shown in the figure. The type of the system is



a) 0 b) 1
c) 2 d) 3

Q.113 A minimum phase unity feedback system has a Bode plot with a constant slope of -20 db/ decade for all frequencies. What is the value of the maximum phase margin for the system?
a) 0^o b) 90^o
c) -90^o d) 180^o

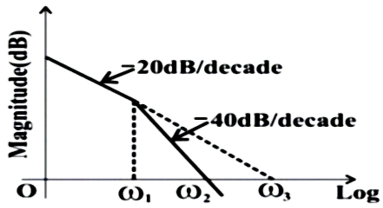
Q.114 At which frequency does the Bode magnitude plot for the function K/s^2 have gain crossover frequency?
a) $\omega = 0$ r/s b) $\omega = \sqrt{K}$ r/s
c) $\omega = K$ r/s d) $\omega = K^2$ r/s

Q.115 Which one of the following is correct?
The slope of the asymptotic Bode magnitude plot is integer multiple of
a) ± 40 db/ decade
b) ± 12 db/ octave
c) $\pm$ 6 db/ octave
d) $\pm$ 3 db/ octave

Q.116 What is the initial slope of Bode magnitude plot of a type – 2 system?
a) – 20 db/decade
b)+20 db/decade
c) -40 db/decade
d)+40 db/decade

Q.117 What is the error in magnitude at the corner frequency for an asymptotic Bode magnitude plot for the term $(1 + s\tau)^{\pm n}$?
a) $\pm$ 20 n db b) $\pm$ 6 n db
c) $\pm$ 3 n db d) $\pm$ 1 n db

Q.118 Consider the following statements regarding the frequency response of a system as shown:
1. The type of the system is one
2. ω_3= static error coefficient (numerically)
3. $\omega_2 = \frac{\omega_1 + \omega_3}{2}$



Select the correct answer using the codes given below:
a) 1, 2 and 3 b) 1 and 2
c) 2 and 3 d) 1 and 3

Q.119 In the Bode plot of a unity feedback control system, the value of phase angle of $G(j\omega)$ is -90^0 at the gain cross over frequency of the Bode plot, the phase margin of the system is:
a) -180^0 b) $+180^0$
c) -90^0 d) $+90^0$

Q.120 What are the gain and phase angle of a system having the transfer function G(s)=(s+1) at a frequency of 1 rad/sec?
a) 0.41 and 0^0 b) 1.41 and 45^0
c) 1.41 and -45^0 d) 2.41 and 90^0

Q.121 Consider the following statements in connection with frequency domain specifications of a control system:

1. Resonant peak and peak overshoot are both functions of the damping ratio ξ only.
2. The resonant frequency $\omega_r = \omega_n$ for $\xi > 0.707$.
3. Higher the resonant peak, higher is the maximum overshoot of the step response.

Which of the statements given above are correct?

a) 1 and 2 only b) 2 and 3 only
c) 1 and 3 only d) 1, 2 and 3

Q.122 A minimum phase transfer function has

a) poles in the right half of s-plane
b) zeros in the right half of s-plane
c) poles in the left half of s-plane and zeros in the right half of s-half
d) no poles or zeros in the right half of s-plane or on the j ω-axis excluding the origin

Q.123 Which of the following transfer functions is/are minimum phases transfer function(s).

1. $\frac{1}{(s-1)}$ 2. $\frac{(s-1)}{(s+3)(s+4)}$
3. $\frac{(s+2)}{(s+3)(s-4)}$

Select the correct answer using the code given below:

a) 1 and 3 b) 1 only
c) 2 and 3 d) None

Q.124 The transfer function of a system is $\frac{1-s}{1+s}$. The system is then which one of the following?

a) Non-minimum phase system
b) Minimum phase system
c) Low-pass system
d) second-order system

Q.125 The low frequency and high frequency asymptotes of Bode magnitude plot are respectively -60 db/decade and -40 db/decade. What is the type of the system?

a) Type 0 b) Type I
c) Type II d) Type III

Q.126 The poles and zeros of an all-pass network are located in which part of the s-plane?

a) Poles and zeroes are in the right half of s-plane
b) Poles and zeroes are in the left half of s-plane
c) Poles in the right half and zeroes in the left half of s-plane
d) Poles in the left half and zeroes in the right half of s-plane

Q.127 Consider the following statements regarding the asympotic Bode plots used for frequency response analysis:

1. The deviation of the actual magnitude response for a zero on real axis is 3 dB at the corner frequency.
2. The phase angle for a pair of complex conjugate poles at undamped frequency depends upon the value of damping ratio

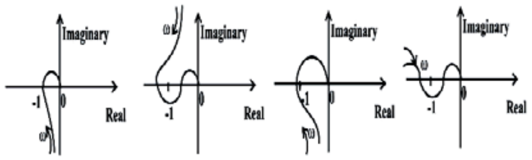
Which of the statements given above is/ are correct

a) Only 1 b) Only 2
c) Both 1 and 2 d) Neither 1 nor 2

Q.128 For the Bode plot of the system G(s) $= \frac{10}{0.66s^2 + 2.33s + 1}$ the corner frequencies are:

a) 0.66 and 0.33 b) 0.22 and 2.00
c) 0.30 and 2.33 d) 0.50 and 3.00

Q.129 Consider the following Nyquist plots of different control systems:



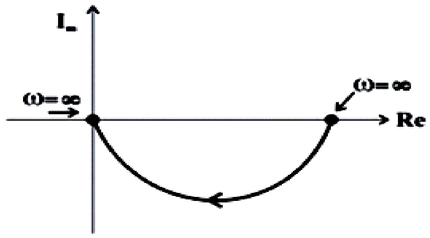
Which of these plot(s) represents (s) a stable system?

a) 1 alone b) 2, 3 and 4
c) 1, 3 and 4 d) 1, 2 and 4

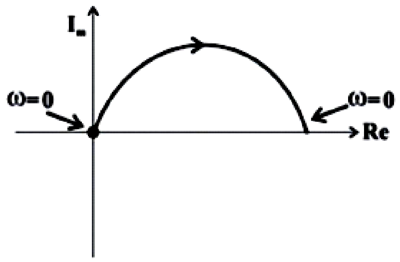
Q.130 The transfer function of a certain system is given by $G(s)=\frac{s}{(1+s)}$.

The Nyquist plot to the system is

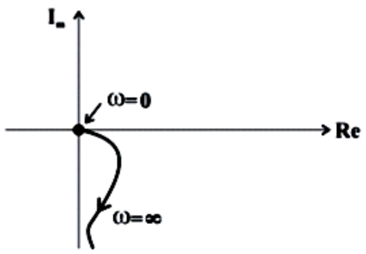
a)



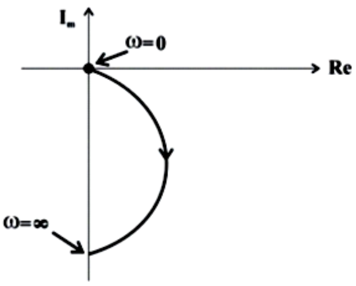
b)



c)

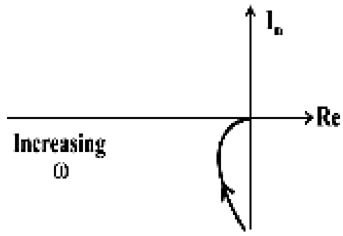


d)

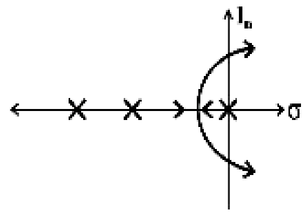


Q.131 The Nyquist plot of a servo system is shown in the figure-I. the root loci for the system would be

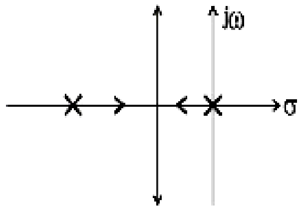
a)



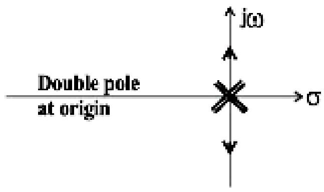
b)



c)



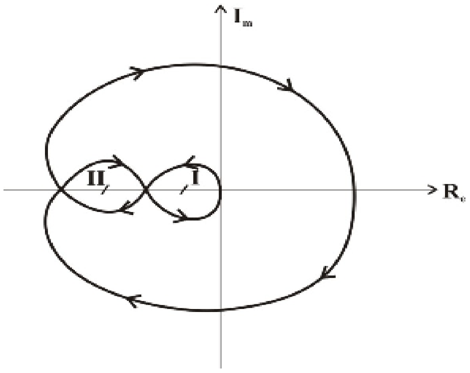
d)



Q.132 If the Nyquist plot cuts the negative real axis at a distance of 0.4, then the gain margin of the system is

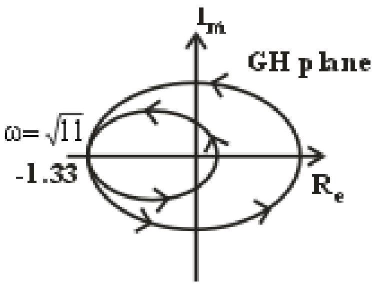
a) 0.4 b) -0.4
c) 4% d) 2.5

Q.133 Consider the Nyquist diagram for given KG(s) H(s). The transfer function KG(s)H(s) has no poles and zeros in the right half of plane. If the (-1+j0) point is located first in region I and then in region II, the change in stability of the system will be from



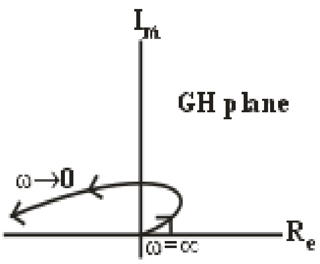
a) Unstable to stable
b) Stable to stable
c) Unstable to unstable
d) Stable to unstable

Q.134 The Nyquist plot of a unity feedback system having open loop transfer function $G(s)=\dfrac{K(s+3)(s+5)}{(s-2)(s-4)}$ for K = 1 is as shown below. For the system to be stable, the range of values of K is



a) 0 < K < 1.33 b) 0 < K <1/1.33
c) K > 1.33 d) K > 1/1.33

Q.135 The Nyquist plot of a control system is shown below. For this system, G(s) H(s) is equal to



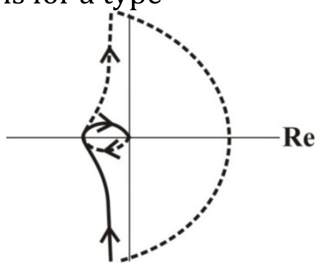
a) $\dfrac{K}{s(1+sT_1)}$

b) $\dfrac{K}{s^2(1+sT_1)}$

c) $\dfrac{K}{s^3(1+sT_1)(1+sT_2)}$

d) $\dfrac{K}{s^2(1+sT_1)(1+sT_2)}$

Q.136 Nyquist plot shown in the given **figure** is for a type



a) Zero system b) One system
c) Two system d)Three system

Q.137 The open loop transfer function of a unity feedback control system is given as $G(s)=\dfrac{1}{s(1+sT_1)(1+sT_2)}$. The phase crossover frequency and the gain margin are, respectively,

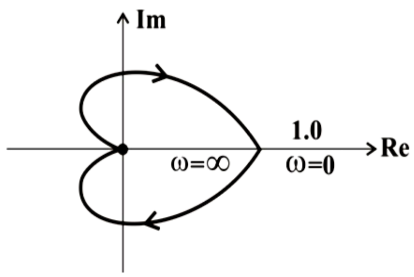
a) $\dfrac{1}{\sqrt{T_1T_2}}$ and $\dfrac{T_1+T_2}{T_1T_2}$

b) $\sqrt{T_1T_2}$ and $\dfrac{T_1+T_2}{T_1T_2}$

c) $\dfrac{1}{\sqrt{T_1T_2}}$ and $\dfrac{T_1T_2}{T_1+T_2}$

d) $\sqrt{T_1T_2}$ and $\dfrac{T_1T_2}{T_1+T_2}$

Q.138 The Nyquist plot shown, matches with the transfer function



a) $\dfrac{1}{(s+1)^3}$ b) $\dfrac{1}{(s+1)^2}$

c) $\dfrac{1}{(s^2+2s+2)}$ d) $\dfrac{1}{(s+1)}$

Q.139 The forward path transfer function of a unity feedback system is given by $G(s)=\frac{1}{(1+s)^2}$

What is the phase margin for this system?

a) $-\pi$rad　　b) 0rad

c) $\pi/2$rad　　d) π rad

Q.140 What is the gain margin of a system when the magnitude of the polar plot at phase cross over is 'a'?

a) 1/a　　b) –a

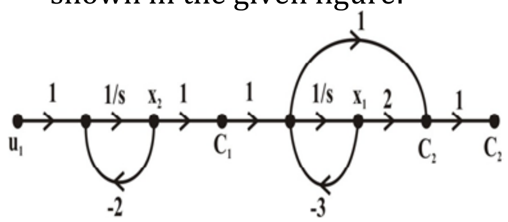
c) Zero　　d) a

Q.141 A system with gain margin close to unity or a phase margin close to zero is

a) Relatively stable　　b) Oscillatory

c) Stable　　d) Highly stable

Q.142 The state diagram of a system is shown in the given figure:



The system is

a) Controllable and observable

b) Controllable but not observable

c) Observable but not controllable

d) Neither controllable nor observable

Q.143 The state-variable description of a linear autonomous system is $\dot{X}=AX$ where X is a state vector and $A=\begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$.

The poles of the system are located at

a) -2 and +2　　b) -2j and +2j

c) -2 and -2　　d) +2 and +2

Q.144 A particular control system is described by the following state equations:

$\dot{X}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}X+\begin{bmatrix}0\\ 1\end{bmatrix}U$ and

$Y\begin{bmatrix}2 & 0\end{bmatrix}X$

The transfer function of this system is:

a) $\frac{Y(s)}{U(s)}=\frac{1}{2s^2+3s+1}$

b) $\frac{Y(s)}{U(s)}=\frac{2}{2s^2+3s+1}$

c) $\frac{Y(s)}{U(s)}=\frac{1}{s^2+3s+2}$

d) $\frac{Y(s)}{U(s)}=\frac{2}{s^2+3s+2}$

Q.145 The state-space representation in phase-variable form for the transfer function $G(s)=\frac{2s+1}{s^2+7s+9}$ is

a) $\dot{x}=\begin{bmatrix}0 & 1\\ -9 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u;\ y=\begin{bmatrix}1 & 2\end{bmatrix}x$

b) $\dot{x}=\begin{bmatrix}1 & 0\\ -9 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u;\ y=\begin{bmatrix}0 & 1\end{bmatrix}x$

c) $\dot{x}=\begin{bmatrix}-9 & 0\\ 0 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u;\ y=\begin{bmatrix}2 & 0\end{bmatrix}x$

d) $\dot{x}=\begin{bmatrix}9 & -7\\ 1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u;\ y=\begin{bmatrix}1 & 2\end{bmatrix}x$

Q.146 Let $\dot{X}=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}X+\begin{bmatrix}0\\ 1\end{bmatrix}$

$Y=\begin{bmatrix}b & 0\end{bmatrix}X$

Where b is an unknown constant.

This system is

a) Observable for all values of b

b) Unobservable for all values of b

c) Observable for all non-zero values of b

d) Unobservable for all non-zero values of b

Q.147 The state-space representation of a system is given by T

$\dot{X} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}X + \begin{bmatrix}1\\ 0\end{bmatrix}U \text{ and } Y = \begin{bmatrix}1\\ 1\end{bmatrix}X$

Then the transfer function of the system is

a) $\dfrac{1}{s^2+3s+2}$ b) $\dfrac{1}{s+2}$

c) $\dfrac{s}{s^2+3s+2}$ d) $\dfrac{1}{s+1}$

Q.148 A linear time-invariant system is described by the following dynamic equation:

$\dfrac{dx(t)}{dt} = Ax(t) + Bu(t)$

$y(t) = Cs(t)$

where

$A = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}, B = \begin{bmatrix}0\\ 1\end{bmatrix}, C = \begin{bmatrix}1 & 1\end{bmatrix}$

The system is

a) Both controllable and observable
b) Controllable but unobservable
c) Observable but uncontrollable
d) Both uncontrollable and unobservable

Q.149 What is represented by state transition matrix of a system?

a) Free response
b) Impulse response
c) Step response
d) Forced response

Q.150 The system matrix of a continuous time system is given by A = $\begin{bmatrix}0 & 1\\ -3 & -5\end{bmatrix}$. Then the characteristic equation is

a) $s^2 + 5s + 3 = 0$ b) $s^2 - 3s - 5 = 0$
c) $s^2 + 3s + 5 = 0$ d) $s^2 + s + 2 = 0$

Q.151 Let $\dot{x} = \begin{bmatrix}1 & 2\\ 0 & b\end{bmatrix}x + \begin{bmatrix}0\\ 1\end{bmatrix}u$ where b is an unknown constant. This system is

a) Uncontrollable for b = 1
b) Uncontrollable for b = 0
c) Uncontrollable for all values of b
d) Controllable for all values of b

Q.152 Given the matrix A = $\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}$

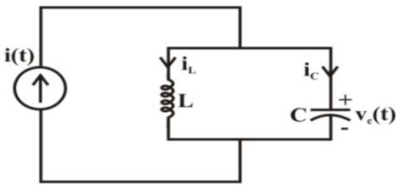
The eigen values of A are:

a) $-1, -2, -3$ b) $-1, 2, 3$
c) $0, 0, -6$ d) $-6, -11, -6$

Q.153 System transformation function H(z) for a discrete time LTI system expressed in state variable form with zero initial conditions is

a) $c(zI - A)^{-1}b + d$ b) $c(zI - A)^{-1}$
c) $(zI - A)^{-1}z$ d) $(zI - A)^{-1}$

Q.154 Which one of the following is the state-space model of the circuit shown below?



a) $\begin{bmatrix}\dot{x}_1 t\\ \dot{x}_2 t\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{L}\\ -\frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{C}\end{bmatrix}u(t)$

$y(t) = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix}$

b) $\begin{bmatrix}\dot{x}_1 t\\ \dot{x}_2 t\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{C} & -\frac{1}{L}\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$

$y(t) = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix}$

c) $\begin{bmatrix}\dot{x}_1 t\\ \dot{x}_2 t\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{L}\\ -\frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u(t)$

$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix}$

d) $\begin{bmatrix}\dot{x}_1 t\\ \dot{x}_2 t\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{L}\\ -\frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$

$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 t\\ x_2 t\end{bmatrix}$

Q.155 A linear time invariant system with input u(t) and output y(t) is described by the state-space representation as given below.

$\dot{x}_1(t) = x_2(t)$

$\dot{x}_2(t) = x_1(t) + x_2(t) + u(t)$

and $y(t) = x_1(t) + 3x_2(t)$

The transfer function of the system is

a) $\frac{s+3}{s^2-s-1}$ b) $\frac{s+3}{s^2+s+1}$

c) $\frac{3s+1}{s^2+s+1}$ d) $\frac{3s+1}{s^2-s-1}$

Q.156 The transfer function of a phase lead compensator is given by $\frac{1+aTs}{1+Ts}$ where a>1 and T>0. The maximum phase shift provided by such a compensator is

a) $\tan^{-1}\left(\frac{a+1}{a-1}\right)$ b) $\tan^{-1}\left(\frac{a-1}{a+1}\right)$

c) $\sin^{-1}\left(\frac{a+1}{a-1}\right)$ d) $\sin^{-1}\left(\frac{a-1}{a+1}\right)$

Q.157 Indicate which one of the following transfer functions represents phase lead compensator?

a) $\frac{s+1}{s+2}$ b) $\frac{6s+3}{6s+2}$

c) $\frac{s+5}{3s+2}$ d) $\frac{s+8}{s^2+5s+6}$

Q.158 A property of phase-lead compensation is that the

a) Overshoot is increased

b) Bandwidth of closed loop system is reduced

c) Rise-time of closed loop system is reduced

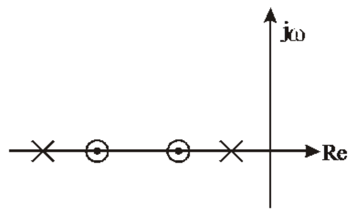
d) Gain margin is reduced

Q.159 Which one of the following is the correct expression for the transfer function of an electrical RC phase-lag compensating network?

a) $\frac{RCS}{(1+RCS)}$ b) $\frac{RC}{(1+RCS)}$

c) $\frac{1}{(1+RCS)}$ d) $\frac{1}{(1+RCS)}$

Q.160 The pole-zero plot shown in the figure is that of which one of the following?



a) Integrator

b) PD controller

c) PID controller

d) Lag-lead compensator

Q.161 What is the effect of phase lead compensator on gain cross – over frequencies and on the bandwidth (ω_b)?

a) Both are increased

b) ω_{cg} is increased but ω_b is decreases

c) ω_{cg} is decreased but ω_b is increased

d) Both are decreased

Q.162 The transfer function of a P-1 controller is:

a) $K_p + K_i s$ b) $K_p + (K_i/s)$

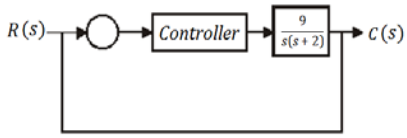
c) $(K_p/s) + K_i s$ d) $K_p s + (K_i/s)$

Q.163 The transfer function of a phase-lead compensator is given by:

$G(s) = \frac{1+3Ts}{1+Ts}$ where T > 0. What is the maximum shift provided by such a compensator?

a) 90° b) 60°
c) 45° d) 30°

Q.164 In the control system shown above the controller which can give zero steady-state error to a ramp input is of



a) Proportional type
b) Integral type
c) Derivative type
d) Proportional plus derivative type

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
(b)	(a)	(d)	(b)	(d)	(c)	(c)	(b)	(c)	(d)	(a)	(a)	(b)	(a)
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
(b)	(a)	(b)	(b)	(a)	(d)	(a)	(b)	(c)	(d)	(a)	(a)	(a)	(b)
29	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
(b)	(c)	(c)	(b)	(a)	(b)	(a)	(b)	(d)	(b)	(b)	(d)	(d)	(c)
43	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**
(c)	(a)	(d)	(c)	(c)	(b)	(d)	(b)	(b)	(a)	(a)	(a)	(d)	(b)
57	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
(d)	(d)	(b)	(b)	(a)	(b)	(a)	(c)	(c)	(b)	(a)	(b)	(c)	(b)
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**
(c)	(c)	(c)	(c)	(b)	(c)	(a)	(c)	(d)	(a)	(d)	(b)	(a)	(c)
85	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**
(b)	(c)	(c)	(c)	(d)	(c)	(c)	(d)	(a)	(a)	(a)	(b)	(c)	(a)
99	**100**	**101**	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**
(c)	(a)	(c)	(d)	(b)	(c)	(c)	(d)	(a)	(c)	(a)	(b)	(c)	(a)
113	**114**	**115**	**116**	**117**	**118**	**119**	**120**	**121**	**122**	**123**	**124**	**125**	**126**
(b)	(b)	(c)	(c)	(c)	(b)	(d)	(b)	(c)	(d)	(d)	(a)	(d)	(d)
127	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
(a)	(d)	(d)	(b)	(d)	(d)	(d)	(d)	(d)	(b)	(a)	(b)	(c)	(a)
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**
(b)	(a)	(b)	(d)	(a)	(c)	(d)	(b)	(a)	(a)	(d)	(a)	(a)	(a)
155	**156**	**157**	**158**	**159**	**160**	**161**	**162**	**163**	**164**				
(d)	(d)	(a)	(c)	(c)	(d)	(a)	(b)	(d)	(b)				

EXPLANATIONS

Q.1 (b)

$$\frac{C_n(s)}{N(s)} = \frac{-G(s)H_2(S)}{1+G(s)H_1(S)H_2(S)}$$

$$= -\frac{1}{H_1(S)} \text{ for } |G(s)H_1(S)H_2(S)| \text{ ? } |$$

Q.2 (a)

Forward paths

$P_1 = abcef \quad P_3 = abn\lg f$

$P_2 = ajigf \quad P_4 = ajkmef$

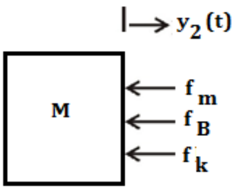
Loop gain

$L_1 = cd \quad L_3 = nmd$

$L_2 = hi \quad L_4 = klh$

Q.3 (d)

Free body diagram of M is shown below:

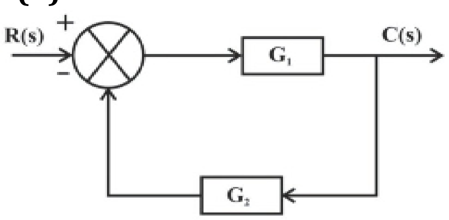


By network law

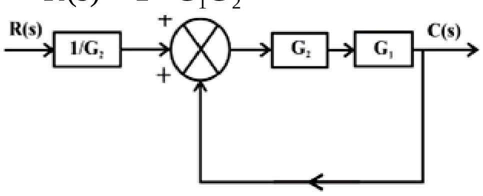
$f_M + f_B + f_k = 0$

$$M\frac{d^2y_2(t)}{dt^2} + B\frac{dy_2(t)}{dt} + k(y_2(t) - y_1(t)) = 0$$

Q.4 (b)

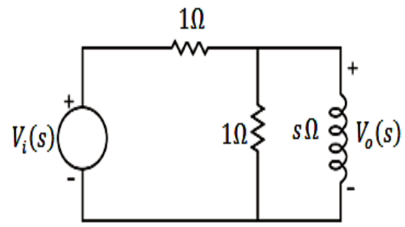


$$\frac{C(s)}{R(s)} = \frac{G_1}{1-G_1G_2}$$



$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1-G_1G_2}\cdot\frac{1}{G_2} = \frac{G_1}{1-G_1G_2}$$

Q.5 (d)



$$\frac{V_o(s)}{V_1(s)} = s\,||\,1+1 = \frac{S}{1+s}+1 = \frac{2S+1}{s+1}$$

Q.6 (c)

Forward paths Individual Loops

$P_1 = adfi$

$L_1 = bL_2 = dc$

$P_2 = aefi$

$L_3 = ec$

$P_3 = ahi$

$L_4 = fgL_5 = hgc$

Non touching Loop pairs: L_1L_4

Q.7 (c)

Negative feedback decreases the distortion.

Q.8 (b)

$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3G_2}{1+G_1G_2H_1 + G_2G_3H_1 - G_4}$$

Q.9 (c)

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1-G_2H_2 + G_1G_2H_1H_2}$$

Q.10 (d)

1. $x_2 = a_1x_1 + a_9x_3$
2. $x_3 = a_2x_2 + a_8x_4$
3. $x_4 = a_3x_3 + a_5x_2 + a_7x_4$
4. $x_5 = a_4x_4 + a_6x_2$

Q.11 (a)

$$\frac{G_1G_2 + G_3G_2}{1 + G_2H_1}$$

Q.12 (a)

$$\frac{C}{R} = \frac{G}{1 + GH - FG}$$

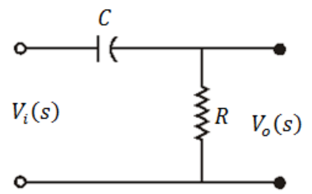
Q.13 (b)

$$\frac{C(s)}{R(s)} = \frac{1 + S^{-1}a}{1 + S^{-1}b} = \frac{s + a}{s + b}$$

Q.14 (a)

$$\frac{Y(s)}{x(s)} = \frac{G_1G_2}{1 - G_2H_1 + G_1G_2H_2}$$

Q.15 (b)



$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{SC}} = \frac{SRC}{1 + SRC}$$

Q.16 (a)

For block diagram 1

$$\frac{C(s)}{R(s)} = \frac{k}{S + 1} + 1$$

$$= \frac{S + 1 + k}{S + 1}$$

For k=1 both are same

Q.17 (b)

Number of loops are 3

Number of forward paths is also 3

$$\frac{C}{R} = \frac{8 + 12 + 20}{1 + 24 + 16 + 40} = \frac{40}{81}$$

Q.18 (b)

$$\frac{C}{R} = \frac{G + H_2}{1 + GH_1} \qquad \text{(Using SFG)}$$

Q.19 (a)

$$\frac{C(s)}{R(s)} == \frac{G_1G_2}{1 + G_1H_1 - G_2H_2} \text{(Using SFG)}$$

Q.20 (d)

1. Feedback reduce system error
2. Feedback can make a stable system as unstable

Q.21 (a)

Forward path gain

Individual Loop gain

$P_1 = rsu$

$L_1 = st$

$P_2 = efh$

$L_2 = fg$

$$\frac{x_2}{x_1} = \frac{rsu(1 - gf) + efh(1 - st)}{1 - st - fg + stfg}$$

$$\frac{x_2}{x_1} = \frac{rsu(1 - fg) + efh(1 - st)}{(1 - st)(1 - fg)}$$

$$= \frac{rsu}{1 - st} + \frac{efh}{1 - fg}$$

Q.22 (b)

For negative feedback

$$\text{Gain,} \quad A_F = \frac{A}{1 + AB} = \frac{10 \times 10 \times 10}{1 + AB}$$

$$B = 9 \times 10^{-3}$$

Q.23 (c)

Consider R(s)=0

$$\frac{C(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)}$$

For , $G_1(s)G_2(s)$? 1

$$C(s) = \frac{1}{G_1(s)} D(s)$$

Step disturbance can be reduced by increasing the gain of $G_1(s)$

Q.24 (d)

Transfer function,

$$T(s) = \frac{y(s)}{R(s)} = \frac{Ak}{S + a + Ak\beta}$$

$$\text{Sensitivity} = \frac{\%\text{change in } T(s)}{\%\text{change in parameter}}$$

$$S_k^T = \frac{\%\text{change in } T(s)}{\%\text{change in k}} = \frac{\partial T}{\partial k} \times \frac{k}{T}$$

$$= \frac{S+a}{s+a+Ak\beta}$$

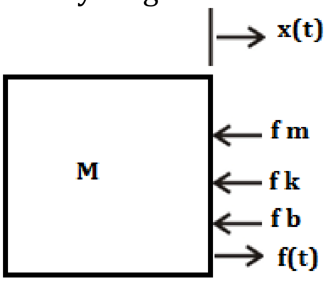
$$S_{\beta}^{T} = \frac{\partial T}{\partial \beta} \times \frac{\beta}{T} = -\frac{Ak\beta}{(s+a+Ak\beta)}$$

$$S_{A}^{T} = \frac{\partial T}{\partial A} \times \frac{A}{T} = \frac{S+a}{s+a+Ak\beta}$$

S_{β}^{T} is negative hence it doesn't reduce the closed loop sensitivity.

Q.25 (a)

Free body diagram of mass is



By Newton's law

$$f_m + f_k + f_b = f(t)$$

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = f(t)$$

Q.26 (a)

Response, $c(t) = te^{-t}$ for $t \geq o$

$$c(s) = \frac{1}{(s+1)^2}$$

Input, $r(t) = tu(t)$

$$R(s) = \frac{1}{S}$$

$$\frac{C(s)}{R(s)} = \frac{s}{(s+1)^2}$$

Q.27 (a)

$$M = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$S_{G}^{M} = \frac{\partial M}{\partial G} \times \frac{G}{M} = \frac{1}{1+G(s)H(s)}$$

Q.28 (b)

Step response, $c(t) = t^2e^{-t}$

$$C(s) = 2/(s+1)^3$$

Input $r(t) = u(t)$

$$R(s) = \frac{1}{s}$$

$$\text{Transferfunction} = \frac{C(s)}{R(s)}$$

$$= \frac{2s}{(s+1)^3}$$

Q.29 (b)

Closed loop system is less sensitive to parameter variations.

Closed loop system has the ability to control the system transient response.

Q.30 (c)

$$c(t) = 1 - 10e^{-t}$$

$$C(s) = \frac{1}{s} + \frac{10}{s+1}$$

$$= \frac{s+1-10s}{s(s+1)} = \frac{1-9s}{s(s+1)}$$

Q.31 (c)

The characteristics equation is given by

$$(s+a)^2 + b^2 = 0$$

$$S^2 + 2as + a^2 + b^2 = 0$$

Comparing with standard 2nd order equations,

We get

$$\omega_n^2 = a^2 + b^2 \Rightarrow \omega_n = \sqrt{a^2+b^2}$$

$$2\xi\omega_n = 2a \Rightarrow \xi = \frac{a}{\sqrt{a^2+b^2}}$$

Q.32 (b)

Given $r(t) = 1 + t + \frac{t^2}{2}$

$$R(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \frac{1}{10} = 0.1$$

Q.33 (a)

$$G(s)=\frac{10(s+1)}{s^2(s+2)}$$

$$r(t)=1+2t \Rightarrow R(s)=\frac{1}{s}+\frac{2}{s^2}$$

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}=0$$

Q.34 (b)

$$4\frac{d^2c(t)}{dt^2}+8\frac{dc(t)}{dt}+16c(t)$$

$$=16u(t)$$

Applying LT both sides we get

$$4s^2C(s)+85C(s)+16c(s)$$

$$=16R(s)$$

(Since u(t) is input)

$$\frac{c(s)}{R(s)}=\frac{16}{4s^2+8s+16}=\frac{4}{s^2+2s+16}$$

$$\omega_n^2=4 \Rightarrow \omega_n=2\frac{rad}{}$$

$$\xi=\frac{1}{2}$$

Q.35 (a)

$$\text{OLTF}, G(s)=\frac{k}{s(s+1)}$$

$$\text{CLTF}, \frac{C(s)}{R(s)}=\frac{k}{s^2+s+k}$$

$\xi=1$ for critically damped system

$$\omega_n^2=k \Rightarrow \omega_n=\sqrt{k}$$

$$2\xi\omega_n=1 \Rightarrow \omega_n=\frac{1}{2}$$

$$\therefore k=\omega_n^2=\frac{1}{4}$$

Characteristic equation is given by

$$s^2+s+\frac{1}{4}=0 \Rightarrow s=-0.5,-0.5$$

Q.36 (b)

The CE is given by

$$s^2+13.25+121=0$$

$$\omega_n^2=121 \Rightarrow \omega_n=11$$

$$2\xi\omega_n=13.2 \Rightarrow \xi=\frac{13.2}{22}<1$$

Hence, the system is under damped

Setting time

$$t_s=4T=\frac{4}{\xi\omega_n}=\frac{4}{6.6}=0.6s$$

Q.37 (d)

$$G(s)=\frac{10}{s^2(s+4)}$$

$$r(t)=2+3t+4t^2$$

$$e_{ss}=\frac{lt}{s\to 0}\frac{s\,R(s)}{1+G(s)H(s)}=3.2$$

Q.38 (b)

Given, Impulse response,

$$c(t)=\frac{1}{6}e^{-0.8t}\text{Sin}(0.6t)$$

In general the impulse response is

$$ke^{-\xi\omega_n t}\sin(\omega_d t)$$

$$\xi\omega_n=0.8$$

$$\omega_d=\omega_n\sqrt{1-\xi^2}=0.6$$

Solving above equations we get

$$\omega_n=1\text{rad}/\text{sec}$$

$$\xi=0.8$$

Q.39 (b)

$$M(j\omega)=\frac{100}{-\omega^2+10\sqrt{2}j\omega+100}$$

$$\omega_n^2=100, 2\xi\omega_n=10\sqrt{2}$$

$$\omega_n=10\xi=\frac{1}{\sqrt{2}}$$

$$M_p=\frac{1}{2\xi\sqrt{1-\xi^2}}=1$$

Q.40 (d)

$$\text{Given } G(s)=\frac{5(1+0.1s)}{s(1+5s)(1+20s)}$$

The above system is type -1

For step input

$$e_{ss}=\frac{A}{1+k_p}$$

$$k_p=\lim_{s\to 0}G(s)=\infty$$

$$\therefore e_{ss}=0$$

For ramp input

$$e_{ss} = \frac{A}{k_v}$$

$$k_v = \lim_{s\to 0} sG(s) = 5$$

$$\therefore e_{ss} = \frac{10}{5} = 2$$

For accelerations input

$$e_{ss} = \frac{A}{k_a}$$

$$k_a = \int_{s\to 0}^{t} s^2 G(s) = 0$$

$$e_{ss} = \infty$$

Q.41 (d)

Given, $\omega_n = 10\frac{rad}{sec}, \xi = 0.1$

For 2% $t_s = 4T = \frac{4}{\xi\omega_n} = 4\sec$

Q.42 (c)

$$E(s) = R(s) - C(s)$$

$$E(s) = R(s) - \frac{K}{s+5}E(s)$$

$$\frac{E(s)}{R(s)} = \frac{s+5}{s+5+K}$$

Q.43 (c)

Unit step response,

$$c(t) = \left[1 - e^{-t}(1+t)\right]u(t)$$

Impulse response

$$h(t) = \frac{d}{dt}c(t) = te^{-t}u(t)$$

Transfer function, $\frac{C(s)}{R(s)} = \frac{1}{(S+1)^2}$

Since poles lie on –ve real axis are repeated hence system is critical stable

Q.44 (a)

$$G(s) = \frac{k}{s(s+a)}$$

$$r(t) = tu(t)$$

$$e_{ss} = \lim_{s\to 0}\frac{s\,R(s)}{1+G(s)H(s)} = \frac{a}{k}$$

$$s_k^{e_{ss}} = \frac{\partial e_{ss}}{\partial k}\times\frac{k}{e_{ss}} = \frac{-a}{k^2}\times\frac{k}{a/k} = -1$$

$$s_k^{e_{ss}} = \frac{\partial e_{ss}}{\partial a}\times\frac{a}{e_{ss}} = \frac{1}{k}\times\frac{a}{a/k} = 1$$

Q.45 (d)

Given, $C(s) = \frac{k}{S+\propto}$

$$C(t) = ke^{-\propto t}$$

At $t = t_2, C(t_2) = 0.37k$

$$0.37k = ke^{-\propto t_2}$$

At $\propto = \frac{1}{t_2}$ above equation will be satisfied.

Q.46 (c)

$$\frac{400}{s^2+12s+400} \to \omega_n = 20, \xi = 0.3$$

Under damped

$$\frac{900}{s^2+90s+900} \to \omega_n = 30, \xi = 1.5$$

Over damped

$$\frac{225}{s^2+30s+225} \to \omega_n = 15, \xi = 1$$

Critically damped

$$\frac{625}{s^2+625} \to \xi = 0$$

Undamped

Q.47 (c)

In general peak time is given by

$$t_p = \frac{n\pi}{\omega_d} = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$$

For 2nd peak, n=3 hence,

$$t_p = \frac{3\pi}{\omega_n\sqrt{1-\xi^2}}$$

Q.48 (b)

$$G(s) = \frac{k}{s(s+1)}$$

$$\frac{C(s)}{R(s)}=\frac{k}{s^2+s+k}$$
$$\omega_n^2=k\Rightarrow\omega_n=\sqrt{k}$$
$$2\xi\omega_n=1\Rightarrow\xi=\frac{1}{2\sqrt{k}}$$
$$\%M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}\times100\%=50$$
$$e^{-\pi\xi/\sqrt{1-\xi^2}}=\frac{1}{2}$$
Substituting the value of ξ ,we get, k = 5.39

Q.49 (d)

$$c(t)=1-e^{-5t}-5te^{-5t}$$
$$c(s)=\frac{25}{S(s+5)^2}\ \text{(by L.T)}$$
$$R(s)=\frac{1}{S}$$
$$\frac{C(s)}{R(s)}=\frac{25}{s^2+10s+25}$$
$$\omega_n^2=25\Rightarrow\omega_n=5$$
$$2\xi\omega_n=10\Rightarrow\xi=1$$
Impulse response,
$$=\frac{d}{dt}(1-e^{-5t}-5te^{-5t})$$
$$=5e^{-5t}-5e^{-5t}+25te^{-5t}$$
$$=25te^{-5t}$$

Q.50 (b)

Given $\xi=0.6,\omega_n=2rad/sec$

Peak time,
$$t_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}=1.95sec$$
Setting time $t_s=\frac{4}{\xi\omega_n}=\frac{4}{0.6\times2}$
$$=3.33sec$$

Q.51 (b)

Given $\frac{y(s)}{x(s)}=\frac{S}{1+S}=H(s)$
$$H(j\omega)=\frac{j\omega}{1+j\omega}$$
$$x(t)=\sin t,\omega=1rad/sec$$
$$H(j\omega)|_{\omega=1}=\frac{j}{1+j}=\frac{1}{\sqrt{2}}\angle45°$$
$$y(t)=\frac{1}{\sqrt{2}}\sin(t+45°)$$

Q.52 (a)

$$\text{Phase},\phi=-\omega T-\frac{\pi}{2}$$
$$-\tan^{-1}(\omega)$$
At $\omega=\omega_1,\varnothing=0$
$$-\omega T-\tan^{-1}(\omega)=\frac{\pi}{2}$$
Taking tan both sides, weget
$$\frac{\tan(\omega_1T)+\omega_1}{1-\tan(\omega_1T)\omega_1}=\infty$$
$$\omega_1=\frac{1}{\tan(\omega_1T)}=\cot(\omega_1T)$$

Q.53 (a)

$$G(s)=\frac{k}{s(s+k)}\ \frac{C(s)}{R(s)}=\frac{k}{s^2+s+k}$$
$$\omega_n^2=k\Rightarrow\omega_n=\sqrt{k}$$
$$2\xi\omega_n=1\Rightarrow\xi=\frac{1}{2\sqrt{k}}$$
As $k\to\infty,\xi=0$

Q.54 (a)

$$c(t)=12.5e^{-6t}\sin8t$$

$\xi\omega_n=6$	$\omega_d=\omega_n\sqrt{1-\xi^2}=8$
$\xi=\frac{6}{10}(\omega_n=10)$	$\omega_n^2-\omega_n^2\xi^2=64$
$=0.6$	$\omega_n^2=100$
	$\omega_n=10$

Q.55 (d)

$$e_{ss}=\frac{1}{1+K_p}+\frac{1}{K_v}+\frac{1}{K_a}$$

Q.56 (b)

For dominant pole the transients die out slowly.

Q.57 (d)

$\omega_n = 3 rad/sec$

$\xi = 0.5$

$\omega_r = \omega_n\sqrt{1-2\xi^2} = 3\sqrt{1-0.5} = \frac{3}{\sqrt{2}}$

$= 2.12 rad/sec$

$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3}} = 1.16$

Q.58 (d)

Delay time $\rightarrow$ Rise time $\rightarrow$ Peak time $\rightarrow$ settling time

Q.59 (b)

$G(s) = \frac{20}{s^2+5s+20}$

$\omega_n^2 = 20 \Rightarrow \omega_n = \sqrt{20}$

$2\xi\omega_n = 5 \Rightarrow \xi = \frac{5}{2\sqrt{20}}$

Setting time for $\pm 2\%$ is

$t_s 4\tau = \frac{4}{5/2} = \frac{8}{5} = 1.6 sec$

Q.60 (b)

$\frac{C(s)}{R(s)} = \frac{k}{s^2+4s+k} \rightarrow$ CLTF

$\omega_n^2 = k \Rightarrow \omega_n = \sqrt{k}$

$2\xi\omega_n = 4$

For $\xi = 0.5$

$\omega_n = 4$

Since $k = \omega_n^2 \Rightarrow k = 16$

Q.61 (a)

Since the input is sinusoidal hence the steady state error is zero.

Q.62 (b)

For 3rd order system the rise time would be smaller that the equivalent 2nd order system

Q.63 (a)

$\frac{C(s)}{R(s)} = \frac{1}{S}$

$c(t) = 1$ which is constant

Q.64 (c)

For a critically damped system the poles lies on –ve real axis, are repeated.

Q.65 (c)

Since the given system is under damped and damping ratio ξ is negative hence the roots will be complex conjugates on the right half of s-plane.

Q.66 (b)

We know that

Steady state error, $e_{ss} = \frac{A}{k}$

By increasing system gain k, e_{ss} can be reduced

Q.67 (a)

Impulse response of 1st block is

$C_1(t) = \frac{d}{dt}[-0.5(1+e^{-2t})] = e^{-2t}$

$C_1(s) = \frac{1}{s+2}$

Impulse response of 2nd block is

$C_2(t) = e^{-t}$

$C_2(s) = \frac{1}{s+1}$

The overall transfer function

$C(s) = C_1(s)C_2(s) = \frac{1}{(s+2)(s+1)}$

Q.68 (b)

The transfer function of the diaphragm is

$\frac{C(s)}{R(s)} = \frac{1}{s^2+2s+2}$

For unit step input $R(s) = \frac{1}{S}$

$\therefore C(s) = \frac{1}{s(s^2+2s+2)}$

The steady state value is given by

$$C(\infty) = \lim_{s\to 0} sC(s) = \frac{1}{2} = 0.5$$

Q.69 (c)

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Given $M_r = 1$

$$\therefore \frac{1}{2\xi\sqrt{1-\xi^2}} = 1$$

$$4\xi^2\left(1-\xi^2\right) = 1$$

$$4\xi^2 - 4\xi^4 = 1$$

$$4\xi^4 - 4\xi^2 + 1 = 0$$

$$\left(2\xi^2 - 1\right)\left(2\xi^2 - 1\right) = 0$$

$$\xi = \frac{1}{\sqrt{2}}$$

Q.70 (b)

Given $t_s = 6 \sec \text{ for } \pm 5\%$

$$\xi = 0.707 = \frac{1}{\sqrt{2}}$$

$$t_s = \frac{3}{\xi\omega_n} = 0.6 \Rightarrow \xi\omega_n = 5$$

$$\omega_n = 5\sqrt{2}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{2}.\frac{1}{\sqrt{2}} = 5$$

In general the poles location is

$S = -\xi\omega_n \pm j\omega_d = -5 \pm j5$

Q.71 (c)

$$-\frac{C(s)}{R(s)} = \frac{k}{S^3 + 6S^2 + 5s + k}$$

s^3	1	5
s^2	6	k
s^1	$\frac{30-k}{6}$	
s^0	k	

So, $k > 0$ & $\frac{30-k}{6} > 0$ system is stable

$\therefore 0 < k < 30$

For $k = 30$, System is marginally stable

Q.72 (c)

s^4	3	5	2
s^3	10	0	0
s^2	5	2	
s^1	-4		
s^0	2		

One sign change in first column so, unstable system.

Q.73 (c)

$$\frac{C(s)}{R(s)} = \frac{k}{S^3 + 7S^2 + 6s + k}$$

s^3	1	6
s^2	7	k
s^1	$\frac{42-k}{7}$	
s^0	k	

So $k > 0$ and

$\frac{42-k}{7} > 0$ for stability

$\therefore 0 < k < 42$

Q.74 (c)

$$\frac{C(s)}{R(s)} = \frac{k(1+Ts)}{S^3 + S^2 + kTs + k}$$

s^3	1	kT
s^2	1	k
s^1	$kT - k$	
s^0	k	

So $kT - k > 0$

$T - 1 > 0$

$T > 1$

Q.75 (b)

s^4	2	3	10
s^3	1	5	
s^2	-7	10	
s^1	$45/7$		
s^0	10		

2 sign changes so two roots lie in right half $s-\text{plane}$

Q.76 (c)

If poles of a system lie on the imaginary axis the system will be marginally stable.

Q.77 (a)

$$\begin{array}{c|cc} s^3 & 1 & 9 \\ s^2 & 6 & 4 \\ s^1 & \frac{50}{6} & \\ s^0 & 4 & \end{array}$$

No sing changes so all three poles lie in L.H.S of $s-\text{plane}$.

Q.78 (c)

$$\begin{array}{c|ccc} s^4 & 1 & 3 & k \\ s^3 & 2 & 2 & \\ s^2 & 2 & k & \\ s^1 & 2-k & & \\ s^0 & k & & \end{array}$$

For the system to be oscillatory

$2-k=0 \Rightarrow k=2$

The Auxiliary equation is

$2s^2+k=0$

$s^2=-1 \Rightarrow \omega = 1\text{rad/sec}$

Q.79 (d)

Closed loop $eq^n \rightarrow \dfrac{k}{s^3+as^2+k}$

$$\begin{array}{c|cc} s^3 & 1 & 0 \\ s^2 & a & k \\ s^1 & -\frac{k}{a} & \\ s^0 & k & \end{array}$$

For $-\infty < k < \infty$ system is unstable

Q.80 (a)

Characteristic

$eq^n s^3+6s^2+5s+k=0$

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 6 & k \\ s^1 & \frac{30-k}{6} & \\ s^0 & k & \end{array}$$

K=30

For sustained oscillation

Auxiliary equation is

$6s^2+k=0$

$s^2=-5$

$\omega=\sqrt{5}\text{rad/sec}$

Q.81 (d)

$$\begin{array}{c|ccc} s^4 & 1 & 24 & k \\ s^3 & 1 & 3/2^4 & \\ s^2 & 20 & k & \\ s^1 & \frac{80-k}{20} & & \\ s^0 & k & & \end{array}$$

For $k>80$ there will be sign change in the first column

Q.82 (b)

$$\begin{array}{c|cc} s^3 & a_0 & a_2 \\ s^2 & a_1 & a_3 \\ s^1 & \frac{a_1a_2-a_0a_3}{a_1} & \\ s^0 & a_3 & \end{array}$$

So, $\dfrac{a_1a_2-a_0a_3}{a_1} \geq 0$

$\therefore a_1a_2 \geq a_0a_3$

Q.83 (a)

$G(s)=T_1T_2S^3+(T_1T_2)S^2+S+k=0$

$$\begin{array}{c|cc} s^3 & T_1T_2 & 1 \\ s^2 & T_1+T_2 & k \\ s^1 & \frac{T_1+T_2-T_1T_2k}{T_1+T_2} & \\ s^0 & k & \end{array}$$

So, $\dfrac{T_1+T_2-T_1T_2k}{T_1+T_2} \geq 0$

$\therefore T_1+T_2 \geq T_1T_2k$

$$k < \frac{T_1 + T_2}{T_1 T_2}$$

Q.84 (c)

Characteristics eq^n

$$s^3 + (a+b)s^2 + abs + k = 0$$

$$\begin{array}{c|cc} s^3 & 1 & ab \\ s^2 & a+b & k \\ s^1 & \frac{(a+b)ab-k}{(a+b)} & \\ s^0 & 0 & \end{array}$$

So, $\frac{(a+b)ab-k}{(a+b)} \geq 0$

$(a+b)ab \geq k$

$\therefore 0 < k < (a+b)ab$

Q.85 (b)

$$s^3 + 25s^2 + 10s + 50 = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & 25 & 50 \\ s^1 & 8 & \\ s^0 & 2 & \end{array}$$

Since there are sign change in the first column hence no rot will lie on RHS of s- plane Also there is no occurrence of Auxiliary equation hence no roots on jωaxis.

Q.86 (c)

$$s^4 + s^3 + 25s^2 + 25 + 3 = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 2 & 3 \\ s^3 & 1 & 2 & \\ s^2 & 0/\xi & 3 & \\ s^1 & \frac{2\xi-3}{\xi} & & \\ s^0 & 3 & & \end{array}$$

Replace zero by $'\xi'$ in s^2row

For $\xi = 0 \frac{2\xi - 3}{\xi} = -\infty$

Since two sing changes, so two roots in RHS of s- plane

Q.87 (c)

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & 4 & 11 \\ s^1 & \frac{29}{4} & \\ s^0 & 11 & \end{array}$$

So coefficient in first column have same sing so, system is stable.

Q.88 (c)

$$\begin{array}{c|cccc} s^6 & 1 & 8 & 20 & 16 \\ s^5 & 2 & 12 & 16 & \\ s^4 & 2 & 12 & 16 & \\ s^3 & 0 & 0 & 0 & \\ s^3 & 1 & 3 & & \\ s^2 & 3 & 8 & & \\ s^1 & 1/3 & & & \\ s^0 & 8 & & & \end{array}$$

Auxiliary equation is

$A(s) \rightarrow s^4 + 6s^2 + 8 = 0$

$\frac{dA(s)}{ds} = 4s^3 + 125$

Solving Auxiliary equation we get

$s = \sqrt{2}j, -\sqrt{2}j, 2j, -2j$

Q.89 (d)

CE is given by

$(s^2 + 2s + 2)(s+2) + k = 0$

$s^3 + 4s^2 + 6s + 4 + k = 0$

$$\begin{array}{c|cc} s^3 & 1 & 6 \\ s^2 & 4 & 4+k \\ s^1 & \frac{20-k}{4} & \\ s^0 & 4+k & \end{array}$$

For the system to be marginally stable

$\frac{20-k}{4} = 0 \Rightarrow k = 20$

Auxiliary equation is given by

$4s^2 + 4 + k = 0$

$4s^2 + 24 = 0 \Rightarrow \omega = \sqrt{6}\text{rad/sec}$

Q.90 **(c)**
The stability of the given system depends on T alone because as T increase the system decay slowly and stability decreases.

Q.91 **(c)**
High gain result instability problem

Q.92 **(d)**
CE for the inner loop is given by

$$(s-a)(s+2a)(s+3a)+k=0$$

$$s^3+4as^2+\begin{pmatrix}-2a^2-3a^2\\+6a^2\end{pmatrix}s$$

$$-6a^3+k=0$$

$$s^3+4as^2+a^2s-6a^3+k=0$$

$$\begin{array}{c|cc} s^3 & 1 & a^2 \\ s^2 & 4a & -6a^3+k \\ s^1 & \dfrac{4a^3+6a^3-k}{4a} & \\ s^0 & -6a^3+k & \end{array}$$

For stability

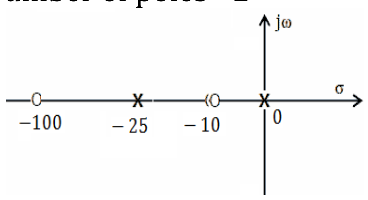
$6a^3<k<10a^3$ $\qquad x<k<y$

CE for overall system is

$$s^3+4as^2+a^2s-6a^3+2k=0$$

$$\frac{x}{2}<k<y/2$$

Q.93 **(a)**
Number of zeros =3
Number of poles =2

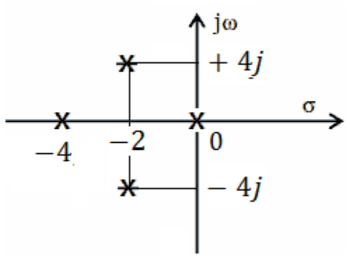


So number of loci terminating at ∞ is 0

Q.94 **(a)**

$$G(s)H(s)=\frac{k}{s(s+4)(s^2+4s+20)}$$

Poles $=0,-4,-2+4j,-2-4j$



Since there is real axis loci between 0 an -4
Hence, number of B.A.P is one

Q.95 **(a)**
Close loop system eq^n can be written as

$$G(s)H(s)=\frac{k(s+1)}{s(s+4)(s^2+2s+2)}$$

No of zero z=1
Number of poles p = 4
No of asymptotes $N=p-z=3$

Angle of asymptotic $=\dfrac{(2q+1)180°}{p-z}$

When $q=0,\theta=60°$

$q=1,\theta=180°$

$q=2,\theta=300°$

Q.96 **(b)**
The number of separate root loci is m= number of poles.
The number of zeros at infinity is $m-n$ and hence $m-n$ root loci will approach infinity.

Q.97 **(c)**

$$s^3+5s^2+(k+6)s+k=0$$

$$s^3+5s^2+6s(S+1)k=0$$

$$G(s)H(s)=\frac{(s+1)k}{s^3+5s^2+6s}$$

$$=\frac{k(s+1)}{s(s+3)(s+2)}$$

Asymptotes meet at centroid

Centroid, $\sigma=\dfrac{-3-2-(-1)}{2}$

$=-\dfrac{4}{2}=-2$ $(-2,0)$ is the centroid.

Q.98 (a)

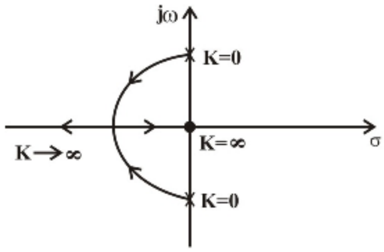
The loop gain is given by

$$G(s)H(s)=\frac{5ks}{s^2+10}$$

Poles $s=\pm j\sqrt{10}$

Zero, $s=0$

One zero is considered at ∞



Q.99 (c)

$$G(s)H(s)=\frac{k(s+2)}{s(s+1)(s+4)(s+4)}$$

No of asymptote N = p – 2

$N=4-1$

$N=3$

Centroid

$$=\frac{\sum \text{real part of poles}-\sum \text{real part of zero}}{p-z}$$

$$=\frac{(-1)+(-4)+(-4)-(-2)}{3}=\frac{-7}{3}$$

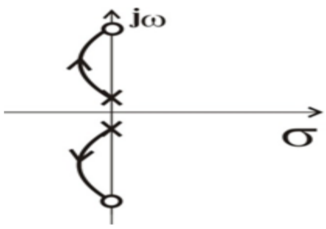
Q.100 (a)

$$G(s)=\frac{k(s^2+64)}{s(s^2+16)}$$

Zeros $s=\pm j8$

Poles, $s=0,\pm j4$

One zero is considered at infinity



Angle of departure = o°

Q.101 (c)

For the given transfer function there are 4 poles and 1 zero. So there will be three zeros at infinity.

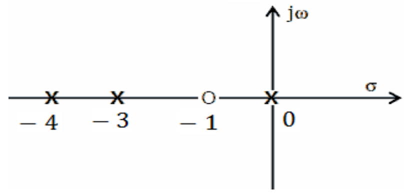
Q.102 (d)

As shown in fig, two poles are at origin and one pole is at –ve real axis so,

$$T.F=\frac{k}{s^2(sT_1+1)}$$

Q.103 (b)

$$G(s)H(s)=\frac{k(s+1)}{s(s+3)(s+4)}$$



Root locus of the system can lie on the real axis between s=0 and $s=-4$

Q.104 (c)

Put $s=z-1$ we get

$$z^5+10z^4+35z^3+50z^2+24\ z=0$$

Therefore 4 roots lie to the left of line $s+1=0$ and rot lies on $s+1=0$

z^5	1	35	24
z^4	0	50	
z^4	1	5	
z^3	30	24	
z^3	5	4	
z^2	$\frac{21}{5}$		
z^1	4		
z^0	0		

There are 4 roots lie to the left of line s+1=0 and 1 root lies on s+1=0.

Q.105 (c)

$$G(s)=\frac{k(s+3)}{s(s+1)(s+2)(s+5)}$$

real axis intercept

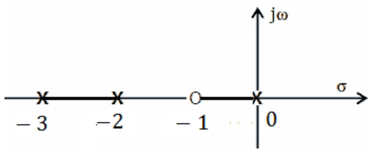
$$=\frac{\sum \text{real port of pole}-\sum \text{real port of zero}}{p-2}$$

$$\sigma=\frac{(-1-2-5)-(-3)}{3}$$

$$\sigma = \frac{-5}{3}$$

Q.106 (d)

$$G(s) = \frac{k(s+1)}{s(s+2)(s+3)}$$



There are 3 root loci branches

Q.107 (a)

The addition of open loop zero pulls the rot loci towards the left and therefore system becomes more stable.

Q.108 (c)

$$G(s)H(s) = \frac{k(s+1)}{s(s+4)(s^2+2s+2)}$$

Poles $s = 0, -4, -1 \pm j1$

Zeros, $s = -1$

Centroid, $\sigma = \frac{-4-1-1-(-1)}{3} = -\frac{5}{3}$

Q.109 (a)

$$G(s)H(s) = \frac{k(s+2)}{s^2+2s+2}$$

To find break point consider

$G(s)H(s) = -1$

$$\therefore k = \frac{s^2+2s+2}{s+2}$$

$$\frac{dk}{ds} = \frac{(s+2)(2s+2)-(s^2+2s+2)}{(s+2)^2} = 0$$

$$\Rightarrow 2(s^2+3s+2) - s^2 - 2s - 2 = 0$$

$$\Rightarrow s^2 - 4s + 2 = 0$$

$s = -0.58, -3.41$

$s = -3.41$ lies on root locus hence it is valid break point.

Q.110 (b)

Each pole produces a slope of -20dB/dec

Each zero produces a slope of 20dB/dec

$14p \rightarrow -280\text{dB/dec}$

$2z \rightarrow +40\text{dB/dec}$

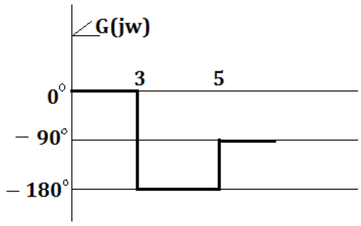
Total = -240dB/dec

∴ slope of highest frequency asymptotic is -240dB/dec

Q.111 (c)

$$G(s) = \frac{s+5}{s^2+4s+9}$$

Phase plot of the above system is shown below:



Hence angle varies between 0° to -180°

Q.112 (a)

Type of the system is determined by the number of poles at origin. Since the initial angle is 0° hence there are no poles at origin Hence Type = 0

Q.113 (b)

$$G(s) = \frac{1}{s}$$

$\omega_{gc} = 1\text{rad/sec}$

$$\therefore 180^\circ + \varnothing|_{\omega=\omega_{gc}} = 180 - 90 = 90$$

Q.114 (b)

$$G(s) = \frac{k}{s^2}$$

At $\omega = \omega_{gc}, |G(j\omega)| = 1$

$$|g(j\omega)| = \frac{k}{\omega_{gc}^2} = 1$$

$\omega_{gc} = \sqrt{k}\text{rad/sec}$

Q.115 (c)

The slope of asymptotic bode plot is the integer multiple of $\pm 20\text{dB/dec}$ or $\pm 6\text{dB/octave}$

Q.116 (c)

Type $2 \Rightarrow$ 2 poels at origin.
One pole gives $-20\text{dB}/\text{decade}$ hence Two poles give $-40\text{dB}/\text{decade}$ of Initial slope

Q.117 (c)

In general error at corner frequency is given by $\pm 3n$ dB

Q.118 (b)

1. Initial slope $-20\text{dB}/\text{decade}$. Hence it is Type 1 system
2. $k = (\omega_3)^{-1} = \dfrac{1}{\omega_3}$

   $e_{ss} = \dfrac{1}{k} = \omega_3$
3. $\omega_2 \neq \dfrac{\omega_1 + \omega_3}{2}$

Q.119 (d)

$PM = 180° + \Phi|_{\omega=\omega_{gc}}$

$\Phi|_{\omega=\omega_{gc}} = -90°$ (given)

$\therefore PM = 180° - 90° = 90°$

Q.120 (b)

$G(s) = s + 1$

$G(j\omega)` = j\omega + 1$

$M = |G(j\omega)| = \sqrt{\omega^2 + 1}$

$M|_{\omega=1} = \sqrt{2} = 1.41$

$\Phi = \angle G(j\omega) = \tan^{-1}(\omega)$

$\Phi|_{\omega=1} = \tan^{-1}(1) 45°$

Q.121 (c)

1. Resonant peak, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

   Peak overshoot,

   $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

   Both are function of ξ only
2. $\omega_r = \omega_n\sqrt{1-\xi^2}$

   For $\xi = \dfrac{1}{\sqrt{2}}$

   $\omega_r = 0$
3. As M_r increases, ξ decreases and M_p increase hence it is true.

Q.122 (d)

A minimum phase transfer function is one whose all the poles and zeros are on the left half of s-plane

Q.123 (d)

None of the transfer function are minimum phase. Because none of them have all their poles and zeros on the left half of s-plane.

Q.124 (a)

$\dfrac{1-s}{1+s}$ is non minimum phase system.

Q.125 (d)

Low frequency asymptote decide the number of poles at origin which is nothing but type of system
$-60\text{dB}/\text{dec} \Rightarrow 3$ poles are origin
Hence Type III system,

Q.126 (d)

For an all pass network the poles lie on the left half of s-plane and zero lies symmetrically on the right half of s-plane

Q.127 (a)

1. In general for n zeros the error at corner frequency $= +3n\text{dB}$
   For $n = 1$ error $= +3\text{ndB}$
2. Phase angle for complex conjugate poles doesn't depend on damping ratio.

Q.128 (d)

$G(s) = \dfrac{10}{0.66s^2 + 2.33s + 1}$

The corner frequencies are nothing but the magnitude of poles and zeros.
The poles are given by the roots of

$0.66s^2 + 2.33s + 1 = 0$

$0.66s^2 + 0.33s + 2s + 1 = 0$

$(2s+1) + 1(2s+1) = 0$

$(0.33s+1)(2s+1) = 0$

$s = -\frac{1}{0.33}, \frac{-1}{2}$

Hence corner frequencies are 3, 0.5

Q.129 (d)

Gain margin, GM

$= \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = -20\log\log(0.64) = 3.86dB$

Q.130 (b)

$G(j\omega) = \frac{j\omega}{1+j\omega}$

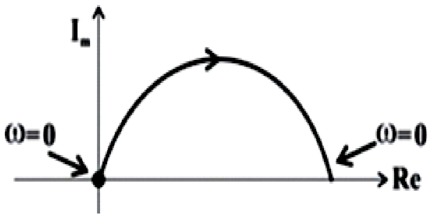
For $\omega = 0, G(j\omega)$

$= 0\angle 0°(M_1\angle\Phi_1)$

For $\omega = \infty, G(j\omega) = 1\angle 0°(M_2\angle\Phi_2)$

$\Phi_1 - \Phi_2 = 0$ hence ending direction is not considered

Since finite poles is neat to the imaginary axis hence starting direction is clockwise



Q.131 (d)

For a stable system both gain margin & phase margin must be positive.

Q.132 (d)

$GM = \frac{1}{a} = \frac{1}{0.4} = 2.5$

Q.133 (d)

When $-1 + j0$ is located in region I, the number of encirclements N = 0
Hence system is stable
When $-1 + j0$ is located in region II then, $N = -2$
Hence system is unstable.

Q.134 (d)

$G(s) = \frac{k(s+3)(s+5)}{(s-2)(s-4)}$

The number of right half poles is 2
For system to be stable the number of rotation around $-1 + j0$ should be 2 in anticlockwise direction.

At $\omega = \sqrt{11}$

$|G(j\omega)| = k\frac{4}{3} = k(1.33)$

For $k > \frac{1}{1.33}$ the number of encirclements around $-1 + j0$ would be 2 in anticlockwise direction hence system is stable.

$N = P = 2$

Q.135 (d)

$Type = 2, Order = 4$

Q.136 (b)

The number of half circle represents the type of system .Here number of half circles =1 Hence Type =1

Q.137 (a)

At $\omega = \omega_{pc}, \angle G(j\omega) = -180°$

$\angle G(j\omega) = -90° - \tan^{-1}(\omega T_1)$

$-\tan^{-1}(\omega T_2)$

$-180° = -90° - \tan^{-1}(\omega_{pc} T_1)$

$-\tan^{-1}(\omega_{pc} T_2)$

$\Rightarrow \omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$

Gain Margin $= \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = \frac{T_1 + T_2}{T_1 T_2}$

Q.138 (b)

At $\omega = 0, G(j\omega) = 1\angle 0^\circ$

At $\omega = \infty, G(j\omega) = 1\angle -180^\circ$

$\therefore G(s) = \frac{1}{(s+1)^2}$

Q.139 (c)

$|G(j\omega)| = \frac{1}{\omega^2 + 1}$

At $\omega = \omega_{gc}, |G(j\omega)| = 1$

$\Rightarrow \omega_{gc}^2 + 1 = 0$

$\omega_{gc} = 1 rad/sec$

$\varnothing = \angle G(j\omega)|_{\omega_{gc}=1rad/sec}$

$= -2\tan^{-1}(1) = -90^o$

Phase margin $= 180^o - 90^o = 90^o$

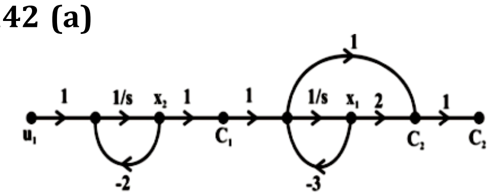
Q.140 (a)

$GM = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = \frac{1}{a}$

Q.141 (b)

Gain margin close to unity or phase margin close to zero is oscillatory.

Q.142 (a)



State Model is obtained by Diagram

$\dot{x}_1 = -3x_1 + x_2 C_2 = x_1$

$\dot{x}_2 = u_1 - 2x_2$

$\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1$

$C_2 = [1 \quad 0]x$

$Q_c = \begin{bmatrix} A & AB \\ 0 & 1 \\ 1 & -2 \end{bmatrix} |Q_c| = -1 \neq 0$

$Q_0 = \begin{matrix} C \\ CA \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} |Q_0| = 1 \neq 0$

$|Q_c| \neq 0, |Q_0| \neq 0, |A| \neq 0$

hence controllable and observable

Q.143 (b)

Characteristic eq^n $\Rightarrow SI - A = 0$

$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = 0$

$\begin{bmatrix} s & -2 \\ -2 & s \end{bmatrix} = 0$

$\Rightarrow s^2 - 4 = 0$

$s = \pm 2j$

So poles are $s = \pm 2j$ and $-2j$

Q.144 (d)

$T.F = C\frac{Adj[SI - A]}{|SI - A|}B + D$

$= \frac{[2 \quad 0]\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix}}$

$T.F = \frac{Y(s)}{U(s)} = \frac{2}{s^2 + 3s + 2}$

Q.145 (a)

Let $G(s) = \frac{Y(s)}{U(s)} = \frac{25 + 1s^\circ}{s^2 + 75 + 9s^\circ}$

Let $s^\circ \to x_1$

$s^1 \to \dot{x}_1 = x_2$

$s^2 \to \dot{x}_2$

$\therefore \quad Y(s) = 2x_2 + 1x_1$..........(1)

$U(s) = \dot{x}_2 + 7x_2 + 9x_1$

So, $\dot{x}_2 = U(s) - 7x_2 + 9x_1$............(2)

$\dot{x}_1 = x_2$............(3)

From eq^n (1) (2) & (3)

$x = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \; y = [1, \quad 2]x$

Q.146 (c)

We have $C = [b, \quad 0]$ and $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Observability

$Q_0 = \begin{bmatrix} C^T & C^T A^T \end{bmatrix}$

$Q_0 = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix}$

$|Q_0| = 2b^2$

System is observable when $|Q_0| \neq 0$ so given system observable for all non zero values of b

Q.147 (d)

$$T.F = \frac{C\left[Adj(SI-A)\right]B+D}{|SI-A|}$$

$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{(s+2)(s+1)} = \frac{1}{(s+1)}$$

Q.148 (b)

Controllability $Q_c = \begin{bmatrix} B, & AB \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

$|Q_c| = -1 \neq 0$

$|A| = 2 \neq 0$

Therefore the system is controllable

Since $e|Q_b| = e(A)$

System observability

$Q_b = \begin{bmatrix} C^T & C^T A^T \end{bmatrix}$

$Q_b = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$

$|Q_b| = 0 \text{but} |A| = 2 \neq 0$

System is not observable

Since $e|Q_b| \neq e(A)$

Q.149 (a)

State transition matrix is given by e^{At} It is also called zero input response It represents transient response or force for response.

Q.150 (a)

Characteristic

$eq^n \Rightarrow |SI - A| = 0$

$S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} = 0$

$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} = 0$

$s^2 + 5s + 3 = 0$

Q.151 (d)

Controllability $Q_c = \begin{bmatrix} B, & AB \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 2 \\ 1 & b \end{bmatrix}$

$|Q_c| = -2 \neq 0, |A| = b$

So system is uncontrollable for b = 0, since $P(Q_c) \neq e(A)$

Q.152 (a)

Characteristics $eq^n \Rightarrow |SI - A| = 0$

$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = 0$

$\therefore s^3 + 6s^2 + 11s + 6 = 0$

Then $s = -1, -2, -3$

Since, closed loop poles are nothing but eigen value.

Q.153 (a)

$c\left[ZI - A\right]^{-1} b + d$

Q.154 (a)

$X_1 = i_{L_1} x_2 = V_c i(t) = u(t)$

Applying KCL we get

$-i(t) + i_L + i_c = 0$

$i_c = u(t) - X_1$

$c\frac{dv_c}{dt} = u(t) - X_1$

$\dot{x}_2 = \frac{u}{c} - \frac{X_1}{c}$........(1)

Applying kVL we get

$V_L(t) = V_c(t)$

$L\frac{di_L}{dt} = X_2$

$\dot{x}_1 = \frac{X_2}{L}$ ....(2)

$Y = V_0(t) = V_c(t) = X_2$ ....(3)

From (1), (2) (3) we get

$$\dot{x} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{c} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/c \end{bmatrix} u$$

$Y = [0 \quad 1] x$

Q.155 (d)

Given $\dot{x}_1 = X_2$

$\dot{x}_2 = X_1 + X_2 + u$

$Y = X_1 + 3X_2$

$$\frac{Y(s)}{U(s)} = \frac{3s+1}{s^2 - s - 1}$$

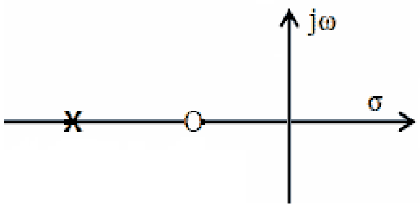
Q.156 (d)

The maximum phase provided is

$$\Phi_{max} = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

Q.157 (a)

For phased lead Compensator pole zero diagram is



$\frac{S+1}{S+2}$ satisfies the above condition

Zero, i.e. $S = -1$ is neat to imaginary axis compate to pole i.e. $S = -2$

Q.158 (c)

Phase lead compensator affects the transient performance of the system.

Q.159 (c)

For phase lag network pole must be near to the imaginary axis

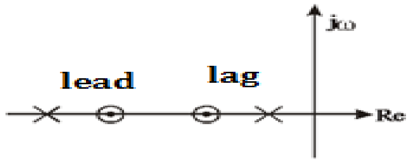
For $\frac{1}{1+RCS}$

Poles, $S = -\frac{1}{RC}$

Zero, $s = \infty$

Q.160 (d)

The pole zero plot of lag lead compensator is



Q.161 (a)

Both ω_{gc} and BW are increased by phase lead compensator.

Q.162 (b)

The transfer function of PI Controller is $k_p + \frac{k_i}{s}$

Q.163 (d)

$$\Phi_{max} = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

$$= \sin^{-1}\left(\frac{3-1}{3+1}\right) = 30°$$

Q.164 (b)

Without controller

$G(s) = \frac{9}{s(s+2)}$ Type -1

For ramp input type -1 system gives constant steady state error

If integral controller is employed the

$G(s) = \frac{9K_i}{s^2(s+2)}$ Type -2

$e_{ss} = 0$